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COMPARISON OF INVARIANTS FOR TRIPLES OF HILBERT SPACES

P. A. Chalov

Abstract

In [2,3] the sequence of invariant characteristics (μ_m) for the finite families of Hilbert spaces were considered. Here we make a comparison of these invariants among themselves. We construct some examples of triples of Hilbert spaces, which show that each system of the first $r+1$ characteristics is stronger than the system of the first r of them. Moreover we show that there exist triples of Hilbert spaces which on the one hand are not quasidiagonally isomorphic, but on the other hand they cannot be distinguished by any function μ_m, $m \in \mathbb{N}$.

1. Introduction

In this paper we continue to study the invariant characteristics μ_m of families of Hilbert spaces, considered in [2,3]. Let us note that these characteristics appeared as natural modification of generalized linear topological invariants (Mityagin [5,4], Zhariuta [6,7,8]), which in their turn developed the classical approximative and diametral dimensions (Kolmogorov, Pelczynski,...). Here we restrict ourselves by considering of triples of Hilbert spaces only. Some examples of pairs of triples will be constructed to show that each system of the first $r+1$ functions μ_m is a stronger invariant than the system of the system of the first r of them. Another example represents a pair of triples, which cannot be distinguished by any characteristic μ_m but are not quasidiagonally isomorphic. The Central Lemma describing some special finite-dimensional triples plays a crucial role, since both examples are constructed with finite-dimentional blocks from this lemma.

2. Preliminaries

A family E of locally convex spaces E_j, $j = 0, 1, \dots, r$, with linear continuous injections $E_{j+1} \subset E_j, j = 0, 1, \dots, r-1$, will be denoted by $E = [E_0, E_1, \dots, E_r]$.

As usual, we denote by $l_2(a)$, $a = (a_i,\ i \in \mathbb{N})$, $a_i \geq 1$, $i \in \mathbb{N}$, the weighted l_2-space of all sequences $x = (\xi_i)$ with the finite norm

$$\| x \|_{l_2(a)} = \left(\sum_{i=1}^{\infty} \xi_i^2 a_i^2 \right)^{\frac{1}{2}}$$

and by $l_2^{(n)}(a)$, $a = (a_i,\, i = 1,2,\ldots,n)$, $a_i \geq 1$, $i = 1,2,\ldots,n$, the space of all n-dimensional vectors $x = (\xi_i,\, i = 1,2,\ldots,n)$ with the norm

$$\| x \|_{l_2^{(n)}(a)} = \left(\sum_{i=1}^{n} \xi_i^2 a_i^2 \right)^{\frac{1}{2}}.$$

Suppose $E = [E_0, E_1, \ldots, E_r]$; then if $E_j = l_2\left(a^{(j)}\right)$, $j = 0,1,\ldots,r$, we write $E = l_2[a^{(0)}, a^{(1)}, \ldots, a^{(r)}]$ and $E = l_2^{(n)}[a^{(0)}, a^{(1)}, \ldots, a^{(r)}]$, if $E_j = l_2^n\left(a^{(j)}\right)$, $j = 0,1,\ldots,r$.

Two families $E = [E_0, E_1, \ldots, E_r]$ and $F = [F_0, F_1, \ldots, F_r]$ are called *isomorphic* if there exists an isomorphism $T : E_0 \to F_0$, whose restriction on each space E_j is also isomorphism from E_j onto F_j, $j = 1,2,\ldots,r$.

A system $\{x_n,\, n \in \mathbb{N}\} \subset E_r$ is said to be an *unconditional basis for a family* $E = [E_0, E_1, \ldots, E_r]$ if this system constitutes an unconditional basis for each space E_j, $j = 0,1,\ldots,r$.

By $\{e_u\}$ we denote the canonical basis for the family $E = l_2[a^{(0)}, a^{(1)}, \ldots, a^{(r)}]$ i.e. $e_i = (\delta_{ik},\, k \in \mathbb{N})$, $i \in \mathbb{N}$.

Two unconditional bases $\{x_i\}$ and $\{y_i\}$ for families E and F, respectively, are called *quasiequivalent* if there exists an isomorphism $T : E \to F$ such that $Tx_i = \lambda_i y_{\sigma(i)}$, $i \in \mathbb{N}$, where $\{\lambda_i\}$ is a certain positive sequence and $\sigma : \mathbb{N} \to \mathbb{N}$ is a certain bijection.

In this case the isomorphism T is said to be quasidiagonal and the families E and F are called quasidiagonally isomorphic.

Hereafter $|M|$ denotes the cardinality for a finite set M and $+\infty$ for an infinite set M and $\mathbb{1}$ stands for the sequence $(1,1,\ldots,1,\ldots)$ or for a finite-dimensional vector $(1,1,\ldots,1)$.

For a given family $E = l_2[\mathbb{1}, a^{(1)}, \ldots, a^{(r)}]$ (or $E = l_2^{(n)}[\mathbb{1}, a^{(1)}, \ldots, a^{(r)}]$) and for a natural number m we define the function (which will be called *m-rectangle characteristic*)

$$\mu_m(E;\tau,t) = \left| \bigcup_{k=1}^{m} \bigcap_{j=1}^{r} \{ i : \tau_j^{(k)} < a_i^{(j)} \leq t_j^{(k)} \} \right|, \tag{1}$$

$\tau = \left(\tau_j^{(k)}, j = 1,2,\ldots,r; k = 1,2,\ldots,m\right), t = \left(t_j^{(k)}, j = 1,2,\ldots r; k = 1,2,\ldots,m\right)$.

A set $P(\tau,t) = \{(x,y) : \tau_1 < x \leq t_1, \tau_2 < y \leq t_2\}$ is called a rectangle if $\tau_1 < t_1$ and $\tau_2 < t_2$. It follows from the definition of the function $\mu_m(E;\tau,t)$ for the triple E that

$$\mu_m(E;\tau,t) = |\bigcup_{k=1}^{m}\{i : \left(a_i^{(1)}, a^{(2)}\right) \in P\left(\tau^{(k)}, t^{(k)}\right)\}|$$
$$= |\{i : \left(a_i^{(1)}, a_i^{(2)}\right) \in \bigcup_{k=1}^{m} P\left(\tau^{(k)}, t^{(k)}\right)\}|,$$

where $\tau = \left(\tau_j^{(k)}, j = 1,2; = 1,2,\ldots,m\right), t = \left(t_j^{(k)}, j = 1,2; k = 1,2,\ldots,m\right)$, $\tau^{(k)} = \left(\tau_1^{(k)}, \tau_2^{(k)}\right)$ and $t^{(k)} = \left(t_1^{(k)}, t_2^{(k)}\right), k = 1,2,\ldots,m$, hence the function (1) calculates, how many points $\left(a_i^{(1)}, a_i^{(2)}\right)$ are contained in the union of m rectangles.
We say that the functions $\mu_m(E;\tau,t)$ and $\mu_m(F;\tau,t)$ are equivalent and write $\mu_m^E \approx \mu_m^F$ if a positive constant $\alpha = \alpha(m)$ exists such that the following inequalities

$$\mu_m(E;\tau,t) \le \mu_m\left(F;\frac{\tau}{\alpha},\alpha t\right), \quad \mu_m(F;\tau,t) \le \mu_m\left(E;\frac{\tau}{\alpha},\alpha t\right)$$

hold for any $\tau = \left(\tau_j^{(k)}, j = 1,2,\ldots r; k = 1,2,\ldots,m\right)$ and any
$t = \left(t_j^{(k)}, j = 1,2,\ldots,r;\ k = 1,2,\ldots,m\right)$.

Proposition 1 *([3], Theorem 8). If families $E = l_2[1\!\!1, a^{(1)},\ldots,a^{(r)}]$ and $F = l_2[1\!\!1, b^{(1)},\ldots,b^{(r)}]$ are isomorphic, then $\mu_m^E \approx \mu_m^F$ for each m.*

It follows from this proposition that if there exists $m \in \mathbb{N}$ such that $\mu_m^E \not\approx \mu_m^F$, then the triples E and F are nonisomorphic.

We say that the systems $(\mu_m(E;\tau,t))$ and $(\mu_m(F;\tau,t))$ are equivalent and write $(\mu_m^E) \approx (\mu_m^F)$ if a positive constant β exists such that the following in equalities

$$\mu_m(E;\tau,t) \le \mu_m\left(F;\frac{\tau}{\beta},\beta t\right), \quad \mu_m(F;\tau,t) \le \mu_m\left(E;\frac{\tau}{\beta},\beta t\right)$$

hold for each $m \in \mathbb{N}$ and every $\tau = \left(\tau_j^{(k)}, j = 1,2,\ldots,r;\ k = 1,2,\ldots,m\right)$,
$t = \left(t_j^{(k)}, j = 1,2,\ldots,r; k = 1,2,\ldots,m\right)$.

Proposition 2 *([3], Theorem 6). For the families $E = l_2[1\!\!1, a^{(1)},\ldots,a^{(r)}]$ and $F = l_2[1\!\!1, b^{(1)},\ldots,b^{(r)}]$ the following statements are equivalent:*

(a) families E and F are quasidiagonally isomorphic;

(b) $(\mu_\mu^E) \approx (\mu_m^F)$.

Further, we shall consider only triples of Hilbert spaces with an unconditional basis.

3. The main Results

The following statement shows that the equivalence of any finite set of characteristics (1) lacks to provide isomorphism for arbitrary pair of families of Hilbert spaces.

Theorem 3 *(cf. [1], Theorems 1,2). For each natural number m there exist triples $E = l_2[\mathbb{1}, a^{(1)}, a^{(2)}]$ and $F = l_2[\mathbb{1}, b^{(1)}, b^{(2)}]$ satisfying the following conditions:*

(i) $\mu_l^E \approx \mu_l^F$ for each $l = 1, 2, \ldots, m$;

(ii) $\mu_{m+1}^E \not\approx \mu_{m+1}^F$.

The next theorem shows that even with the equivalence of all functions (1) the families of Hilbert spaces need not be quasidiagonally isomorphic.

Theorem 4 *(cf. [1], Theorem 3). There exist two triples $E = l_2[\mathbb{1}, a^{(1)}, a^{(2)}]$ and $F = l_2[\mathbb{1}, b^{(1)}, b^{(2)}]$ satisfying the following conditions:*

(iii) $\mu_m^E \approx \mu_m^F$ for each $m \in \mathbb{N}$;

(iv) $(\mu_m^E) \not\approx (\mu_m^F)$.

To prove Theorems 3 and 4 we shall cite the examples which show the correctness of these theorems. In each example infinite-dimensional spaces will be collected from special blocks of finite-dimensional spaces.

The following Central Lemma gives a construction of such blocks.

Lemma 5. *Let m be any natural number, α and β arbitrary numbers such that $1 < \alpha < \beta$. Then there exists a natural number $n = n(\alpha, \beta, m)$ and two triples of n-dimensional spaces $E = l_2^{(n)}[\mathbb{1}, a^{(1)}, a^{(2)}]$ and $F = l_2^{(n)}[\mathbb{1}, b^{(1)}, b^{(2)}]$ satisfying the followign conditions:*

(v) for each $l \leq m$ the inequalities

$$\mu_l(E; \tau, t) \leq \mu_l\left(F; \frac{\tau}{\alpha}, \alpha t\right), \tag{2}$$

$$\mu_l(E; \tau, t) \leq \mu_l\left(E; \frac{\tau}{\alpha}, \alpha t\right), \tag{3}$$

hold for any $\tau = \left(\tau_j^{(k)}, j = 1, 2; k = 1, 2, \ldots, l\right)$ and any $t = \left(t_j^{(k)}, j = 1, 2; k = 1, 2, \ldots, l\right)$;

(vi) the below mentioned statement is valid for $c = \beta$ but it fails for any $c < \beta$: the inequalities

$$\mu_{m+1}(E; \tau, t) \leq \mu_{m+1}\left(F; \frac{\tau}{c}, ct\right), \tag{4}$$

$$\mu_{m+1}(F; \tau, t) \leq \mu_{m+1}\left(E; \frac{\tau}{c}, ct\right), \tag{5}$$

hold for any $\tau = \left(\tau_j^{(k)}, j = 1, 2; k = 1, 2, \ldots, m+1\right)$ *and any* $t = \left(t_j^{(k)}, j = 1, 2;\right.$ $\left.k = 1, 2, \ldots, m+1\right)$.

4. Proof of Central Lemma

First we introduce a notion of an α-dense set. Let A and B be subsets of $\mathbb{R}^2$. We say that the set A is α-dense in B if for each point $(x, y) \in B$ there is a point $(\tilde{x}, \tilde{y}) \in A$ such that

$$\frac{\tilde{x}}{\sqrt{\alpha}} \leq x \leq \sqrt{\alpha}\tilde{x}, \quad \frac{\tilde{y}}{\sqrt{\alpha}} \leq y \leq \sqrt{\alpha}\tilde{y}.$$

We take any natural number 8 m, arbitrary numbers α and β such that $1 < \alpha < \beta$, and arbitrary positive numbers X_1 and Y_1. Denote by $X_2, X_3, Y_2, Y_3, \ldots, Y_{2m}$ numbers defined by the following formulae: $X_i = \beta^{i-1}X_1$, $i = 2, 3; Y_j = \beta^{j-1}Y_1, j = 2, 3, \ldots, 2m$.

In the first quadrant of the plane xOy we draw two "combs" each of them has m "cogs". The bases of these combs are parallel to the axis Oy and the cogs are parallel to axis Ox (see Figure 1).

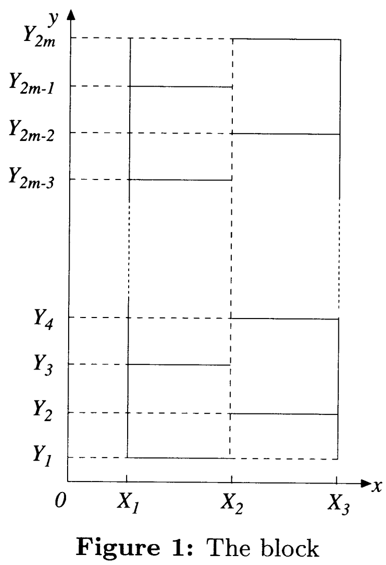

Figure 1: The block

On these combs we select a bounded α-dense point set M such that all the points (X_i, Y_j), $i = 1, 2, 3$; $j = 1, 2, \ldots, 2m$, are contained in M.

Suppose M consists of $(n-1)$ points, then we can write $M = \{(x_i, y_i), i = 1, 2, \ldots, n-1\}$. We take else two additional points $N_1(\dot{x}, \dot{y})$ on the left comb and $N_2(\ddot{x}, \ddot{y})$ on the right comb.

Let us define the vectors $a^{(j)} = \{a_i^{(j)}, i = 1, 2, \ldots, n\}$ and $b^{(j)} = \{b_i^{(j)}, i = 1, 2, \ldots, n\}$, $j = 1, 2$. By definition, put $a_i^{(1)} = b_i^{(1)} = x_i, a_i^{(2)} = b_i^{(2)} = y_i, i = 1, 2, \ldots, n-1$; $a_n^{(1)} = \dot{x}, a_n^{(2)} = \dot{y}, b_n^{(1)} = \ddot{x}, b_n^{(2)} = \ddot{y}$.

Suppose $E = l_2^{(n)}[\mathbb{1}, a^{(1)}, a^{(2)}]$ and $F = l_2^{(n)}[\mathbb{1}, b^{(1)}, b^{(2)}]$. We assert that the triples E and F satisfy the conditions (v) and (vi).

Indeed, let l be a natural number less than or equal to m. Let us take arbitrary $\tau = \left(\tau_j^{(k)}, j = 1, 2; k = 1, 2, \ldots, l\right)$, $t = \left(t_j^{(k)}, j = 1, 2; k = 1, 2, \ldots, l\right)$, and show that the inequality (2) holds.

Since $a_i^{(j)} = b_i^{(j)}$, $j = 1, 2$, for all $i = 1, 2, \ldots, n-1$, we can see that there is nothing to prove as

$$\left(a_n^{(1)}, a_n^{(2)}\right) \notin \bigcup_{k=1}^{l} P\left(\tau^{(k)}, t^{(k)}\right),$$

where $\tau^{(k)} = \left(\tau_1^{(k)}, \tau_2^{(k)}\right), t^{(k)} = \left(t_1^{(k)}, t_2^{(k)}\right), k = 1, 2, \ldots, l$.

Supporse $\left(a_n^{(1)}, a_n^{(2)}\right) \in \bigcup_{k=1}^{l} P\left(\tau^{(k)}, t^{(k)}\right)$; then the following two case may occur:

1) there is $\left(a_i^{(1)}, a_i^{(2)}\right) \notin \bigcup_{k=1}^{l} P\left(\tau^{(k)}, t^{(k)}\right)$ on the left comb;

2) all points $\left(a_i^{(1)}, a_i^{(2)}\right)$ situated on the left comb are contained in the set

$$\bigcup_{k=1}^{l} P\left(\tau^{(k)}, t^{(k)}\right).$$

In the case 1, taking into account α-density of M, we get that there exists a point

$$\left(b_i^{(1)}, b_i^{(2)}\right) \in \bigcup_{k=1}^{l} [P(\frac{\tau^{(k)}}{\alpha}, \alpha t^{(k)}) \backslash P\left(\tau^{(k)}, t^{(k)}\right)]$$

on the left comb. Therefore, in this case the inequality (2) holds.

In the case 2 it will be observed that there is i such that

$$(X_2, Y_{2i}) \in \bigcup_{k=1}^{l} P\left(\tau^{(k)}, t^{(k)}\right).$$

Indeed, each of the combs has m cogs and number of the rectangles $P\left(\tau^{(k)}, t^{(k)}\right)$ is less than or equal to m. Hence, there exists i such that both points (X_2, Y_i) and (X_2, Y_{i+1})

are contained in one of the rectangles $P\left(\tau^{(k)}, t^{(k)}\right)$. Since the points $(X_2, Y_i) \in M$ for any $i = 1, 2, \ldots, 2m$, there exists

$$\left(b_i^{(1)}, b_i^{(2)}\right) \in \bigcup_{k=1}^{l} [P(\frac{\tau^{(k)}}{\alpha}, \alpha t^{(k)}) \backslash P\left(\tau^{(k)}, t^{(k)}\right)]$$

on the right comb. Whence, in this case the inequality (2) holds too.

By analogy we can get that the inequality (3) also holds. This completes the proof of the condition (v). Before starting with the condition (vi) we note that the inequalities (4) and (5) are valid obviously as $c = \beta$. Therefore to prove the condition (vi) we take an arbitrary c such that $1 < c < \beta$ and put $\tau_1^{(k)} < X_1, k = 1, 2, \ldots, m+1$; $t_1^{(k)} = X_2, k = 1, 2, \ldots, m$; $t_1^{(m+1)} = X_1$; $\tau_2^{(1)} = \tau_2^{(m+1)} < Y_1; \tau_2^{(k)} = cY_{2k-2}$, $k = 2, 3, \ldots, m; t_2^{(k)} = Y_{2k-1}, k = 1, 2, \ldots, m$; $t_2^{(m+1)} = Y_{2m-1}$. Then we have

$$\left(a_i^{(1)}, a_i^{(2)}\right) \in \bigcup_{k=1}^{m+1} P\left(\tau^{(k)}, t^{(k)}\right)$$

if and only if $\left(a_i^{(1)}, a_i^{(2)}\right)$ is a point on the left comb, and

$$\left(b_i^{(1)}, b_i^{(2)}\right) \in \bigcup_{k=1}^{m+1} P\left(\frac{\tau^{(k)}}{c}, ct^{(k)}\right)$$

if and only if $\left(b_i^{(1)}, b_i^{(2)}\right)$ is a point on the left comb.

Hence, according to the definition of the vectors $a^{(j)}, b^{(j)}$, $j = 1, 2$, we have

$$\begin{aligned}\mu_{m+1}(E; \tau, t) &= |\{i : \left(a_i^{(1)}, a_i^{(2)}\right) \in \bigcup_{k=1}^{m+1} P\left(\tau^{(k)}, t^{(k)}\right)\}| \\ &> |\{i : \left(b_i^{(1)}, b_i^{(2)}\right) \in \bigcup_{k=1}^{m+1} P\left(\frac{\tau^{(k)}}{c}, ct^{(k)}\right)\}| = \mu_{m+1}\left(F; \frac{\tau}{c}, ct\right).\end{aligned}$$

This means that the inequality (4) is not true if $c < \beta$. Lemma 5 is proved.

Remark. It follows from the proof of Lemma 5 that we can constuct a block beginning at any point (X_1, Y_1) from the first quadrant of the plane xOy.

5. Proofs of Theorems 3 and 4

Let m_0 be an arbitrary natural number, $\alpha > 1$ an arbitrary number. Let us take an arbitrary sequence $(\beta_k, k \in \mathbb{N}) \uparrow +\infty$ such that $\beta_1 > \alpha$ and arbitrary positive numbers X and Y.

We define the sequence $\left(Y^{(k)}, k \in \mathbb{N}\right)$ by the folllowing formulae: $Y^{(1)} = Y, Y^{(k)} = \beta_{k-1}^{2m_0-1}\beta_k Y^{(k-1)}$ for all $k = 2, 3, \ldots$.
Taking into account Remark for each $k \in \mathbb{N}$ we apply Lemma 5 with $m = m_0$, $\beta = \beta_k, X_1 = X, Y_1 = Y^{(k)}$.

Thus, we have the sequence of natural numbers $(n_k,\ k \in \mathbb{N})$ and two sequences of finite dimentional triples $(E^{(k)} = l_2^{(n_k)}[1\!\!1, a^{(1)}(k), a^{(2)}(k)])$ and $(F^{(k)} = l_2^{(n_k)}[1\!\!1, b^{(1)}(k), b^{(2)}(k)])$, where $a^{(j)}(k) = (a_i^{(j)}(k), i = 1, 2, \ldots, n_k), b^{(j)}(k) = (b_i^{(j)}(k),\ i = 1, 2, \ldots, n_k), j = 1, 2$.

We define the sequences $a^{(j)} = (a_i^{(j)}, i \in \mathbb{N})$ and $b^{(j)} = (b_i^{(j)}, i \in \mathbb{N}), j = 1, 2$, by the following formulae: $a_i^{(j)} = a_s^{(j)}(k), b_i^{(j)} = b_s^{(j)}(k), j = 1, 2$, if $\sum\limits_{l=0}^{k-1} n_l < i \le \sum\limits_{l=0}^{k} n_l$, where $n_0 = 0,\ s = i - \sum\limits_{l=0}^{k-1} n_1,\ k \in \mathbb{N}$.

By the choice of the numbers X, Y and by the definitions of the sequences $\left(Y^{(k)}\right), a^{(j)}, b^{(j)}, j = 1, 2$, the following linear continuous injections $l_2\left(a^{(2)}\right) \subset l_2\left(a^{(1)}\right) \subset l_2$ and $l_2\left(b^{(2)}\right) \subset l_2\left(b^{(1)}\right) \subset l_2$ hold. Therefore, we have two triples $E = l_2 1\!\!1, a^{(1)}, a^{(2)}]$ and $F = l_2[1\!\!1, b^{(1)}, b^{(2)}]$.

The condition (i) follows now from the condition (v) of Lemma 5. Since each comb has m_0 cogs the condition (vi) of Lemma 5 implies the condition (ii). By Proposition 1 we deduce that the triples E and F are nonisomorphic. This example proves Theorem 3.

If we define the sequence $\left(Y^{(k)}, k \in \mathbb{N}\right)$ by the following formulae: $Y^{(1)} = Y, Y^{(k+1)} = \beta_k^{2k-1}\beta_{k+1}Y^{(k)}$ for all $k \in \mathbb{N}$ and then apply Lemma 5 with $m = k, \beta = \beta_k, X_1 = X, Y_1 = Y^{(k)}$, we obtain an example which proves Theorem 4.

Indeed, in this case, the number of cogs in each pair of combs increases together with the number of blocks. Therefore the condition (iv) follows from the condition (vi) of Lemma 5.

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References

[1] P. A. Chalov. Triples of Hilbert spaces, (in Russian). Manuscript 1387-81 Dep., deposited at VINITI, 1981.

[2] P. A. Chalov, V. P. Zahariuta. On linear topological invariants on some class of families of Hilbert spaces, (in Russian). Manuscript 3862- B86, deposited at VINITI, 1986.

[3] P. A. Chalov, V. P. Zahariuta. Invariants and quasiequivalence property for families of Hilbert spaces, to appear.

[4] B. S. Mityagin. Approximative dimension and bases in nuclear spaces, (in Russian). Usp. Mat. Nauk, 16(100), 63-132, 1961.

[5] B. S. Mityagin. The equivalence of bases in Hilbert scales, (in Russian). Studiaa Math., 37(2), 111-137, 1971.

[6] V. P. Zahariuta. Linear topological invariants and isomorphisms of spaces of analytic functions, (in Russian). Matem. analiz i ego pril., Rostov-on-Don, Rostov Univ., 2,3-13, 1970; 3, 176-180, 1971.

[7] V. P. Zahariuta. On isomorphisms and quasiequivalence of bases of power Köthe spaces (in Russian) Proceedings of 7th winter school in Drogobych, 101-126, 1976.

[8] V. P. Zahariuta. Synthetic diameters and linear topological invariants, (in Russian). School on theory of operators in functional spaces (abstracts of reports), 51-52, Minsk, 1978.

3'LÜ HİLBERT UZAYLARININ, İNVAZİAUTLARININ KARŞILAŞTIRILMASI

Özet

Bu çalışmada [2] ve [3] nolu kaynaklarda tanımlanan invariantlar kendi aralarında karşılaştırılmış ve her mertebedeki invaziantların farklılıkları ayrımında önem taşıdığı gösterilmiştir.

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