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$\overline{NC}$-p-GROUPS WITH NILPOTENT CENTRALIZERS

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Abstract

In this work a sufficient condition is given for an $\overline{NC}$-p-group to have an epimorphic image which is an $\overline{NF}$-p-group.

1. Introduction

A group G is called **locally graded** if every nontrivial finitely generated subgroup of G has a proper subgroup of finite index. Bruno in [5] called a locally graded group G a **minimal non nilpotent-by-finite group** (**$\overline{NF}$-group** for short) if every proper subgroup of G is **nilpotent-by-finite** (**NF-group)** but G itself does not have this property. She showed that if G is a non perfect $\overline{NF}$-group then $G/G' \cong C_{p^\infty}$ and G' is nilpotent. Later Otal and Peña in [13] defined $\overline{NC}$-groups by replacing "finite" by "Černikov" above and obtained similar properties for them. Recently these groups have also been studied by Arıkan and Asar in [1].

The existence of nonperfect $\overline{NF}$-p-groups has been known for a long time. The group constructed by Heineken and Mohamed in [9] is the first example of such a group. Later similar constructions have been given in [6], [7], [10] and [11]. So an important problem here is whether or not perfect $\overline{NF}$-p-groups and $\overline{NC}$-p-groups exist. $\overline{NC}$-p-group have been studied in [3] and [4] especially under the additional condition of "the normalizer condition". But the problem even under the above additional condition still remains open. In the present work we have considered $\overline{NC}$-p-groups in which every proper centralizer is nilpotent.

The main result of this work is the following theorem.

Theorem. *Let G be an $\overline{NC}$-p-group such that the following conditions hold:*

(i) For each normal nilpotent subgroup K of G and for every noncentral element xK of G/K, $C_{G/K}(xK)$ is nilpotent.

(ii) $E^G < G$, for every proper subgroup E of G of finite exponent.

Then G has an epimorphic image which is an $\overline{NF}$-group.

2. Some General Properties of Locally Nilpotent p-Groups

Heineken and Mohamed obtained the following important property on p. 370 of [9]:

$$\text{“}(H')^{m^{n-1}} \leq C_H(x)\text{”}$$

An easy generalization of this property has been given in Lemma 2.1 of [3] and a slightly more general form of this lemma is Lemma 2.2 of [4] which is stated below for convenience.

Lemma 2.1 *Let H be a normal subgroup and a be an element of a group G such that $[H,a] \leq Z(H)$. Let m,n be positive integers such that $a^m \in C_G(H)$ and $< a >$ has subnormal index n in $H < a >$. Then*

$$H^{m^n} \leq C_G(a)$$

Proof. See Lemma 2.1 of [4]. □

Lemma 2.2. *Let H be a perfect locally nilpotent p-group such that $Z(H) = 1$. Let N be a normal nilpotent subgroup of H such that $Z(N)$ has infinite exponent. Let E be a proper subgroup of H such that EN is nilpotent and EN/N has finite exponent. Then $E^H < H$.*

Proof. Let $Z = Z(N)$. let $c \geq 0$ be the nilpotency class of EN and m be the exponent of EN/N. Then for any $a \in E, < a >$ has subnormal index at most c in EN which implies by Lemma 2.1 that

$$Z^{m^c} \leq C_H(a)$$

for all $a \in E$ and thus

$$E \leq C_H(Z^{m^c})$$

But since $Z^{m^c} \neq 1$ and normal in H and $Z(H) = 1$ it follows that $E^H < H$, which was to be shown. □

3. Centralizers of Elements in $\overline{NC}$-p-Groups

Lemma 3.1 *Let G be an $\overline{NC}$-p-group and let $x \in G\backslash Z(G)$ such that $C_G(x)$ is nilpotent. Then $C_{G/Z(G)}(xZ(G))$ is also nilpotent.*

Proof. Let $Z = Z(G)$. Then $Z(G/Z) = 1$ since G is perfect by Theorem A (i) of [3]. Let $U/Z = C_{G/Z}(xZ)$. Then $U/Z \neq G/Z$ since $x \notin Z$ and $Z(G/Z) = 1$. We claim that U is nilpotent. Let K be the kernel of the homomorphism given by $u \to [u, x]$, for all $u \in U$. Then $K = C_G(x)$, so K is nilpotent by hypothesis and normal in U. Also $U/K \cong [U, x] \leq Z$. Let $o(x) = m$. Then for any $u \in U$

$$1 = [u, x^m] = [u, x]^m$$

which implies that $U/K \cong [U, x]$ has finite exponent at most m.

Furthermore as $U \neq G, U$ is an NC-group. Therefore is has a normal nilpotent subgroup Y such that U/Y is Černikov. Clearly then U/KY is finite since it is Černikov and has finite exponent. Thus U has a finite subgroup F such that $U = F(KY)$. Now KY is a normal nilpotent subgroup of U and F^G is nilpotent by Lemma 2.2 of [4]. Therefore U is nilpotent since $U = (U \cap F^G)KY$ which was to be shown. □

Lemma 3.2 *Let G be an $\overline{NC}$-p-group such that $C_G(x)$ is nilpotent for every $x \in G \setminus Z(G)$. Then $G/Z(G)$ has no nontrivial radicable abelian subgroups.*

Proof. Let $D/Z(G)$ be a radicable abelian subgroup of $G/Z(G)$. First suppose that $Z(G) = 1$. Let N be a normal nilpotent subgroup G and let $1 \neq x \in D$. Then $< x > N$ is nilpotent by (1) Lemma of [12] since $< x >$ is subnormal in G by Theorem A (i) of [3]. Let $Y = Z(< x > N) \cap N$. Then $Y \neq 1$ and $YD \leq C_G(x)$ which is nilpotent and so D centralizes $C_G)(x)$ by Lemma 2.2 (iii) of [2]. Now since $DN \leq C_G(Y)$ which is nilpotent, it follows that D centralizes N. Clearly then D centralizes G by Theorem A(i) of [3] and so $D = 1$ since $Z(G) = 1$.

Next suppose that $Z(G) \neq 1$. Put $\overline{G} = G/Z(G)$. Then $Z(\overline{G}) = 1$ as before and $\overline{G}$ satisfies the hypothesis by Lemma 3.1. Therefore $\overline{G}$ has no nontrivial radicable abelian subgroups by the first case of the proof. □

4. HM^*-Groups

HM^*-groups have been introduced in [3] as a generalization of Heineken-Mohamed groups constructed in [7], [9] and [10]. They play a central role in the study of $\overline{NC}$-p-groups. Many properties of these groups have been obtained in [3] and [4].

Definition 4.1 *$X \neq 1$ be a locally nilpotent p-group. If*

(i) X' is nilpotent

and

(ii) $X/X' \cong C_{p^\infty} \times \cdots \times C_{p^\infty} = C_{p^\infty}^{(n)}$ for some $n \geq 1$

then X is called an **HM*-group**. *If $n = 1, X' \neq 1$ and every proper subgroup of X is subnormal then X is called* **a group of Heineken-Mohamed type (HM-group** *for short). Note that in an HM-group every proper subgroup is nilpotent by (1) Lemma of [12] and Lemma 4.2 (iii) below. Some of the elementary properties of HM^*-group are contained in the following Lemma.*

Lemma 4.2 *Let X be an HM^*-group for a prime p. Then the following hold.*

(i) $X' = [X, X']$

(ii) There does not exist any proper normal subgroup N of X satisfying $X = X'N$

(iii) If X satisfies the normalizer condition, then there does not exist any proper subgroup Y of X satisfying $X = X'Y$.

Proof. See Lemma 3.1 of [4]

For the existence of an HM^*-group in an NC-p-group see Lemma 3.3 of [3]. Here we obtain further properties of these groups. But first the following is needed. □

Lemma 4.3 *Let H be a nilpotent group such that $H/Z(H)$ has finite exponent. Then H' has finite exponent.*

Proof. Let $c \geq 1$ be the nilpotency class of H. We may use induction on c. If $c = 1$ then H is abelian and the assertion is trivial So suppose that $c > 1$ and the assertion is true for $c - 1$.

It is well-known that

$$K_c(H) = < [x_1, \ldots, x_c] : x_1, \ldots, x_c \in H >$$

and also $K_c(H) \leq Z(H)$. Let m be the exponent of $H/Z(H)$ and let $t = [x_1, \ldots, x_c]$. Since $t \in Z(H)$, it is easy to see that

$$\begin{aligned} t^m &= [x_1, \ldots, x_c]^m \\ &= [x_1, \ldots, x_c^m] \\ &= 1 \end{aligned}$$

which implies that $K_c(H)$ has finite exponent since it is abelian. Let $\overline{H} = H/K_c(H)$. Then $\overline{H}$ has nilpotency class $c - 1$ and so

$$(H/K_c(H))' = H'K_c(H)/K_c(H)$$

has finite exponent by induction hypothesis then also H' has the same property since $K_c(H)$ does. □

Lemma 4.4 *Let T be a locally nilpotent p-group such that $T/Z(T)$ is HM^*-group. If $(T/Z(T))'$ has finite exponent, then T' has finite exponent.*

Proof. Let $Z = Z(T)$ and put $H = T'Z$. By hypothesis $H/Z(H)$ has finite exponent which implies that H' has finite exponent by Lemma 4.3. In particular then T'' has finite exponent. So without loss of generality we may suppose that $T'' = 1$ and thus suppose that T' is abelian

Let $T'Z/Z$ have exponent m and let $a \in T'$. Since

$$1 = [a^m, t] = [a, t]^m$$

for all $t \in T$, it follows that $[a, T]$ has finite exponent. Consequently $[T', T]$ has finite exponent since a is any element of T'. Now since T/Z is a HM^*-group, Lemma 4.2 (i) gives that

$$(T'Z)/Z = [T', T]Z/Z$$

whence

$$T'Z = [T', T]Z$$

and hence

$$T' = [T', T](T' \cap Z).$$

Now let $\overline{T} = T/[T', T]$. Since $\overline{T}' \leq \overline{Z}$, $\overline{T}/\overline{T}'\overline{Z} = \overline{T}/\overline{Z}$ is radicable abelian and this implies that $\overline{T}$ is abelian and so $\overline{T}' = 1$, that is $T' \leq [T', T]$ which implies that T' has finite exponent. □

Lemma 4.5 *Let T be a locally nilpotent p-group and N be a normal nilpotent subgroup of T such that T/N is an HM^*-group. Let K be a normal subgroup of T contained in N such that N/K has finite exponent. If W/K is the unique maximal HM^*-subgroup of T/K such that T/W has finite exponent, then*

$$(T/K)' = (W'N')K/K.$$

Proof. Let $\overline{T} = T/K$. Since $\overline{T}/\overline{W}$ has finite exponent, the group

$$\left(\frac{\overline{T}/\overline{N}}{(\overline{T}/\overline{N})'}\right)/\frac{(\overline{WN}/\overline{N})(\overline{T}/\overline{N})'}{(\overline{T}/\overline{N})'}$$

is radicable abelian and has finite exponent, which is possible only if

$$\frac{\overline{T}/\overline{N}}{(\overline{T}/\overline{N})'} = \frac{(\overline{WN}/\overline{N})(\overline{T}/\overline{N})'}{(\overline{T}/\overline{N})'}$$

which gives that

$$\overline{T}/\overline{N} = (\overline{WN}/\overline{N})(\overline{T}/\overline{N})'.$$

So applying Lemma 4.2 (ii) we get that

$$\overline{T}/\overline{N} = \overline{WN}/\overline{N}$$

which gives that $\overline{T} = \overline{WN}$.

Now the group $\overline{T}/\overline{W}' = (\overline{W}/\overline{W}')(\overline{NW}'/\overline{W}')$ is nilpotent since each factor on the right side is nilpotent and normal. In particular then $\overline{W}/\overline{W}' \leq Z(\overline{T}/\overline{W}')$ by Lemma 2.2 (iii) of [2]. Therefore $[\overline{N},\overline{W}] \leq \overline{W}'$ and hence

$$\begin{aligned}\overline{T}'/\overline{W}' = (\overline{T}/\overline{W}')' = (\overline{NW}/\overline{W}')' &= \overline{N}'\overline{W}'[\overline{N},\overline{W}]/\overline{W}' \\ &= \overline{N}'\overline{W}'/\overline{W}'\end{aligned}$$

which gives that

$$\overline{T}' = \overline{N}'\overline{W}'$$

which was to be shown. □

Lemma 4.6. *Let H be $\overline{NC}$-p-group and let N be a normal nilpotent subgroup of H satisfying Theorem A(i) of [3]. Suppose that for every proper subgroup E of finite exponent of H one has $E^H < H$. Then the following holds:*
If T/N is an HM^*-subgroup of H/N, then $T'^H < H$.

Proof. Let T/N be an HM^*-group of H/N. It follows from the choice of N and Lemma 3.2 of [4] that $(T/N)' = T'N/N$ has finite exponent.

Let $Z = Z(H)$. Without loss of generality $Z \leq N$. First suppose that $Z = 1$. If $Z(N)$ has infinite exponent, then $T'^H < H$ by Lemma 2.2. So suppose that $Z(N)$ has finite exponent. Then N must have finite exponent by Theorem 2.23 of [14] which implies that T' has finite exponent since $T'N/N$ does, so $T'^H < H$ by hypothesis.

Now suppose that $Z \neq 1$. Clearly $Z(H/Z) = 1$ since H is perfect by Theorem A of [3], so if $Z(N/Z)$ is infinite then $(T'Z/Z)^{H/Z} < H/Z$ by the first case of the proof which implies that $T'^H < H$. Therefore we may suppose that $Z(N/Z)$ and hance also N/Z has finite exponent. In particular then $(T'N)/Z$ has finite exponent and also $T/T'N$ is Černikov. Therefore T/Z has a unique maximal HM^*- subgroup V/Z such that T/V has finite exponent by Lemma 3.3 of [3].

Now applying Lemma 4.5 we get

$$(T/Z)' = (V/Z)'(N/Z)'$$

and hence

$$T' = V'N'(T \cap Z).$$

We see from this that $T'^H < H$ if $V'^H < H$. But since $(V/Z)'$ has finite exponent $(V'Z/Z \leq T'N/Z)$, applying Lemma 4.4 gives that V' has finite exponent and hence $V'^H < H$ by hypothesis. Clearly it follows from this that $T'^H < H$ which was to be shown. □

Lemma 4.7 *Let G be an $\overline{NC}$-p-group and N be a normal nilpotent subgroup of G given in Theorem A(i) of [3]. Let H/N be the subgroup of G/N generated by all the HM^*-subgroups of G/N. Then either $H = G$ or H contains a normal nilpotent subgroup K of G such that G/K is an $\overline{NF}$-group.*

Proof. Assume that $H \neq G$. Clearly H is normal in G. First we show that G/H is an $\overline{NF}$-group. Assume not. Then G/H contains a proper subgroup X/H which is not an NF-group. Then X/N contains a unique maximal HM^*-subgroup T/N such that X/T has finite exponent by Lemma 3.3 of [3]. But since $T/N \leq H/N$, X/H must have finite exponent and so it must be nilpotent, since it has a normal nilpotent subgroup of finite index and finite subgroups are subnormal. This is a contradiction and so it follows that G/H is $\overline{NF}$-group.

Next as H is an NC-group it contains a normal nilpotent subgroup K such that H/K is Černikov. By Lemma 4.7(i) of [8] we may assume that K is normal in G. Let $\overline{G} = G/K$. By Theorem 3.29 of [14]

$$\overline{G}/C_{\overline{G}}(\overline{H})$$

must be Černikov since $\overline{H}$ is Černikov. But since G is perfect, this gives that $\overline{G} = C_{\overline{G}}(\overline{H})$ and hence $\overline{H} \leq Z(\overline{G})$, since two normal NC-subgroup of $\overline{G}$ generate and NC-subgroup of $\overline{G}$. Clearly now it is easy to see from this that every proper subgroup of $\overline{G}$ is an NF-group which was to be shown. □

Lemma 4.8 *Let G be an $\overline{NC}$-p-group. Suppose that*

$$G = \bigcup_{i=1}^{\infty} T_i',$$

where for each $i \geq 1$, T_i is a normal HM^*-subgroup of G such taht $T_i' \leq T_{i+1}'$. Then G contains a normal nilpotent subgroup K such that $K \leq T_i'$ for some $i \geq 1$ and G/K contains a noncentral element having a nonnilpotent centralizer.

Proof. Clearly $G = G'$ by Theorem A of [3]. Let $k \geq 1$ be the smallest integer such that $T_k' \neq 1$ and put $S = T_k$. Since S is normal in G, S/S' is a radicable abelian and Černikov subgroup of G/S' which implies that $S/S' \leq Z(G/S')$ by Theorem 3.29 (2) of [14] since G is perfect. Thus $[S, G] \leq S'$.

Choose $i > k$ and put $T = T_i$. First suppose that S' is elemartary abelian. Let $D = [S', T']$ and put $\overline{G} = G/D$. Then $[\overline{S}', \overline{T}'] = 1$ and also $[\overline{S}, \overline{G}] \leq \overline{S}', [\overline{T}, \overline{G}] \leq \overline{T}'$ as was shown above. Choose $s \in S$ and $u \in T'$. Then

$$[\bar{s}, \bar{u}^p] = [\bar{s}, \bar{u}]^p = 1$$

since $[\bar{s}, \bar{u}] \in \overline{S}'$ which is elementary abelian. Hence it follows that

$$[\overline{S}, (\overline{T}')^p] = 1,$$

since s and u are chosen orbitrarily.

Furthermore it follows from Lemma 4.2 (ii) that

$$G^p = \bigcup_{i=1}^{\infty} (T_i')^p = G,$$

which allows us to choose $i > k$ such that $(\overline{T}_i')^p \not\leq Z(\overline{G})$. Now if $\bar{t} \in (\overline{T}_i')^p \setminus Z(\overline{G})$, then it follows from above that

$$\overline{S} \leq C_{\overline{G}}(\bar{t}) < \overline{G}$$

and $\overline{S}$ is not nilpotent since $\overline{S}' \neq 1$. Also $D = [S', T'] \leq S'$ which is nilpotent.

Next let $\overline{G} = G/S''(S')^p$. Then $\overline{S}'$ is elementary abelian. First we show that $\overline{S}' \neq 1$. Suppose if possible that $S' = S''(S')^p$. Then $S' = S'^p$ by Theorem A of [3] and so S'/S'' is radicable abelian which gives that S' radicable abelian by Theorem 9.23 of [14]. Moreover any finite extension of S' in S is nilpotent since finite subgroups of S are subnormal which implies that $S' \leq Z(S)$ by Lemma 2.2 (iii) of [2] and hence $S' = 1$ which is a contradiction. Now, as in the first case, we can find an $i > k$ such that if $\overline{D} = [\overline{S}, \overline{T}_i']$, then $\overline{G}/\overline{D}$ contains a noncentral element having a nonnilpotent centralizer. Also $D = [S', T']S''(S')^p \leq S'$ which is nilpotent. Thus in both cases we may let $K = D$, since $D \leq S'$ which is nilpotent. □

5. Proof of The Theorem

Proof of the Theorem. Assume that the assertion is false. By Theorem A of [3] $G = G'$. Let N be a normal nilpotent subgroup of G given in Theorem A(i) of [3].

First we show that HM^*-subgroups exist in G/N. Since G/N is not an $\overline{NF}$-group, it has a proper subgroup X/N which is not an NF-group. By Theorem A(i) of [3] X contains a normal nilpotent subgroup Y such that X/Y is infinite Černikov and YN/N has finite exponent. Therefore X/N contains a unique maximal HM^*-subgroup W/N such that X/W has finite exponent by Lemma 3.3 of [3].

Next we show that every HM^*-subgroup of G/N is contained in a normal HM^*-subgroup of G/N. So let T/N be an HM^*-subgroup of G/N. Let $L = T'^G N$ and put $\overline{G} = G/L$. Then $\overline{G} \neq 1$ by hypothesis and Lemma 4.6 and $\overline{T}$ is radicable abelian. Moreover L is nilpotent. To see this note that $T'N/N \leq YN/N$ by Lemma 3.2 of [4] which implies that $T'N/N$ has finite exponent. Therefore L/N is nilpotent of finite exponent by Lemma 2.2 of [4]. But since L is an NC-group this gives that L is nilpotent. Consequently $\overline{T} \leq Z(\overline{G})$ by Lemma 3.2. Thus in particular LT is normal in G.

Let W/N be the unique maximal HM^*-subgroup of LT/N which exists by Lemma 3.3 of [3]. Clearly then $T/N \leq W/N$ and W/N is normal in G/N by its uniqueness.

Now let P be the set of all the normal HM^*-subgroups of G/N. Put

$$H/N = < T/N : T/N \in P >$$

Then H is a normal subgroup of G such that every HM^*-subgroup of G/N is contained in H/N. If $H \neq G$, then we get a contradiction by Lemma 4.7 and our assumption. So suppose that $H = G$.

Now put $\overline{G} = G/N$. Assume that there exists an infinite properly ascending chain

$$\overline{T}_1 < \overline{T}_2 < \cdots < \overline{T}_n < \cdots \tag{1}$$

of elements of P. Let

$$\overline{T} = \bigcup_{n=1}^{\infty} \overline{T}_n.$$

First suppose that $\overline{T} = \overline{G}$. Then

$$\overline{G} = \bigcup_{n=1}^{\infty} \overline{T}'_n$$

since $\overline{G} = \overline{G}'$. Also $\overline{T}'_n \leq \overline{T}'_{n+1}$ for all $n \geq 1$ by Lemma 3.2 of [4]. Therefore $\overline{G}$ contains a normal nilpotent subgroup $\overline{K}$ such that $\overline{G}/\overline{K}$ contains a noncentral element having a nonilpotent centralizer and $\overline{K} \leq \overline{T}'_n$ for some $n \geq 1$ by Lemma 4.8. But also it can be shown as in the case of L above that $T'_n N$ is nilpotent, which contradicts (i) of the hypothesis. Consequently it follows that $\overline{T} \neq \overline{G}$.

Now since $\overline{T}$ is an NC-group by the preceding paragraph, the definiton of N and Lemma 3.3 of [3] ensures that $\overline{T}$ has a unique maximal HM^*-subgroup $\overline{W}$ which implies that $\overline{T} = \overline{W}$, that is $\overline{T}$ is an HM^*-group. Now $\overline{T}_n\overline{T}'/\overline{T}'$ is radicable abelian and

$$\overline{T}_n\overline{T}'/\overline{T}' \leq \overline{T}_{n+1}\overline{T}'/\overline{T}'$$

for all $n \geq 1$. But since $\overline{T}/\overline{T}'$ is radicable abelian and Černikov there must exist an $n \geq 1$ such that

$$\overline{T}_n\overline{T}'/\overline{T}' = \overline{T}/\overline{T}'$$

and hence

$$\overline{T} = \overline{T}_n\overline{T}' = \overline{T}_n$$

by Lemma 4.2 (ii) which constradicts (1). Therefore any chain of the form (1) must be finite which means that any element of P is contained in a maximal element of P.

Since $\overline{G} = \overline{H}, \overline{G}$ is generated by the elements of P which implies that P contains at least two distinct maximal elements $\overline{U}$ and $\overline{V}$. Let $\overline{Y} = \overline{UV}$. Since $\overline{Y} \neq \overline{G}$ it contains a unique maximal HM^*-subgroup $\overline{R}$ by Lemma 3.3 of [3]. Also $\overline{R}$ is normal in $\overline{G}$ since $\overline{Y}$ is normal in $\overline{G}$ and so $\overline{R} \in P$. But since, $\overline{U}, \overline{V} \leq \overline{R}$, it follows that $\overline{U} = \overline{V}$ which is a contradiction.

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Nilpotent Merkezleyenli $\overline{NC}$-p-Grupları

Özet

Bu çalışmada bir $\overline{NC}$-p-grubunun bir epimorfik görüntüsünün $\overline{NF}$-grubu olması için bir yeter şart verilmiştir.

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