Turkish Journal of Mathematics

Volume 20 | Number 4 Article 5

1-1-1996

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Tr. J. of Mathematics
20 (1996) , 481 – 491.

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Abstract

It is considered the relationship between spaces $L_f(\lambda,\sigma)$ and subspaces of the space $A_1(\bar{A}_1)$ of analytic functions in the open (closed) unit disc, generated by systems $F(\alpha_n z), n \in N$, if they constitute a basis in their closure.

1. Introduction

0. Let F be an entire function, $\alpha = (\alpha_n)_{n=1}^{\infty}$ a certain sequence of different points of the complex plane C, and $|\alpha_n| \uparrow \infty$. Let A_r be the space of all functions, analytic on the disk $\{|z| < r\}, 0 < r \leq \infty$, endowed by the topology of uniform convergence on compact subsets. We consider the closed subspace $A_r(F,\alpha)$ defined by a sequence of functions

$$F(\alpha_n z), n \in N. \tag{1}$$

If the sequence (1) forms a basis in $A_r(F,\alpha)$ then $A_r(F,\alpha)$ is called a generalized Dirichlet series space (GDS). Analogously GDS $\bar{A}_r(f,\alpha)$ is defined as the corresponding subspace of the space $\bar{A}_r$ of all functions, analitic on the closed disk $\{|z| \leq r\}, 0 \leq r < \infty$, considered with the usual inductive topology.

In the present paper we consider a connection between GDS of types $A_\infty, \bar{A}_0$ on the one hand and Köthe spaces $L_f(\lambda,\sigma)$ on the other hand. Let us recall that $L_f(\lambda,\sigma)$ is the Köthe space defined by the following Köthe matrix:

$$a_{p,n} = \exp f(\lambda_n \sigma_p), n, p \in N$$

where $f = f(u)$ is non-decreasing odd function, defined on R and logarithmically convex as $u > 0; \lambda = (\lambda_n)$ is a positive increasing sequence; (σ_p) is an increasing sequence such that $\sigma_p \uparrow \sigma, -\infty < \sigma \leq \infty$. Thus $L_f(\lambda,\sigma)$ is the locally convex space of all sequences $t = (t_n), t_n \in C$, such that

$$\| t \|_p := \sum_{n=1}^{\infty} |t_n| a_{p,n} < \infty, p \in N$$

with the topology defined by the system of seminorms $\{\| t \|_p, p \in N\}$.

Given f denote by $(f)_0$ (respectively $(f)_\infty$)) the class of all F-spaces, isomorfic to $L_f(\lambda, 0)$ (respectively $L_f(\lambda, \infty)$) with some λ. It is known [3] that classes $(f)_0, (g)_\infty$ are disjoint for any f, g.

The function f is called rapidly increasing if $f(\theta u)/f(u) \uparrow \infty$ as $u \to \infty$ for all $\theta > 1$. If this condition fails then $(f)_0 = (I)_0$ and $(f)_\infty = (I)_\infty$, where $I(u) \equiv u$; othervise $(f)_0 \cup (I)_0 = \emptyset$ and $(f)_\infty \cup (I)_\infty = \emptyset$.

It is known that every non-degenerated F-space (i.e. it is isomorphic neither to the space ω nor to a Banach space, nor to their Cartesian product) has a subspace which belongs to some class $(f)_\infty$ and has a quotient space which belongs to some class $(f)_0$ [1] (see also [2, 6, 4]). Our purpose is to show that GDS $A_r(F, \alpha), \bar{A}_r(F, \alpha)^\star$ are wide-spread quite as well as spaces of classes $(f)_0, (f)_\infty$.

1. The convex function $h(w) = f(e^w), 0 \leq w < \infty$, has following property

$$\lim_{w \to \infty} (h(w)/w) = \infty.$$

Its Young congugate function $h^\star(v) = \max\{wv - h(w)\}$ is a convex function with the same property [8].

Consider the following entire function

$$F_f(z) = \sum_{n=1}^{\infty} e^{-h^\star(n-1)} z^{n-1}.$$

Let us use the notation

$$|g|_r = \max_{|z| \leq r} |g(z)|.$$

Lemma 1 *The functions $|F_f|_u$ and $\exp f(u)$ satisfy the following relations:*

$$\forall \theta > 1 \exists c > 0 \forall u \geq 0 \frac{1}{c} \exp f(\frac{u}{\theta}) \leq |F_f|_u \leq c \exp f(u\theta). \tag{2}$$

Proof. First we have

$$\begin{aligned} |F_f|_u &= \sum_1^{\infty} e^{-h^{\star(n-1)}} u^{n-1} \\ &\leq \max_n (e^{-h^{\star(n-1)}} (u\theta)^{n-1}) \sum_{n=1}^{\infty} \theta^{-n+1} \\ &\leq \frac{\theta}{\theta - 1} \exp \max\{-h^\star(n-1) + (n-1)\ln(ue)\} \\ &\leq \frac{\theta}{\theta - 1} \exp h(\ln(u\theta)) = c_1 \exp f(u\theta), 0 < u < \infty, \end{aligned}$$

where c_1 depends on θ only. Since h is Young conjugate for h^*, there exists $v_0 > 0$ such that

$$h(\ln u) = \max_v \{v \ln u - h^*(v)\} = v_0 \ln u - h^*(v_0).$$

Let n_0 be an integer such that $n_0 - 1 \leq v_0 < n_0$. Then

$$\begin{aligned} |F_f|_u &\geq \exp(-h^*(n_0-1) + (n_0-1)\ln u) \\ &\geq \exp(-h^*(v_0) + v_0 \ln u - \ln u) \\ &= \exp(h(\ln u) - \ln u) = f(u) \exp\left(1 - \frac{\ln u}{h(\ln u)}\right) \\ &\geq \frac{1}{c_2} f(u) \geq \frac{1}{c_2} f(\frac{u}{\theta}). \end{aligned}$$

Taking $c = \max\{c_1, c_2\}$ we complete the proof.

Denote

$$x_n = F_f(nz), n \in N, \tag{3}$$

and

$$\begin{aligned} X_f &= \left\{ t = (t_n) : \| t \|_p = \sum_1^\infty |t_n| |x_n|_p < \infty, p \in N \right\}, \\ X'_f &= \left\{ u = (u_n) : \exists p | \sup_n \frac{|u'_n|}{|x_n|_p} < \infty \right\} \\ A'_\infty &= \left\{ u = (u_n) : \lim \sup_{n\to\infty} |u_n|^{1/n} < \infty \right\}. \end{aligned}$$

From Lemma 1, we get the relation

$$\forall p \exists q : \sum_1^\infty \frac{|x_n|_p}{|x_n|_q} < \infty, \tag{4}$$

which means that the Köthe space X_f is nuclear [9]. Therefore the strong dual space X_f^* can be realized as the space X'_f using the duality

$$< t, u > = \sum_1^\infty t_n u_n, \tag{5}$$

where $t = (t_n) \in X_f, u = (u_n) \in X'_f$. Analogously we deal with the spaces A'_∞ (duality between them is given by the same formula (5) with $\sum_n t_n z^n \in A_\infty$ and $u = (u_n) \in A'_\infty$).

□

Lemma 2 *(cf [5]) The sequence (3) forms a basis in the space $A_\infty(F_f, N)$ if and only if for every sequence $t = (t_n) \in X'_f$ there exists a function*

$$\varphi(z) = \sum_1^\infty u_n e^{-h^*(n-1)} z^{n-1}, \tag{6}$$

such that $(u_n) \in A'_\infty$ and $\varphi(n) = t_n, n \in N$.

Proof. For arbitrary $t = (t_n) \in X_f$ we put $T(t) = \sum_1^\infty t_n x_n$. The linear operator $T : X_f \to A_\infty$ is continuos and so its conjugate operator $T' : A'_\infty \to X'_f$ is. Let $\{x_n\}$ be a basis in $A_\infty(F_f, N)$, then it is absolute, because of nuclearity of the space and T is an isomorphism as well as $T'(A'_\infty) = X'_f$. Hence

$$< T', e_n >=< u, Te_n >=< u, x_n >= \sum_{k=1}^\infty u_k e^{-h^*(k-1)} n^k = \varphi(n) = t_n, n \in N.$$

Thus the necessity is proved. To prove the sufficiency we have to repeat the above considerations in the inverse order.

□

Theorem 3 *The sequence (3) forms a basis in the space $A_\infty(F_f, N)$.*

Proof. By Lemma 1, for arbitrary sequence $(t_n) \in X'_f$ there exist constants p_1 and c_1 such that the inequality

$$|t_n| \leq c_1 \exp f(p_1 n), n \in N \tag{7}$$

holds. We construct a function (6) by means of interpolational Lagrange series

$$\varphi(z) = \frac{1}{\pi} \sin \pi z \sum_{k=1}^\infty (-1)^{k-1} (\frac{z}{k})^{\nu_k} \frac{t_k}{z-k}, \tag{8}$$

where ν_k is the integer part of $f(p_1 k), k \in N$. For any $p_2 > \pi$ we have

$$\exists c_2 > 0 : \sup_k \left| \frac{\sin \pi z}{\pi(z-k)} \right|_r \leq c_2 \exp(p_2 z), 0 < r < \infty. \tag{9}$$

Consequently

$$|\varphi|_r \leq c_2 \exp(p_2 r) \sum_{k=1}^{\infty} (\frac{r}{k})^{\nu_k} |t_k| \leq c_1 c_2 c_3 \sup_k (\frac{r\theta e}{k})^{f(p_1 k)}, \tag{10}$$

where $\theta > 1$ is arbitrary and the constant

$$c_3 = \sum_{k=1}^{\infty} (\theta)^{f(p_1 k)} < \infty$$

does not depend on r. To estimate the function

$$\varphi_r(u) = (\frac{r\theta e}{u})^{f(p_1 u)} \tag{11}$$

we consider two cases. First, let $f(u)$ be rapidly increasing. Then its maximum cannot be realized outside of the interval $(r\theta_1 e, r\theta e)$ for some $\theta_1 < \theta$ and large enough n. Therefore the following estimate

$$\max_u \varphi_r(u) < \exp f(p_1 \theta e r) \tag{12}$$

holds asymptotically, i.e. for large enough r. If $f(u)$ is slowly increasing, we use the following its property:

$$\frac{u f'(u)}{f(u)} \uparrow \alpha, \tag{13}$$

where $f'(u)$ means a left-side derivative, $0 < \alpha < \infty$. Since $\varphi_r'(u) > 0$ in some neighbourhood of any point $u < r\theta e^{1-1/\alpha}$, the function $\varphi_r(u)$ attains its maximum in the interval $(r\theta_2 e, r\theta e)$, where $\theta_2 < \theta e^{1-1/\alpha}$. Therefore we have the estimate:

$$\max_u \varphi_r(u) < \exp \frac{f(p_1 \theta e r)}{\alpha} \tag{14}$$

Taking into account (13) there exists $p > p_1 \theta e$ such that

$$p_2 r + \frac{f(p_1 \theta e r)}{\alpha} < f(pr)$$

holds for big enough r. Combining this with (7)-(12) and (14), we get the following estimate (with constants c and p, independent of r):

$$|\varphi|_r \leq c e^{f(pr)}, 0 < r < \infty.$$

Therefore

$$
\begin{aligned}
|u_n e^{-h^{\star(n-1)}}| &\le \min_r \frac{|\varphi|_r}{r^{n-1}} \le cp^{n-1} \min_r \exp(f(pr) - (n-1)\ln pr) \\
&= cp^{n-1} \exp(-\max_r((n-1)\ln pr - h(\ln pr))) \\
&= cp^{n-1} e^{-h^{\star(n-1)}}, n \in N. \qquad (15)
\end{aligned}
$$

Hence

$$\lim_{n\to\infty} \sup |u_n|^{\frac{1}{n}} \le p < \infty,$$

i.e. $(u_n) \in A'_\infty$. Therewith $\varphi(n) = t_n, n \in N$ by construction. It remains to apply Lemma 2.

As follows from Theorem 3, the formula $T(x_n) = e_n$ generates an isomorphism of the space $A_\infty(F_f, N)$ onto Köthe space X_f.

□

Corollary 4 *The space $L_f(N, \infty)$ and GDS $A_\infty(E_f, N)$ are diagonally isomorphic.*

3. Let us prove an analogue of Theorem 3 for the space $\bar{A}'_0$, dual to $\bar{A}_0$.

Theorem 5 *Let $f(u)$ be rapidly increasing. Then the sequence (3) forms a basis in the space $\bar{A}'_0(F_f, N)$.*

First introduce some notation:

$$
\begin{aligned}
\bar{A}'_0 &= \left\{(u_n) : \lim_{n\to\infty} |u_n|^{\frac{1}{n}} = 0\right\}; \\
\bar{X}_f &= \left\{(t_n) : \exists r > 0 | \sum_1^\infty |t_n||x_n|_r < \infty\right\}; \\
\bar{X}'_f &= \left\{u = (u_n) : \forall r > 0 | \parallel u \parallel_{-r} = \sum_{k=1}^\infty \frac{|u_n|}{|x_n|_r} < \infty\right\}.
\end{aligned}
$$

The formula (5) sets a duality for the pair of spaces $(\bar{X}_f, \bar{X}'_f)$ as well as for the pair $(\bar{A}_0, \bar{A}'_0)$ (in the latter case $\sum_n t_n z^{n-1} \in \bar{A}_0$ and $(u_n) \in \bar{A}'_0$). Hence $\bar{X}_f$ is a strong dual for Köthe space $\bar{X}'_f$ and so is $\bar{A}'_0$ for $\bar{A}_0$.

The following fact can be proved analogically to Lemma 2.

Lemma 6 *For (3) to be a basis in the space $\bar{A}_0(E_f, N)$ it is necessary and sufficient that for every sequence $t = (t_n) \in \bar{X}'_f$ there exists an entire function*

$$\varphi(z) = \sum_{1}^{\infty} u_n e^{-h^{\star(n-1)}} z^{n-1},$$

such that $(u_n) \in \bar{A}'_0$ and $\varphi(n) = t_n, n \in N$.

Proof of Theorem 5. Given a sequence $(t_n) \in \bar{X}'_f$ take the entire function $\varphi(z)$ defined by (8) with ν_k chosen as the entire part of the number $\ln |t_k|$, if it is positive, or 0 otherwise. Let us show that the function $\varphi(z)$ satisfies the condition:

$$\forall \epsilon > 0 \exists c > 0 \parallel \varphi|_r \leq c e^{f(\epsilon r)}, 0 < r < \infty. \tag{16}$$

To do this, for a given ϵ, take n_ϵ such that

$$|t_n| \leq e^{f(\frac{\epsilon n}{2e})}, \text{ for } n > n_\epsilon.$$

Let $\varphi_1(z)$ be the sum of the first n_ϵ members of the series (8). Then, clearly, the estimate:

$$|\varphi_1|_r \leq c_1 e^{4r} \tag{17}$$

holds, where the constant c_1 depends on ϵ. For the sum $\varphi_2(z)$ of the rest of members of (8) we have

$$|\varphi_2|_r \leq c_2 e^{4r} \sup_k (\frac{r\theta e}{k})^{f(\frac{\epsilon n}{2e})} \tag{18}$$

where $1 < \theta < 2$ and the constant c_2 depend on ϵ only. As in the proof of Theorem 1, we were able to ascertain that the maximum $M(r)$ of the function

$$(\frac{r\theta e}{u})^{f(\frac{\epsilon u}{2e})},$$

for big enough r, attains on some interval $(r\theta_1 e, r\theta e)$ with $1 < \theta_1 < \theta$. Therefore we have that, asymptotically, the estimate

$$|\varphi_2|_r < \left(\frac{\theta}{\theta_1}\right)^{f(\frac{\epsilon r \theta}{2})} < e^{f(\epsilon r)}$$

holds. From here and (17), (18) we get (16). Now, by estimation of Taylor cofficients of the function $\varphi(z)$ (cf (15)), we get

$$\limsup_{n \to \infty} |u_n|^{\frac{1}{n}} \leq \epsilon.$$

On account of arbitrariness of ϵ we get $(u_n) \in \bar{A}'_0$. Since the condition $\varphi(n) = t_n, n \in N$ holds by construction, Theorem is proved.

Corollary 7 *Let f(u) be rapidly increasing. Then $L_f(N,0)$ is diagonally isomorphic to the space $\bar{A}'_0(F_f,N)$, dual to GDS $\bar{A}'_0(F_f,N)$.*

4. In connection with Corollaries 4,7 the question aries: is any GDS of type A_∞ (a dual space to GDS of type A_0) isomorphic to some space $L_f(\lambda,\infty)$ (correspondingly, $L_f(\lambda,0)$).

Let $\phi(z)=\exp\varphi(z)$ be an entire function without zeros. Put $f(u)=|\varphi|_u, 0\le u<\infty$. Evidentl, $f(u)$ is increasing and logarithmically convex; it is rapidly increasing if and only if $\phi(z)$ is an entire function of infinite order. It is simple to show that for each $\theta>1$ a constant c exists such that

$$|\phi|_u \le \frac{1}{c}\exp f(\frac{u}{\theta}).$$

Therefore the estimate

$$|\phi|_u > \exp f(\frac{u}{\theta}) \tag{19}$$

holds for big enough u (for each $\theta>1$, if f is rapidly increasing, and for some $\theta>1$, otherwise). On the other hand

$$|\phi|_u \le \exp f(u) \tag{20}$$

for all u. Thus we get the following

Theorem 8 *The space $L_f(\lambda,\infty)$ with $f(u)=|\varphi|_u, 0<u\le\infty$ and $\lambda_n=|\alpha|_n, n\in N$ is diagonally isomorphic to GDS $A_\infty(\phi,\alpha)$. Therewith $A_\infty(\phi,\alpha)\in(I)_\infty$ iff $\phi(z)$ has a finite order.*

It is clear that the restriction on ϕ can be weakened, for example, as follows

$$\exists\theta>1, c>0|\frac{1}{c}f(\frac{u}{\theta})\le\ln|\phi|_u\le cf(u\theta), 0<u<\infty,$$

where $f(u)$ is some logarithmically convex function (in particular, Theorem remains true for entire functions of an order $\rho<1$).

Similarly can be proved the following

Theorem 9 *Let $\phi(z)$ have an infinite order. Then the space $\bar{A}'_0(\phi,\alpha)$, dual to GDS $\bar{A}'_0(\phi,\alpha)$, is diagonally isomorphic to the space $L_f(\lambda,0)$, where $f(u)=|\varphi|_u, 0<u<\infty$, and $\lambda_n=|\alpha_n|, n\in N$.*

As will be seen the restriction on the function ϕ is substantial.

5. Let (X,Y) be an ordered pair of locally convex spaces. We say that X and Y are essentially different (shortly, $(X,Y)\in R$) if each linear continuous operator $T:X\to Y$ is compact. In particular, $(X,Y)\in R$, if $X\in(I)_0$ and $Y\in(I)_\infty$ (Zahariuta, [10]). We use this fact to bring in some necessary supplements to Theorems 5,9.

Theorem 10 *Let a function $\phi(z)$ have a finite order. Then the system of functions $(\phi(\alpha_n z))_{n=1}^{\infty}$ cannot form a basis in the space $\bar{A}_0(\phi,\alpha)$ (in other words, $\bar{A}_0(\phi,\alpha)$ cannot be GDS).*

Proof. In fact, ad absurdum, assuming the opposite and taking into account (19), (20), we get that the dual space $\bar{A}_0'(\phi,\alpha)$ must be isomorphic to the space $L_f(\lambda,0)$ with $f(u)=|\varphi|_u, 0<u<\infty$, and $\lambda=|\alpha_n|, n\in N$. We have $L_f(\lambda,0)\in(I)_0$ since, under the conditions of Theorem, the function $f(u)$ is not rapidly increasing. On the other hand, $\bar{A}_0'\simeq L_f(N,\infty)$, where $f(u)\equiv u$. Thus $\bar{A}_0'(\phi,\alpha)\in(I)_0$, but $\bar{A}_0'\in(I)_\infty$, hence, by [10], we have

$$(\bar{A}_0'(\phi,\alpha),\bar{A}_0')\in R \tag{21}$$

The operator $J^\star$, conjugate to the identical embedding $J:\bar{A}_0'(\phi,\alpha)\to\bar{A}_0'$, is an endomorphism. But this contradicts to (21).

Taking into account results of [10], [4], we can sum up previous results as follows. □

Theorem 11 *(a) The classes $(f)_\infty((I)_\infty$ included) contain GDS of type A_∞; the classes $(f)_0\neq(I)_0$ contain strong dual to spaces GDS of type $\bar{A}_0$.*

(b) The space $\bar{A}_0'(\phi,\alpha)$, a strong dual to GDS $\bar{A}_0(\phi,\alpha)$, and the space $\bar{A}_0'$ (respectively, A_∞ and GDS $A_\infty(\phi,\alpha)$, where $\phi(z)$ has an infinite order) are essentially different. In particular, the space $\bar{A}_0(\phi,\alpha)(A_\infty)$ cannot be isomorphic to subspace of $\bar{A}_0'$ (respectively, $A_\infty(\phi,\alpha)$; the space $A_\infty(\phi,\alpha)(\bar{A}_0'$ cannot be isomorphic to quotient space of A_∞ (respectively, $\bar{A}_0'(\phi,\alpha)$).

(c) There exists an infinite set of GDS of type A_∞ (or strong dual to GDS of type $\bar{A}_0$), linearly ordered by relation $(X,Y)\in R$.

(d) Every space $L_f(\lambda,\infty)$ (respectively, $L_f(\lambda,0)$, non-belonging to $(I)_0$) contains a complemented subspace, isomorphic to some GDS of type A_∞ (respectively, some dual space to GDS of type $\bar{A}_0$).

Proof. It remains to prove (d) only. To do this, from the sequence $\lambda=(\lambda_n)$ we take a subsequence $\mu=(\mu_n)$ such that $\mu_{n+1}-\mu_n\leq 1$ and put $\nu=(\nu_n)$ where ν_n is the entire part of the number $\mu_n, n\in\mathbf{N}$. Then the space $L_f(\mu,\infty)$ can be realized as a topologically complemented subspace of $L_f(\lambda,\infty)$ and, obviously, it is isomorphic to the space $L_f(\nu,\infty)$. By Theorem 3 the sequence $(F_f(nz))$ constitutes a basis in the space $A_\infty(F_f,\mathbf{N})$; by Theorem 8, the space $L_f(\mathbf{N},\infty)$ is quasidiagonally isomorphic to the space $A_\infty(F_f,\mathbf{N})$; its restriction on $L_f(\nu,\infty)$ is an isomorphism of this space onto GDS $A_\infty(F_f,\nu)$. The second part of (d) relating to $L(\lambda,0)$ can be obtained by using of Theorems 5,9 (since $L_f(\lambda,\infty)$ does not belong to $(I)_0$ the function $f(u)$ is rapidly increasing).

In conclusion, consider the following three classes of spaces: (a) strong dual spaces to GDS of type $\bar{A}_0'$, (b) GDS of type A_∞, (c) closed subspaces of the space A_1, generated by non-closed systems of monomials in A_1. Let us recall that F-space X is called non-degenerated if it is isomorphic neither to the space ω of all sequences, nor to a Banach space, nor to their Cartesian product. It is known that each non-degenerated F-space X has a subspace from the class $(f)_\infty$ (Shaginian T. [4], see also [2],[6],[1]) and has a quotient space from the class $(f)_0$ (Ahonen H. [1]). Combining this facts with our considerations we get. □

Theorem 12 *Let X be a non-degenerated F-space and let (a), (b), (c) be the above defined classes. Then X has a subspace, belonging to (b) and X has a quotient space either from(a), or from (c).*

Acknowledgement

Author thanks V. Kashirin and V. Zahariuta for preparing of the English translation and the latter for realizing of the Latex-version of this article.

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GENELLEŞTİRİLMİŞ DİRİCKLET SERİLERİ ÜZERİNE

Özet

Sonsuza artarak yaklaşan bir $\{\alpha_n\}$ dizisi ve bir F tam fonksiyonu için $\{F(\alpha_n z)\}$ fonksiyon dizisinin birim diskteki analitik fonksiyonlar Frecket uzayında gerdiği kapalı alt uzaylar incelenmiş ve bu dizinin alt uzayları incelenmiş ve bu dizinin alt uzaya baz teşkil ettiği hallerde uzayın yapısıyla genel LF uzaylarının yapıları arasındaki ilişkiler araştırılmıştır.

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Received 18.7.1995