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NORMAL BOUNDARY VALUE PROBLEMS FOR DIFFERENTIAL EQUATIONS OF HIGHER ORDER

F. G. Maksudov, Z. I. Ismailov

Abstract

In this work the arbitrary order differential operator expression of the form

$$\iota(u) = \frac{\partial u(t)}{\partial t^n} + Au(t),$$

where A is a bounded normal operator in the Hilbert Space H is considered in the Hilbert Space of vector functions $L_2(H(0, 1))$.

This paper describes all normal boundary value problem for the indicated differential expression in terms of abstract boundary conditions and determines a connection with other typea of boundary value problems.

As is known, the normality of a bounded operator is equivalent to the commutability of two selfadjoint operators i.e, its real and imagenary parts. However, presentation and verification of the commutability of two unbounded operators are accompanied by a series of difficulties. Therefore, the conception of normality is introduced in a different way.

Definition 1 *A linear densely definite formally operator, C , in Hilbert space e with a domain, $D(C)$, is called formally normal if $D(C) \supset D(C^*)$ and $\| Cf \|_E = \| C^* f \|_E$ for any $f \in D(C)$ and maximal formal if it has no such extensions. A formally normal operator is seid to be normal if $D(C) = D(C^*)$.*

The theory of unbounded normal operators depending on the raised problems has been investigated by many suthors (see, a.g. [1,2,5,6,7,10,13,15]). However, the results obtained in these works are less afficient for differential operators varifying, otherwise, that theory was not accommodated to the differential operators theory in Hilbert Space. In this direction only some results of particular cases were published. [3,9,11,14].

Let us give some definitions.

Definition *An arbitrary linear subset $\theta \supset E \otimes E$ is called a linear relation in the Hilbert space E . If θ_1 and θ_2 are linear relations and $\theta_1 \supset \theta_2$ then θ_2 is called an artension of θ_1*

Definition A linear relation θ is called dissipative (accumulative, symmetric) if for any $\{x, x'\} \in \theta$

$$Jm(x', x)_E \geq: (Jm(x', x)_E \leq 0, Jm(x, x)_E = 0).$$

A dissipative (accumulative, symmetric) relation is called maximal dissipative (accumulative, symmetric) if it has no nontrivial dissipative (accumulative, symmetric) extensions, A symmetric relation is called Hermitian if it is maximal dissipative and maximal accumulative simultaneously.

Definition A linear densely definite operator, C , with domain, $O(C)$, in a Hilbert space is called dissipative if

$$Jm(Cf, f)_E \geq 0 \text{ for all } f \in D(C);$$

accumulative if

$$Jm(Cf, f)_E \leq 0 \text{ for all } f \in D(C); \text{ and}$$

accretive if

$$Re(Cf, f)_E \geq 0 \text{ for all } f \in D(C).$$

A dissipative (accumulative, accretive) operator is called maximal dissipative (accumulative, accretive) if it does not have any nontrivial (i.e. different from C itself) dissipative (accumulative, accretive) extensions.

Considered in the present paper is the n^{th} order differential operator expression of the form

$$i(u) = \frac{d^n u(t)}{dt^n} + Au(t), \quad 0 \leq t \leq 1, \tag{1}$$

where A is a bounded normal operator in Hilbert space, H .

Consider the expression formally adjoint to (1):

$$i^+(u) \equiv (-1)^n, \frac{d^n u(t)}{dt^n} + A^*u(t). \tag{2}$$

Definition on the dense in $L_2(H, (0, 1))$ set D_o having elements of the form

$$\sum_{k=1}^m \varphi_k(t) f_k, \quad \varphi_k(t) \in W_2^n(0, 1), \quad f_k \in H$$

the operator $L'_0 : L'_0 u = l(u)$.

It is easy to verify that if $n = 2k$, $Jm A \geq 0$ ($Jm A \leq 0$), then the operator L'_0 is dissipative (accumulative) and, in the case when $n = 2K - 1$, $Re A \geq 0$, the operator

L'_0 is accrative, with $L_0 = (L'_0)$. In a similar way is the niminal operator $L_0^+ = (L'_0)^+$, according to expression (2).

The operator $L(L^+)$, adoint to the operator $L_0^+ = (L_0)$ in the space $L_2(H, (0, 1))$, is called a maximal operator generated by the differential expression (1)((2)).

It is established that the minimaloperator, L_0 , in both cases is formally normal but is not maximal.

This paper describes all normal boundary value problems for the differential expression (1) in terms of abstract boundary conditions and indicates connection with other types of boundary value problems.

Definition [S,p.158]. Let C be a closed symmetric operator in the Hilbert space E with finite or infinite indices defect. The triplet $(\mathfrak{A}, \gamma_1, \gamma_2)$, where \mathfrak{A} is a Hilbert spaces and γ_1 and γ_2 are linear mappings from $D(C^*)$ to \mathfrak{A} is called a space of boundary values of operator C if

1) for any $f, g \in D(C)$,

$$(C^*f, g)_E - (f, C^*g)_E = (\gamma_1f, \gamma_2g)_{\mathfrak{A}} - (\gamma_2f, \gamma_1g)_{\mathfrak{A}};$$

2) for any $F_1, F_2 \in \mathfrak{A}$ there exists vector $f \in D(C^*)$ such that

$$\gamma_1f = F_1, \quad \gamma_2f = F_2.$$

Note that for arbitrary n if $u(t) \in D(L)$, then $u(t) \in W_2^n(H, (0, 1))$. So from the embedding theorem (see [4], Introduction, item 6) it follows that, at thee of the interval $(0, 1)$, the functions $u(t), u'(t), \dots, u^{(n-1)}(t)$ have boundary values in space H .

I in this section we assume that $n = 2k$, $k \in N$, and without loss of generality the operator JmA in H to be positive definite, i.e.

$$i(u) \equiv \frac{\partial^{2k}u(t)}{\partial t^{2k}} + Au(t), \quad 0 \leq t \leq 1. \quad (3)$$

it is easy to see that

$$i(u) \equiv \frac{\partial^{2k}u(t)}{\partial t^{2k}} + ReAu + l JmA u.$$

Theorem 1.1. Whatever the unitary operators W_1 and W_2 in \mathfrak{A} may be, the restriction of the maximal operator L on the set of vector-functions, $u(t) \in D(L)$, satisfying the conditions

$$(W_1 - E)\gamma_1u + l(W_1 + E)\gamma_2u = 0. \quad (4)$$

$$(W_2 - E)\gamma_1((JmA)^{1/2}u) + l(W_2 + E)\gamma_2((JmA)^{1/2}u) = 0 \quad (5)$$

where E is a unity operator in H , presents a normal extension of the minimay operator L_0 . Conversely, any normal extension of operator L_0 is the restriction of the operator L on the set of vector-functions $u(t) \in D(L)$ satisfying uniguely conditions (1.2) and (1.3) and the unitary operators W_1 and W_2 are defined by the extension.

The proof is baserd in general on following three theorems.

Theorem 1.2. [2] a) let \tilde{L} be a normal extension of the minimal operator L_0 , then the closure of operators $Re\tilde{L} = (\tilde{L} + \tilde{L}^*)/2$ and $Jm\tilde{L} = (\tilde{L} - \tilde{L}^*)/2i$ are selfadjoint extensions of minimal operators \bar{A}_0 and \bar{B}_0 , respectively, and on $D(\tilde{L}) = D(\tilde{L}^*) = D(Re\tilde{L}) \cap D(Jm\tilde{L})$ we have

$$(Re\tilde{L}u, Jm\tilde{L}u)_{L^2_{(H,(0,1))}} = (Jm\tilde{L}u, Re\tilde{L}u)_{L^2_{(H,(0,1))}}$$

where $A_0 = (L_0 + L_0^*)/2$, $B_0 = (L_0 - L_0^*)/(2l)$

b) On the other hand, let \bar{A} and \bar{B} be selfadjoint operators such that $\bar{A}_0 \supset \bar{A}$ and $\bar{B}_0 \supset \bar{B}$ and the equality $(Au, Bu) = (Bu, Au)$ holds on $D(\bar{A}) \cap D(\bar{B})$ then the operator $\tilde{L} = A + lB$ with $D(\tilde{L}) = D(\bar{A}) \cap D(\bar{B})$, where A is the restriction of \bar{A} on $D(\tilde{L})$, and B is the restriction of \bar{B} on $D(\tilde{L})$, is the normal extension of the minimal operator L_0 .

Theorem 1.3. [12]. For any symmetric operator with the indices of defect (n, n) ($n \leq \infty$) the space of boundary values

$$(\varkappa, \gamma_1, \gamma_2) \text{ with } \dim \varkappa = n$$

Theorem 1.4. [8]. Whatever unitary operator W in H may be the linear relation $\{x, x'\}$ difined by

$$(W - E)x' + f(W + E)x = 0$$

is a Hermitian relation. On the other hand, any Hermitian relation $\{x, x'\}$ may be presented by the last equation, where W is uniguely determined.

Proof of the Theorem 1.1 Let \tilde{L} be a normal extension of the minimal operator, L_0 . It is a restrection of the maximal operator, L [6]. The operators $Re\tilde{L}$ and $Im\tilde{L}$ are selfadjoint extensions of the minimal operators ReL_0 and ImL_0 (see Theorem 1.2).

Consequently, $Re\tilde{L}$ is descrtibed by the following boundary condition

$$(W_1 - E)\gamma_1 u + l(W_1 + E)\gamma_2 u = 0,$$

where the space of boundary values of minimal operator generated by the expression $\partial^{2k}/\partial t^{2k}$ in $L_2(H, (0, 1))$ is denoted by $(\varkappa, \gamma_1, \gamma_2)$. The existence of $(\varkappa, \gamma_1, \gamma_2)$ follows from the Theorem 1.3.

Note that every time on $D(\text{Re}\tilde{L})$ the operator $\text{Im}\tilde{L}$ will be a selfadjoint extension of the operator $\text{Im}Au(t)$ originally defined on $W_2^{2k}(H(0,1))$.

Since \tilde{L} is normal, then on

$$D(\tilde{L}) = D(\tilde{L}^*) = d(\text{Re}\tilde{L}) \cap D(\text{Im}\tilde{L})$$

the following relation (see Theorem 1.2) holds for any $u(t) \in D(\tilde{L})$:

$$\begin{aligned} & (\text{Re}\tilde{L}u, \text{Im}\tilde{L}u)_{L_2(H,(0,1))} - (\text{Im}\tilde{L}u, \text{Re}\tilde{L}u)_{L_2(H,(0,1))} = \\ & \left(\frac{\partial^{2k}u}{\partial t^{2k}} + \text{Re}Au, \text{Im}Au \right)_{L_2(H,(0,1))} - \left(\text{Im}Au, \frac{\partial^{2k}u}{\partial t^{2k}} + \text{Re}Au \right)_{L_2(H,(0,1))} \\ & = \left(\frac{\partial^{2k}u}{\partial t^{2k}} ((\text{Im}A)^{1/2}u), (\text{Im}A)^{1/2}u \right)_{L_2(h,(0,1))} \\ & - \left((\text{Im}A)^{1/2}u, \frac{\partial^{2k}u}{\partial t^{2k}} ((\text{Im}A)^{1/2}u) \right)_{L_2(H,(0,1))} = (\gamma_1(\text{Im}A)^{1/2}u, \gamma_2(\text{Im}A)^{1/2}u)_{\mathfrak{X}} \\ & - (\gamma_2(\text{Im}A)^{1/2}u, \gamma_1(\text{Im}A)^{1/2}u)_{\mathfrak{X}} = 0 \end{aligned} \quad (6)$$

Hence, it follows that the linear relation

$$\vartheta\{\{\gamma_2((\text{Im}A)^{1/2}u), \gamma_1((\text{Im}A)^{1/2}u)\}, u(t) \in D(\tilde{L})\}$$

is symmetric. Since the operator $(\text{Im}A)^{1/2}u$ is boundedly invertible, then it can be easily proved that the linear relation ϑ is Hermitian. So, there exists unitary operator W_2 in \mathfrak{X} such that the linear relation, ϑ , is described by the following equation (see Theorem 1.4):

$$(W_2 - E)\gamma_1((\text{Im}A)^{1/2}u) + l(W_2 + E)\gamma_2((\text{Im}A)^{1/2}u) = 0, u(t) \in D(\tilde{L})$$

Conversely let \tilde{L} be a restriction of the maximal operator L on the set of vector-functions, $u(t)$ satisfying conditions (1.2) and (1.3). It is an extension of the minimal operator, L_0 . Then the linear relation,

$$\tau = \{\{\gamma_2u, \gamma_1u\}, u \in D(\tilde{L})\},$$

is Hermitian. By virtue of property (1) for $(\mathfrak{X}, \gamma_1, \gamma_2)$, the selfadjointness of the operator $\text{Im}\tilde{L}$ on $D(\tilde{L})$ is obvious. The boundary condition (1.3) means that the linear relations

$$\vartheta = \{\{\gamma_1((\text{Im}A)^{1/2}u), \gamma_2((\text{Im}A)^{1/2}u)\}, u(t) \in D(\tilde{L})\}$$

are Hermitian. So, from relation (1.4), it follows that

$$(\text{Re}\tilde{L}u, \text{Im}\tilde{L}u)_{L_2(H,(0,1))} = (\text{Im}\tilde{L}u, \text{Re}\tilde{L}u)_{L_2(H,(0,1))}, u \in D(\tilde{L}).$$

Remark 1.1. If in the differential expression (1.1) $ImA = 0$ then the minimal operator L_0 is symmetric and all its normal extensions coincide with the selfadjoint ones (there is more general theorem in [5]), i.e. such normal extensions are described by the differential expression (1.1) and the boundary condition (1.2). So, theorem 1.1 is true also in the case when $ImA = 0$. Moreover, this theorem generalizes the analogous theorem on selfadjoint extensions for the considered differential expression (1) (see [8], theorem 1.6, p.159).

Remark 1.2. In (1.1) if ImA is negative definite then one can prove that all normal extensions of the minimal operator L_0 are described by the differential expression (1.1), boundary condition (1.2) and

$$(W_2 - E)\gamma_1((-ImA)^{1/2}u) + l(W_2 + E)\gamma_2((-ImA)^{1/2}u) = 0.$$

Remark 1.3. Generally, if in (1.1) A is any bounded normal operator, then all kind of normal extension of the minimal operator are described by expression (1.1), boundary condition (1.1) and the following modified boundary condition:

$$(W_2 - E_{\infty})\gamma_1((ImA - (m + 1)E_H)^{1/2}u) + l(W_2 + E_{\infty})\gamma_2((ImA - (m + 1)E_H)^{1/2}u) = 0$$

where E_H is the unit operator in H and $m = \inf_{f \in H} (ImAf, f)$.

Corollary 1.1. *If the extension, $\tilde{L}, L_0 \supset \tilde{L} \supset L$, is normal and $ImA \geq 0$ ($ImA \leq 0$), then it is maximal dissipative (maximal accumulative).*

Dissipativity (accumulativity) of the normal extension follows from the following relation

$$Im(\tilde{L}y, y)_{L_2(H, (0,1))} = (ImAy, y)_{L_2(H, (0,1))} \geq 0 (\leq 0) y(t) \in D(\tilde{L}).$$

Remark 1.4. Maximal formally normal extensions of the minimal operator L_0 are described by differential expression (1.1) and boundary conditions (1.2) and (1.3) where W_1 and W_2 are isometric operators in H . The general form of formally normal extensions of the operator L_0 is given by the conditions

$$\begin{aligned} K_1(\gamma_1 u + l\gamma_2 u) &= \gamma_1 u - i\gamma_2 u, \quad \gamma_1 u + i\gamma_2 u \in D(K_1), \\ K_2(\gamma_1(ImA)^{1/2}u) + i\gamma_2((ImA)^{1/2}u) &= \gamma_1((ImA)^{1/2}u) - i\gamma_2((ImA)^{1/2}u), \\ \gamma_1(ImA)^{1/2}u + i\gamma_2((ImA)^{1/2}u) &\in D(K_2), \end{aligned}$$

Where K_1 and K_2 are isometric operators in H .

Corollary 1.2. *If the extension, $\tilde{L}, L_0 \supset \tilde{L} \supset L$, is normal and described by the differential expression (1.1) and boundary conditions (1.2) and (1.3), then the extension $\tilde{L}^*, L_0^+ \supset \tilde{L}^* \supset L^+$, is normal to and described by the differential expression*

$$i^+(\nu(t)) = \frac{\partial^{2k}\nu(t)}{\partial t^{2k}} + A^*\nu(t), \quad 0 \leq t \leq 1.$$

in the space $L_2(H, (0, 1))$ and the boundary conditions of the form

$$\begin{aligned} (W_1^* + E)\gamma_2\nu - t(W_1^* - E)\gamma_1\nu &= 0, \\ (W_2^* + E)\gamma_2((ImA)^{-1/2}\nu) - t(W_2^* - E)\gamma_1((ImA)^{-1/2}\nu) &= 0. \end{aligned}$$

II. In this section we suppose that $n = 2k - 1$, $k \in N$ and, without loss of generality, we will consider ReA positive definite in H , i.e. consider the differential expression of the form

$$l(u) = \frac{\partial^{2k-1}u(t)}{\partial t^{2k-1}} + Au(t), \quad 0 \leq t \leq 1, \quad (7)$$

where A is normal in H and $ReA > 0$.

Let L_0 and L be minimal and maximal operators generated by the expression (2.1) in the space $L_2(H, (0, 1))$ respectively.

t is easy to see that

$$i(u) = ReAu(t) + l(-l \frac{\partial^{2k-1}}{\partial t^{2k-1}} + ImA)u(t).$$

Here again, denote by $(\mathfrak{A}, \gamma_1, \gamma_2)$ the space of boundary values of the minimal operator, generated by the symmetric expression

$$-l \frac{\partial^{2k-1}}{\partial t^{2k-1}}$$

in the space $L_2(H, (0, 1))$. It exists (see [12]).

The following theorem gives a constructive description in the terms of abstract boundary conditions of all kinds of normal extensions of the minimal operator L_0 , which is easily proved with the help of the scheme offered in proving theorem 1.1.

Theorem 2.1. *Whatever unitary operators W_1 and W_2 in \mathfrak{A} may be, the restriction of maximal operator L on the set of vector-functions $u(t) \in D(L)$ satisfying the conditions*

$$(W_1 - E)\gamma_1u + l(W_1 + E)\gamma_2u = 0, \quad (8)$$

$$(W_2 - E)\gamma_1((ReA)^{1/2}u) + l(W_2 + E)\gamma_2((ReA)^{1/2}u) = 0 \quad (9)$$

presents a normal extension of the operator L_0 . On the other hand, any normal extension of the operator L_0 is the restriction of the operator L on the set of vector function $u(t) \in D(L)$ satisfying the conditions (2.2) and (2.3), the unitary operators are defined by the extension uniquely.

Remark 2.1. Depending on the characteristics of A the boundary condition (2.3) is modified, but the boundary condition (2.2) remains invariable.

If $R \in A$ is negative definite, than

$$(W_2 - E_{\infty})\gamma_1((-ReA)^{1/2}u) + t(W_2 + E_{\infty})\gamma_2((-ReA)^{1/2}u) = 0.$$

If A is any bounded normal operator and $m = \inf_{f \in H} (ReAf, f)$, is of the form

$$(W_2 - E_{\infty})\gamma_1((ReA - (m + 1)E_H)^{1/2}u)t(W_2 + E_{\infty})\gamma_2((ReA - (m + 1)E_H)^{1/2}u) = 0$$

If $ReA = 0$, then it can be proved that all normal extensions of the operator L_0 are generated by the differential expression (2.1) and boundary condition (2.2).

Remark 2.2. Maximal formally normal extensions of the minimal operator, L_0 , are described by the differential expression (2.1) and boundary conditions (2.2) and (2.3) in which W_1 and W_2 are isometric operators in H . The general form of formally normal extensions of the operator L_0 is given by the conditions

$$\begin{aligned} K_1(\gamma_1u + l\gamma_2u) &= \gamma_1u - l\gamma_2u, \quad \gamma_1u + l\gamma_2u \in D(K_1), \\ K_2(\gamma_1((ReA)^{1/2}u) + l\gamma_2((ReA)^{1/2}u)) &= \gamma_1((ReA)^{1/2}u) - l\gamma_2((ReA)^{1/2}u), \\ \gamma_1((ReA)^{1/2}u) + l\gamma_2((ReA)^{1/2}u) &\in D(K_2), \end{aligned}$$

where K_1 and K_2 are isometric operators in H .

Corollary 2.1. If the extension $\tilde{L}, L_0 \supset \tilde{L} \supset L$ is normal, then it is maximal accretive. The accretiveness of \tilde{L} follows from relation

$$Re(\tilde{L}y, y)_{L_2(H(0,1))} = (ReAy, y)_{L_2(H(0,1))} \geq 0,$$

which is true for all $y(t) \in D(\tilde{L})$.

Corollary 2.2. If the extension $\tilde{L}, L_0 \supset \tilde{T} \supset L$ is normal, and described by the differential expression (2.1) and boundary conditions (2.2) and (2.3), then the extension $\tilde{L}^*, L_0^+ \supset \tilde{L}_0 \supset \tilde{L}^*P$ is also normal and described by the differential expression

$$i^+(\nu) = \frac{\partial^{2k-1}\nu(t)}{\partial t^{2k-1}} + A^*\nu(t)$$

= and boundary conditions of the form

$$\begin{aligned} (W_1^* + E)\gamma_2\nu - t(W_1^* - E)\gamma_1\nu &= 0, \\ (W_2^* + E)\gamma_2((\operatorname{Re}A)^{1/2}\nu) - (W_2^* - E)\gamma_1((\operatorname{Re}A)^{-1/2}\nu) &= 0 \end{aligned}$$

in the space $L_2(H, (0, 1))$.

III. Example 1. We consider in the space $L_2(H, (0, 1))$ the differential expression of the form

$$\iota(u) = \frac{\partial^2 u}{\partial x^2} + Au, \quad (10)$$

where A is a bounded normal operator in H and $\operatorname{Im}A$ is a positive definite operator.

It is easy to establish that

$$\mathfrak{z} = H \otimes H, \quad \gamma_1 u = \{u(0), u(1)\}, \quad \gamma_2 u = \{u'(0), u'(1)\}$$

is the space of the boundary values for the minimal operator generated by the differential expression $\frac{\partial^2}{\partial x^2}$ in $L_2(H, (0, 1))$. Taking into account

$$\gamma_1 \left[[\operatorname{Im}A]^{1/2} u \right] = \begin{bmatrix} (\operatorname{Im}A)^{1/2} & , 0 \\ 0 & (\operatorname{Im}A)^{1/2} \end{bmatrix} \gamma_1 u, \quad t = 1, 2,$$

and $R(\gamma_1) = R(\gamma_2) = H \otimes H$, we obtain that every kind of normal extensions of the minimal operator are described by the differential expression (3.1) and the boundary condition

$$(W_E)\gamma_1 u + t(W + E)\gamma_2 u = 0,$$

where the operators W and

$$\begin{bmatrix} (\operatorname{Im}A)^{1/2} & , 0 \\ 0 & , (\operatorname{Im}A)^{1/2} \end{bmatrix} W \begin{bmatrix} (\operatorname{Im}A)^{1/2} & , 0 \\ 0 & , (\operatorname{Im}A)^{-1/2} \end{bmatrix}$$

are unitary operators in $H \otimes H$.

Example 2. Consider in $L_2(H, (0, 1))$ the differential expression

$$\iota(u) = \frac{\partial^2 u}{\partial x^2} + Ay(t), \quad (11)$$

where A is bounded and normal in H .

It is easy to verify that

$$\mathfrak{A}H, \gamma_1 u = \frac{u(0) - u(1)}{2}, \gamma_2 u = \frac{u(0) + u(1)}{2t}$$

is the space of boundary values for minimal operator generated by the differential expression

$$i(u) = -t \frac{\partial u(t)}{\partial t}$$

in $L_2(H, (0, 1))$.

From the boundary conditions (2.2) and (2.3) it follows that all normal extensions are described by the following conditions:

$$u(1) = W_1 u(0),$$

$$(ReA)^{1/2} u(1) = W_2 (ReA)^{1/2} u(0)$$

If ReA is positive definite, then from [9]

$$\{u(0) | u(t) \in D(\tilde{L}_{norm})\} = \{u(1) | u(t) \in D(\tilde{L}_{norm})\} = H,$$

It follows that all normal extensions of minimal operator are described by differential expression (3.2) and the boundary condition

$$u(1) = W_1 u(0),$$

where W_1 and $(ReA)^{1/2} W_1 (ReA)^{1/2}$ are unitary in the space H .

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DERECESİ BÜYÜK DİFRANSİYEL DENKLEMLER İÇİN NORMAL SINIR DEĞER PROBLEMLERİ

Özet

Bu çalışmada $l(u) = \frac{d^n u(t)}{dt^n} + Au(t)$, A ber H Hilbert uzayı ve $A \in L_2(H, (0, 1))$ üzerinde sınırlı, normal bir operatör, tipi difransiye denklemlerin normal sınır değer problemleri soyut bir çerçevede ele alınmış ve değişik sınır değer problemleriyle ilişkileri üzerinde durulmuştur.

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