Symbol detection using the differential evolution algorithm in MIMO-OFDM systems

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Abstract: Channel estimation and symbol detection in multiple-input and multiple-output (MIMO)-orthogonal frequency division multiplexing (OFDM) systems are essential tasks. Although the maximum likelihood (ML) detector reveals excellent performance for symbol detection, the computational complexity of this algorithm is extremely high in systems with more transmitter antennas and high-order constellation size. In this paper, we propose the differential evolution (DE) algorithm in order to reduce the search space of the ML detector and the computational complexity of symbol detection in MIMO-OFDM systems. The DE algorithm is also compared to some heuristic approaches, such as the genetic algorithm and particle swarm optimization. According to the simulation results, the DE has the advantage of significantly less complexity and is closer to the optimal solution.

Key words: Differential evolution, particle swarm optimization, genetic algorithm, maximum likelihood algorithm, MIMO-OFDM, symbol detection

1. Introduction

In order to provide higher data rate transmission in modern communication systems, orthogonal frequency-division multiplexing (OFDM) has recently attracted much attention owing to its potential to increase spectral efficiency [1]. Moreover, a significant capacity increase has been provided for OFDM systems by combining them with multiple-input, multiple-output (MIMO) technology in many communication systems, such as WLAN, HIPERMAN, and 4G wireless cellular systems [2].

However, for these systems, channel estimation and symbol detection are required at the receiver for coherent demodulation [3,4]. For this reason, various algorithms, such as the maximum likelihood (ML) and zero forcing (ZF) algorithms, have been proposed to detect symbols. Although the implementation of the ZF algorithm is quite easy and it is a less complex algorithm, it underperforms in fast-fading and time-varying environments [5,6]. In comparison with the ZF algorithm, the ML algorithm reveals excellent performance in these environments, but the main drawback of the ML algorithm is its extremely high computational complexity. It searches the candidate symbol vector on each subcarrier and the Euclidean distance between the received and actual symbols is computed for all of the possible combinations of the transmitted symbols. In addition to this, the search space grows exponentially with the number of transmitter and receiver antennas, and so its

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computational complexity becomes intensive [7].

Some studies have been found in the literature with the aim of reducing the complexity and obtaining an optimal solution from the ML algorithm, which detects symbols. In [8], the use of orthogonal matrix triangularization (QR decomposition) with sort and Dijkstra's algorithm was proposed for decreasing the computational complexity of the sphere decoder that is used for the ML detection of signals on multifading channels. In order to calculate the Euclidean distance of the candidate symbol, the multistage likelihood was presented in [9]. In [10], the sphere detector was proposed to have a polynomial computational reduction, but when the search space is large, it takes much more computational time.

Furthermore, heuristic approaches such as the genetic algorithm (GA) and particle swarm optimization (PSO) are implemented with the ML principle for channel estimation and symbol detection, for their ability of reducing the search space of the ML algorithm and their advantages of computational complexity reduction. In [11] the GA and in [12] PSO were used for channel estimation and data detection based on the ML algorithm in pulse amplitude modulation-based communication systems. Moreover, in [13], a memetic differential evolution (DE) algorithm was proposed for minimum bit error rate detection in multiuser MIMO systems.

In this paper, we propose the DE algorithm in order to reduce the search space of the ML detector and to reduce the computational complexity of the symbol detection in MIMO-OFDM systems. The DE algorithm is also compared to some heuristic approaches, such as the GA and PSO. The paper is structured as follows: the MIMO-OFDM system model and the ML symbol detection algorithm are presented in Section 2. In Section 3, the DE algorithm used for symbol detection is described. The comparative simulation results are given in Section 4, and the paper is concluded in Section 5.

2. MIMO-OFDM system model

Figure 1 shows the simplified block diagram of the MIMO-OFDM system. For this system, we consider the N_{tx} transmit, N_{rx} receive antennas, n OFDM symbols, and K subcarriers.
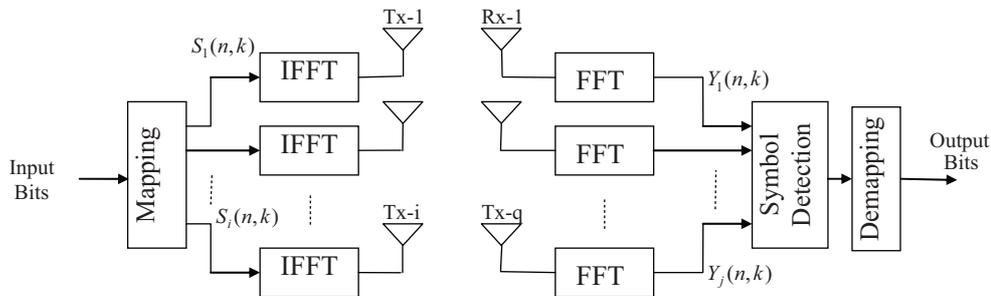

Figure 1. Simplified block diagram of the MIMO-OFDM system.

A vector of the information data is mapped onto complex symbols considering the modulation type. The transmitted symbol vector is expressed as:

$$S[n,k] = [S_1(n,k), ..., S_{N_{tx}}(n,k)]^T \quad k = 0, ..., K-1, \tag{1}$$

where $S_i[n,k]$ is the symbol that is transmitted at the n_{th} symbol, k_{th} subcarrier, and i_{th} antenna, and $[.]^T$ is the transpose operation. By applying inverse fast Fourier transform (IFFT), symbol vectors are turned into

the OFDM symbol:

$$s_n[m] = \frac{1}{\sqrt{KN_{tx}}} \sum_{k=0}^{K-1} S[n,k] e^{j2\pi m/k}, \quad m = 0, ..., K-1. \tag{2}$$

Next, the cyclic prefix (CP) is added to avoid intersymbol interference (ISI) and the signal vectors are fed through the i_{th} transmitter antenna. After removing the CP from the received signal vector at the q_{th} receiver antenna, the fast Fourier transform (FFT) is taken as:

$$Y[n,k] = \frac{1}{\sqrt{K}} \sum_{m=0}^{K-1} y[m] e^{-j2\pi km/K} \quad n = 0, ..., K-1. \tag{3}$$

Next, the received signal vector can be expressed as:

$$Y_q[n,k] = \sum_{i=1}^{N_{tx}} H_i[n,k] S_i[n,k] + W_q[n,k], \tag{4}$$

where $H_i[n,k]$ is the channel impulse response vector and $W_q[n,k]$ is the additive white Gaussian noise [14].

2.1. ML symbol detection in the MIMO-OFDM

The estimations of the data symbols are obtained by maximizing the following metric:

$$S_* \triangleq \arg\max P(Y|S). \tag{5}$$

Next, the ML algorithm detects the symbols by minimizing the squared Euclidian distance to target vector Y over the N_{tx} dimensional discrete search set:

$$S_* = \arg\min \|Y - HS\|^2. \tag{6}$$

For the optimal solution of the ML detection, all possible $M^{N_{tx}}$ combinations of the transmitted symbols must be searched. For this reason, the computational complexity increases with the transmitter antenna [15]. Therefore, we propose heuristic approaches in order to reduce the computational complexity of the symbol detection in the MIMO-OFDM system.

3. DE algorithm for symbol detection

DE is a simple and powerful population-based evolutionary algorithm for global optimization problems. It uses crossover, mutation, and selection operators like the GA, but the GA relies on crossover while DE relies on mutation; hence, DE gives better solutions than the GA in many applications [16,17]. A flow diagram of the symbol detection based on the DE algorithm is shown in Figure 2.

As can be seen from Figure 2, the populations of all of the individuals that represent the solution of the symbols are initialized randomly in the search space and evaluated using the fitness function of each symbol, which can be seen in Eq. (6) for our problem. Next, the algorithm finds the optimum solution by utilizing the differential information of the individuals among the population. The population is improved using the mutation, crossover, and selection operators until the termination criterion, which is determined as 10^{-2}, is carried out.
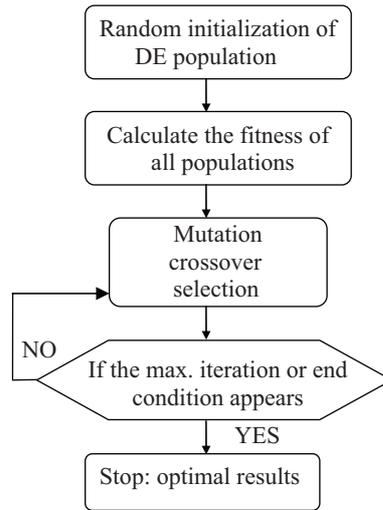
Figure 2. DE flow diagram.

For each target vector $x_i(g) \in (i = 1, 2, ...NP)$ in the D-dimensional search space, a mutant vector in generation g is generated as:

$$v_i(g+1) = x_{r1}(g) + F\left(x_{r2}(g) - x_{r3}(g)\right), \tag{7}$$

where r_1, r_2, and $r_3 \in (1, 2, ...NP)$ are random integers, NP is the population size, and F is the scaling factor that controls the difference of x_{r1} and x_{r2}. After mutation, the crossover operation increases the diversity of the population as:

$$U_i(g+1) = \begin{cases} v_i(g+1) & if \quad rand_j(0,1) \leq C_r \vee j = k \\ 0 & else \end{cases}, \tag{8}$$

where C_r is the constant crossover parameter, $rand_j(0,1)$ is the j_{th} evolution of a random number, and $k \in \{1, 2, ..., D\}$ is the index of random parameters. Next, the selection chooses the vector between the target and the trial vector to create an individual for the next generation [16,17]. When the termination criterion, which is determined as 10^{-2}, is carried out, the individual that best represents the symbols is chosen for the optimal solution in the symbol detection. In order to adapt the OFDM signals to the DE algorithm, each complex of signals is separated into real and imaginary parts, since the OFDM symbols consist of complex signals. The pseudo-code of the DE algorithm for symbol detection is shown in Table 1.

4. Simulation results

We considered the MIMO-OFDM systems with 2×4, 4×4, 8×8, and 16×16 transmitter and receiver antennas in order to evaluate the performances of the heuristic symbol detectors based on the GA, PSO, and DE, with the classical symbol detectors based on the ML and ZF algorithms. The simulation parameters of the MIMO-OFDM system and heuristic approaches are given in Tables 2 and 3, respectively.

The settings of the control parameters for DE were investigated in [17]. Hence, we selected the parameters for DE according to the recommended values of [17].

In the simulations, the same system and heuristic parameters were used to get figures for various transmitter and receiver antennas. However, the iteration numbers needed for convergence to the optimal solution in the heuristic approaches were changed in the simulations to evaluate the performance of the systems. The iteration numbers of the algorithms were averaged after 50 runs of simulations.

Table 1. Pseudo-code of the DE symbol detection.

```
% Initialize populations of all individuals that represent the solution of symbol strings randomly
    while (the termination criteria which is 10⁻²)
    for (i = 0; i < Np; i + +)
    randomly pick r₁, r₂, r₃ ∈ [1, 2, ..., Np]i ≠ r₁ ≠ r₂ ≠ r₃
    randomly pick jrand ∈ [1, 2, .., n]
% mutation operation
        hᵍxᵍᵣ₁ + F(xᵍᵣ₂ − xᵍᵣ₃)
% crossover operation
    randomly set template
    for (j = 0; j < n; j = ++)
        if rand1ᵢⱼ ≤ CR or j = jrand)
            vᵍᵢⱼ = hᵍᵢⱼ
        else vᵍᵢⱼ = xᵍᵢⱼ
    endfor
% selection operation
        if (f(vᵍᵢ < f(xᵍᵢ))
            xᵍ⁺¹ᵢ = vᵍᵢ
    endfor
    endwhile
```

Table 2. MIMO-OFDM simulation parameters.

Parameter	Value
Number of subcarriers	128
Cyclic prefix size	FFT/4 = 32
Modulation type	8PSK
Channel type	Rayleigh fading

Table 3. Control parameters of the heuristic approaches.

DE	PSO	GA
Population size = 25	Swarm size = 30	Population size = 60
Crossover rate = 0.8	Max. velocity = 20	Crossover rate = 0.8
Scaling factor = 0.8	Inertia factor = 0.9 (start); 0.4 (end)	Mutation rate = 0.2
Combination factor = 0.8	Learning factor = 2	

Figure 3 shows the convergence curves of the heuristic algorithms for the 2×4 MIMO-OFDM system at a signal-to-noise ratio (SNR) value of 15 dB. In order to converge to the optimal solution, the required averaged iteration numbers for the DE are less than the others. While the DE requires 19 iterations, PSO and the GA require 24 and 32 iterations, respectively.

In Figure 4, the bit error rate (BER) versus the SNR performance of the symbol detectors in the 2×4 MIMO-OFDM systems is shown. As can be seen from Figure 4, the DE has better performance than the GA, PSO, and ZF algorithms, and the BER performance of the DE is close to that of the ML detectors. For instance, the BER difference between the DE and ZF is more than 10^{-1} at a 15 dB SNR value. Moreover, the DE has about a 2.5 dB and a 1 dB SNR gain at a 10^{-2} BER when compared with the GA and PSO algorithms, respectively.
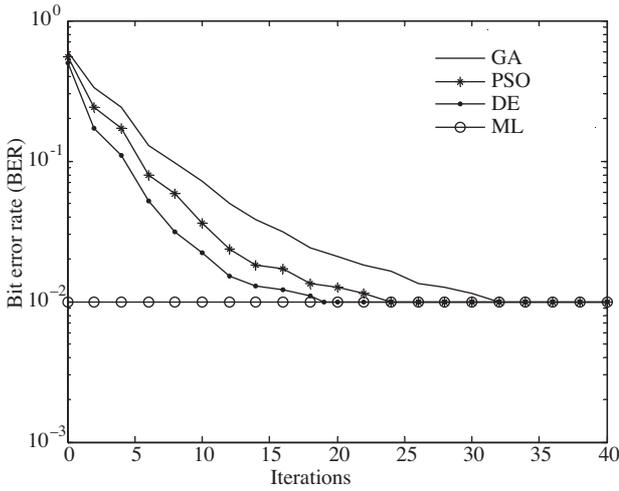
Figure 3. Convergence curves with the averaged iteration of the heuristic algorithms.
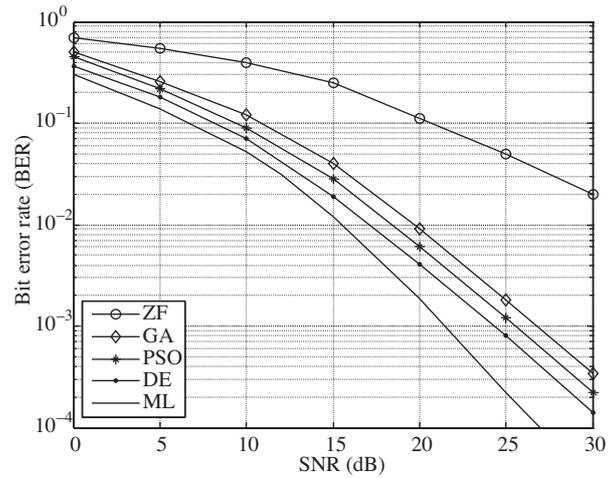

Figure 4. The BER versus the SNR of the detectors for a 2 × 4 MIMO-OFDM system.

In Figures 5 and 6, the BER versus the SNR performance of the symbol detectors in the 4 × 4 and 8 × 8 MIMO-OFDM systems is depicted, respectively, to show the effect of the increasing number of receiver and transmitter antennas on the detection performance. For the 4 × 4 system, 24 iterations in the DE, 30 iterations in PSO, and 41 iterations in the GA are required to converge to optimal solutions. On the other hand, for the 8 × 8 system, 34 iterations in the DE, 42 iterations in PSO, and 64 iterations in the GA are required. When Figure 5 is considered, the DE and PSO algorithms further require about 3 dB and 4 dB SNR values at a 10^{-3} BER, respectively, compared to the optimal ML detector. Despite the fact that the ML algorithm has better performance than the others, the computational complexity of this algorithm is considerably high. According to Figures 5 and 6, it can be seen that an increasing of the number of transmitter and receiver antennas would decrease the BER values of the MIMO-OFDM system. At a 30 dB SNR value, a DE detector with 8 × 8 antennas has about a 10^{-1} BER advantage compared to a DE detector with 4 × 4 antennas.
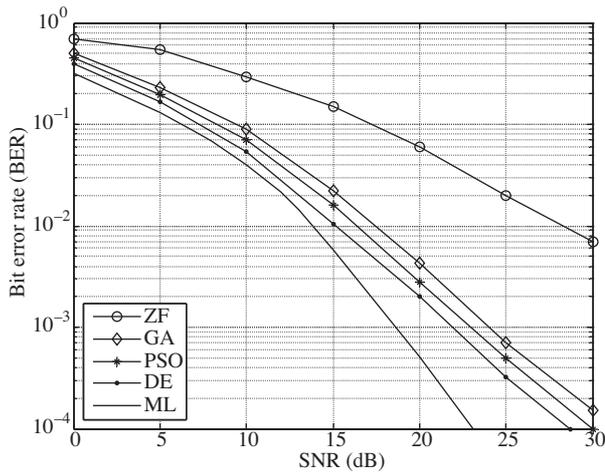

Figure 5. The BER versus the SNR of the detectors for a 4 × 4 MIMO-OFDM system.
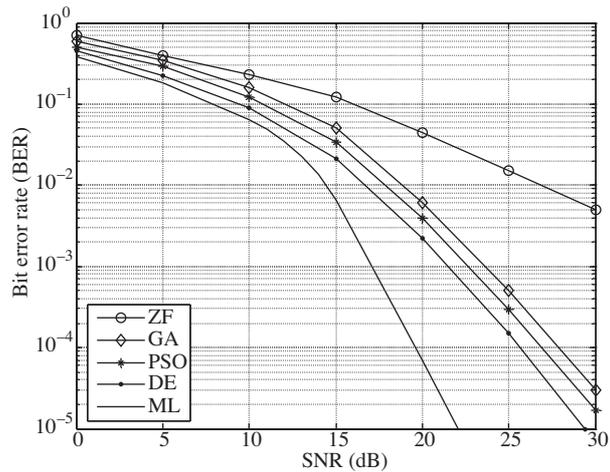

Figure 6. The BER versus the SNR of the detectors for a 8 × 8 MIMO-OFDM system.

The BER versus the SNR performance for 16×16 MIMO-OFDM systems can be seen in Figure 7. According to Figure 7, the DE-based symbol detector has better performance than the other evolutionary-based detectors. For instance, the SNR difference between the DE and PSO is 1.5 dB and the SNR difference between the DE and GA is 2.5 dB at a 10^{-3} BER value.
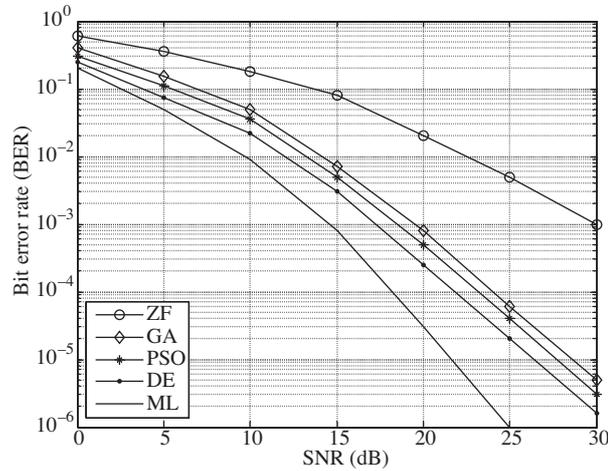

Figure 7. The BER versus the SNR of the detectors for a 16×16 MIMO-OFDM system.

Moreover, in order to reveal the computational advantages of the heuristic approaches, especially the DE algorithm over the ML algorithm, we investigated the computational complexity of the symbol detectors in terms of the N_{rx} (number of receiver antenna), N_{tx} (number of transmitter antenna), N_p (population size), N_{itr} (number of iterations), and M (constellation size). For the ZF detector, there are $4N_{tx}^3 + 2N_{tx}^2 N_{rx}$ operations [18]; in the ML detector, $N_{rx}(N_{tx}+1)M^{N_{tx}}$ operations are required [18]; and in the GA, PSO, and DE algorithms, $N_p(N_{tx}N_{rx} + \mu)N_{itr}$ operations are necessary, where μ is the number of population-updating parameters [18]. The number of μ, which depends on the algorithm, is almost the same for the heuristic approaches. However, the computational complexity does not directly depend on μ, but does directly depend on the number of iterations that provide convergence in the algorithms. Among the simulated algorithms, the number of operations in the DE is less than in the others depending on the number of iteration shortages that were in the convergence. As can be seen from the above analysis, the computational complexity of the ML algorithm is quite high in the case of the transmitter and receiver antennas and the constellation size increase. For this reason, the ML algorithm is not a practical solution for symbol detection in MIMO-OFDM systems that have large antenna and constellation sizes. However, the proposed detector has significantly less computational complexity than the other algorithms.

5. Conclusion

In this paper, we proposed the DE algorithm in order to reduce the search space of the ML detector and to reduce the computational complexity of symbol detection in MIMO-OFDM systems. The DE algorithm was also compared to some heuristic approaches, such as the GA and PSO. According to the simulation results, in spite of the fact that the ML algorithm had the best performance, the computational complexity of the ML algorithm was extremely high when the system had higher-order modulation constellations and large antenna sizes. Among the heuristic approaches, the DE had the advantage over PSO and the GA in terms of not only

the convergence speed but also the BER. It was concluded that the DE algorithm is a satisfactory solution for optimal symbol detection in MIMO-OFDM systems.

Acknowledgments

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