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NEAR ULTRAFILTERS AND $\mathcal{LUC}$-COMPACTIFICATION OF REAL NUMBERS

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Abstract

In this work we will investigate some of the topological properties of the $\mathcal{LUC}$-compactification of real numbers $\mathbf{R}$ in terms of the concept of near ultrafilters.

1. Introduction

By a *compactification* of the topological space $\mathbf{R}$, we shall mean a compact Hausdorff space K with an embedding $e : \mathbf{R} \to K$ with $e[\mathbf{R}]$ is dense in K. We will usually identify $\mathbf{R}$ with $e(\mathbf{R})$ and consider $\mathbf{R}$ as a subspace.

The topological space $\mathbf{R}$ has a compactification $\widetilde{\mathbf{R}}$ with the property that $C(\widetilde{\mathbf{R}})$ is isomorphic to the algebra $\mathcal{LUC}(\mathbf{R})$ of bounded real-valued uniformly continuous functions defined on $\mathbf{R}$. $\widetilde{\mathbf{R}}$ is the spectrum of $\mathcal{LUC}(\mathbf{R})$ furnished with the Gelfand topology (i.e., weak topology from $\mathcal{LUC}(\mathbf{R})^*$) (see [1]). As is well known, this compactification has the property that a bounded continuous function f from $\mathbf{R}$ to $\mathbf{R}$ has a continuous extension $\widetilde{f} : \widetilde{\mathbf{R}} \to \mathbf{R}$ if and only if f is uniformly continuous (see [2]).

The compactification $\widetilde{\mathbf{R}}$ was constructed in terms of the concept of near ultrafilters (see [4]). We shall say that a subset η of $\mathcal{P}(\mathbf{R})$ has the near finite intersection property if η is non-empty and if, for every finite subset F of η and every $W \in \mathcal{B}$, $\bigcap_{Y \in F} (W + Y) \neq \emptyset$. We say that η is a near ultrafilter if η is maximal subject to being a subset of $\mathcal{P}(\mathbf{R})$ with the near finite intersection property. It is clear that every ultrafilter on $\mathbf{R}$ is a near ultrafilter. We take $\widetilde{\mathbf{R}}$ to be set of near ultrafilters on $\mathbf{R}$. For each $Y \subseteq \mathbf{R}$, let $C_Y = \{\eta \in \widetilde{\mathbf{R}} : Y \in \eta\}$. Then $\widetilde{\mathbf{R}}$ is made into a topological space by taking the family of all sets C_Y as a base for the closed sets. With this topology $\widetilde{\mathbf{R}}$ is a compact Hausdorff space and the mapping $e : \mathbf{R} \to \widetilde{\mathbf{R}}$ is defined by $e(x) = \{Y \subseteq \mathbf{R} : x \in \overline{Y}\}$ for each $x \in \mathbf{R}$ is an embedding with $e(\mathbf{R})$ dense in $\widetilde{\mathbf{R}}$ (see [4]). We identify a subset Y of $\mathbf{R}$ with $e(Y)$.

If $\mathbf{X}$ is a topological space, $\beta\mathbf{X}$ will denote the Stone-Čech compactification of $\mathbf{X}$ and $\mathbf{X}^*$ will denote the growth $\beta\mathbf{X}\backslash\mathbf{X}$. $\mathcal{B}$ will denote the set of all neighborhoods of 0 in $\mathbf{R}$. $\mathbf{R}^+$, $\mathbf{R}^-$ denote the set of nonnegative reals and nonpossitive reals, respectively. $\gamma\mathbf{R}$ will denote $\widetilde{\mathbf{R}}\backslash\mathbf{R}$.

We give some of the properties of near ultrafilters that we will use in this paper. More details about near ultrafilters can be found in [4],[2].

1.1. Some Properties of Near Ultrafilters

Let ξ be a near ultrafilter,

1) If F be a finite subset of ξ, then $\bigcap_{Y\in F}(W+Y)\in\xi$ for all $W\in\mathcal{B}$.

2) $Y\in\xi$ if and only if $(W+Y)\bigcap Z\neq\emptyset$ for every $Z\in\xi$ and every $W\in\mathcal{B}$ if and only if $Y\bigcap(Z+W)\neq\emptyset$ for every $Z\in\xi$ and every $W\in\mathcal{B}$.

3) $Y\in\xi$ if and only if $(Y+W)\in\xi$ for every $W\in\mathcal{B}$. Furthermore, this is the case if and only if $\mathrm{cl}_{\mathbf{R}}Y\in\xi$.

4) If $Y_1,Y_2\subseteq\mathbf{R}$, then $Y_1\bigcup Y_2\in\xi$ if and only if $Y_1\in\xi$ or $Y_2\in\xi$.

5) $Y\in\xi$ if and only if $\xi\in\mathrm{cl}_{\widetilde{\mathbf{R}}}Y$.

2. Some Properties of the Space $\widetilde{\mathbf{R}}$

Lemma 2.1. Let $\xi\in\widetilde{\mathbf{R}}$. For any $X\in\xi$ and any $W\in\mathcal{B}$, the set C_{X+W} is a neighborhood of ξ, and the sets of this form provide a basis for the neighborhoods of ξ.

Proof. Let $X\in\xi$ and $W\in\mathcal{B}$ and let $(X+W)^*=\mathbf{R}\backslash(X+W)$ and $C'_y=\widetilde{\mathbf{R}}\backslash C_y$ for a subset y of $\mathbf{R}$. Then it is clear that $\xi\in C'_{(X+W)^*}$ and that $C'_{(X+W)^*}\subseteq C_{(X+W)}$. Therefore, $C_{(X+W)}$ is a neighborhood of ξ.

Now suppose that $Y\subseteq\mathbf{R}$ and that $Y\notin\xi$. Then $Y\cap(W+X)=\emptyset$ for some $X\in\xi$ and some $W\in\mathcal{B}$. Let $W_1\in\mathcal{B}$ symmetric and $W_1+W_1\subseteq W$. Then clearly $C_{(X+W_1)}\subseteq\widetilde{\mathbf{R}}\backslash C_Y$ since $(W_1+X)\cap(W_1+Y)=\emptyset$. □

We remind that a topological space X is called an F-space if for each $f\in C(X)$, the sets $Negf=\{x\in X: f(x)<0\}$ and $Posf=\{x\in X: f(x)>0\}$ are completely separated, that is, there exists a mapping $h\in C(X)$ such that $h(x)=0$ if $x\in Posf$ and $h(x)=1$ if $x\in Negf$.

Theorem 2.1 $\gamma\mathbf{R}$ *is not an F-space.*

Proof. Let $f(x)=\sin x$, and let $\xi\in\gamma\mathbf{R}$ such that $\xi\in\mathbf{cl}_{\widetilde{\mathbf{R}}}\{2n\pi\}_{n\in\mathbf{N}}$. Because of the fact that f is uniformly continuous it extends to a continuous function $\widetilde{f}$ from $\widetilde{\mathbf{R}}$ to $\widetilde{\mathbf{R}}$. Clearly, any neighborhood of ξ contains a point $\eta\in\mathbf{cl}_{\widetilde{\mathbf{R}}}\{2n\pi+\delta\}_{n\in\mathbf{N}}\backslash\mathbf{R}$ and a point $\zeta\in\mathbf{cl}_{\widetilde{\mathbf{R}}}\{2n\pi-\delta\}_{n\in\mathbf{N}}\backslash\mathbf{R}$ for some $\delta\in]0,\pi[$. Since $\widetilde{f}(\eta)>0$ and $\widetilde{f}(\zeta)<0$, $\xi\in\mathbf{cl}_{\widetilde{\mathbf{R}}}\{\mu\in\gamma\mathbf{R}: \widetilde{f}(\mu)>0\}\cap\mathbf{cl}_{\widetilde{\mathbf{R}}}\{\mu\in\gamma\mathbf{R}: \widetilde{f}(\mu)<0\}$ which is a contradiction. □

Theorem 2.2 *Let* $\xi \in \mathbf{cl}_{\widetilde{\mathbf{R}}} \mathbf{R}^+ \setminus \mathbf{R}$ *and* $Y \in \xi$. *Then for any* $k > 0$, *there is a sequence* $(y_r) \subseteq Y$ *such that* $y_{r+1} - y_r > k$ *for every* $r \in \mathbf{N}$ *and* $\{y_r : r \in \mathbf{N}\} \in \xi$.

Proof. Let $m \in \mathbf{N}$ with $m > k$. Then, either

$$\bigcup_{n \in 2\mathbf{N}-1} [nm, (n+1)m] \in \xi \quad \text{or} \quad \bigcup_{n \in 2\mathbf{N}} [nm, (n+1)m] \in \xi.$$

Suppose that $X_1 = \bigcup_{n \in 2\mathbf{N}-1} [nm, (n+1)m] \in \xi$, and that A denote $2\mathbf{N}-1$, then either

$$\bigcup_{n \in A} [nm, (n+\tfrac{1}{2})m] \in \xi \text{ or } \bigcup_{n \in A} [(n+\tfrac{1}{2})m, (n+1)m] \in \xi.$$

Suppose that $X_2 = \bigcup_{n \in A} [nm, (n+\frac{1}{2})m] \in \xi$. If we proceede in this way, we can define a sequence of sets (X_n) with the following properties:

i) $X_n \in \xi$;

ii) each X_n can be written as $\bigcup_{r=1}^{\infty} I_{n.r}$, where $I_{n,r}$ is a closed interval of length $\frac{m}{2^{n-1}}$;

iii) for each n, $d(I_{n,r}, I_{n,r'}) > m$ if $r \neq r'$;

iv) for each n and r, $I_{(n+1),r} \subseteq I_{n,r}$;

v) for each $r = 1, 2, 3, \ldots$, there will be a unique point $x_r \in \bigcap_{n=1}^{\infty} I_{n,r}$.

Let $X = \{x_r : r \in \mathbf{N}\}$. It is clear that $x_r < x_{r+1}$ holds for each $r \in \mathbf{N}$. We claim that $X \in \xi$. To see this, let $Z \in \xi$ and let $\epsilon > 0$. Choose $n \in \mathbf{N}$ so that $\frac{m}{2^{n-1}} < \frac{\epsilon}{2}$. Since $X_n \in \xi$, there will be a point $x \in X_n$ such that $d(x, Z) < \frac{\epsilon}{2}$. If $x \in I_{n,r}$, then $d(x_r, x) < \frac{\epsilon}{2}$. Hence, $d(x_r, Z) < \epsilon$. Thus, $(X + (-\epsilon, \epsilon)) \cap Z \neq \emptyset$ and so $X \in \xi$.

Let $\delta \in \mathbf{R}$ and $0 < \delta < \frac{1}{5}$. Then

$$V = \{x_r : d(x_r, Y) < \delta\} \in \xi.$$

For otherwise we should have

$$V' = \{x_r : d(x_r, Y) \geq \delta\} \in \xi.$$

This is impossible since $((-\frac{\delta}{8}, \frac{\delta}{8}) + V') \bigcap Y = \emptyset$. Now, since the finite set $\{x_r : r \leq \frac{1}{\delta}\} \notin \xi$,

$$X_\delta = \{x_r : x_r \in V, \frac{1}{r} < \delta\} \in \xi.$$

Now for each r, choose $y_r \in Y$ with $d(x_r, y_r) < d(x_r, Y) + \frac{1}{r}$. We shall show that

$$Y_\delta = \{y_r : x_r \in X_\delta\} \in \xi.$$

Let $0 < \epsilon < \delta$. Then if $x_r \in X_\epsilon$, we have that $y_r \in Y_\delta$ and $d(y_r, x_r) < 2\delta$. Thus, $X_\epsilon \subseteq (Y_\delta + (-2\delta, 2\delta))$ and so $(Y_\delta + (-2\delta, 2\delta)) \in \xi$ and that $Y_\delta \in \xi$. Since $x_{r+1} - x_r > m$, $d(y_r, x_r) < 2\delta < \frac{m-k}{2}$ and $d(y_{r+1}, x_{r+1}) < \frac{m-k}{2}$, we have $y_{r+1} - y_r > k$. □

Remark 2.1 It is quite easy to prove that if $\xi \in \mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^- \setminus \mathbf{R}$ and $Y \in \xi$, then for any $k > 0$ there is a sequence $(y_r) \subseteq Y$ such that $y_r - y_{r-1} > k$ for every $r \in \mathbf{N}$, and $\{y_r : r \in \mathbf{N}\} \in \xi$.

A point of $\widetilde{\mathbf{R}}$ is called a remote point if it does not belong to the closure of any discrete subspace of $\mathbf{R}$. As a consequence of Theorem 2.2, $\widetilde{\mathbf{R}}$ has no remote points, but under the continuum hypothesis the set of remote points of $\beta\mathbf{R}$ is dense in $\mathbf{R}^*$ (see[5]).

Theorem 2.3 *If $\xi \in \mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^+ \setminus \mathbf{R}$, then every neighborhood of ξ contains a topological copy of $\beta\mathbf{N}\setminus\mathbf{N}$.*

Proof. We first note that for each $Y \in \xi$ and $W \in \mathcal{B}$

$$C_{(Y+W)} = \{\eta \in \widetilde{\mathbf{R}} : Y + W \in \eta\}$$

is a neighborhood of ξ, and $\{\ C_{(Y+W)} : Y \in \xi,\ W \in \mathcal{B}\}$ forms a base for the neighborhoods system of ξ.

Now let G be a neighborhood of ξ. Then there exists $Y \in \xi$ and $W \in \mathcal{B}$ satisfying $C_{(Y+W)} \subseteq G$. There is a sequence $(x_n) \subseteq Y$ with $x_{n+1} - x_n \geq 1$ by Theorem 2.2. Hence, $C_{((x_n)+W)}$ is a neighborhood of ξ which is contained in G. Let $H = C_{((x_n)+W)}$. It is easy to see that $H \supseteq \mathbf{cl}_{\widetilde{\mathbf{R}}}(x_n)\setminus(x_n)$.

Now we define a mapping h from $\mathbf{N}$ to $\mathbf{R}$ such that $h(n) = x_n$. Clearly, the mapping h is continuous and it extends to a continuous mapping h^β from $\beta\mathbf{N}$ onto $\mathbf{cl}_{\widetilde{\mathbf{R}}}(x_n)$. Let ξ and η be two distinct points in $\beta\mathbf{N}$. Then there will be $U \in \eta$ and $V \in \xi$ satisfying $U \cap V = \emptyset$ and so $h^\beta(U) \cap h^\beta(V) = \emptyset$ because of the fact that h is one to one on $\mathbf{N}$. Let W be the interval $(-\frac{1}{3}, \frac{1}{3})$. Clearly, $(h^\beta(U) + W) \cap (h^\beta(V) + W) = \emptyset$ since $|h^\beta(n) - h^\beta(m)| \geq 1$ for every $n, m \in \mathbf{N}$ with $n \in U, m \in V$. Also,

$$\mathbf{cl}_{\widetilde{\mathbf{R}}} h^\beta(U) \cap \mathbf{cl}_{\widetilde{\mathbf{R}}} h^\beta(V) = \emptyset.$$

Since $\xi \in \mathbf{cl}_{\widetilde{\mathbf{R}}}U$, $h^\beta(\xi) \in \mathbf{cl}_{\widetilde{\mathbf{R}}}h^\beta(U)$ and since $\eta \in \mathbf{cl}_{\widetilde{\mathbf{R}}}V$, $h^\beta(\eta) \in \mathbf{cl}_{\widetilde{\mathbf{R}}}h^\beta(V)$. Therefore, $h^\beta(\xi) \neq h^\beta(\eta)$. Hence, h is one to one on $\beta\mathbf{N}$.

It is a well-known fact that a one to one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism. Therefore, h^β is a homeomorphism between $[\mathbf{cl}_{\widetilde{\mathbf{R}}}(x_n)]\setminus(x_n)$ and $\mathbf{N}^*$. □

Remark 2.2 It can be easily proved that every neighborhood of ξ in $\mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^{-}\backslash\mathbf{R}$ contains a topological copy of $\beta\mathbf{N}\backslash\mathbf{N}$.

Corollary 2.1 No point in $\gamma\,\mathbf{R}$ has a countable base of neighborhoods in $\gamma\mathbf{R}$.

Proof. If $\xi \in \gamma\mathbf{R}$, by Theorem 2.3 there is a subset X of $\mathbf{R}$ such that $\xi \in X$ and X is homeomorphic to $\mathbf{N}^*$.

We may suppose that $\xi \in \mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^{+}\backslash\mathbf{R}^{+}$ and that it has a countable base of neighborhoods of (U_n) in $\gamma\mathbf{R}$. Then $(\cap U_n) \cap X$ is a singleton. But this is a contradiction, since $(\cap U_n) \cap X$ is homeomorphic to a nonempty G_δ-set of $\mathbf{N}^*$, and it is a well-known fact that in $\mathbf{N}^*$ every nonempty G_δ-set has nonempty interior (see[5]). □

It is immediate from Corollary 2.1 that $\widetilde{\mathbf{R}}\backslash\mathbf{R}$ is not metrizable and has not have a countable base.

Theorem 2.4 *If $\eta \in \gamma\mathbf{R}$, there is no sequence (x_n) in $\mathbf{R}$ converging to η.*

Proof. We may suppose that $\eta \in \mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^{+}\backslash\mathbf{R}$ and that there is such a sequence (x_n) in $\mathbf{R}$. By Theorem 2.2, we may suppose that $x_{n+1} - x_n \geq 1$ for all $n \in \mathbf{N}$. Clearly, the sequence (x_n) can not be bounded, otherwise it would have a subsequence (x_{n_r}) which converges to a real number k which is a contradiction since $x_{n_{r+1}} - x_{n_r} \geq 1$ and (x_{n_r}) also converges to η.

We define a function f from $\mathbf{R}$ to $\mathbf{R}$ as follows:

$$f(x_n) = \begin{cases} 0 & \text{if} \quad n \quad \text{even} \\ 1 & \text{if} \quad n \quad \text{odd} \end{cases}$$

and complete the definition by piecewise linearity. It is easy to see that f is uniformly continuous and hence that it extends to a continuous function $\widetilde{f}$ from $\widetilde{\mathbf{R}}$ to $\widetilde{\mathbf{R}}$ (cf.[4]). Therefore, $\widetilde{f}(\eta) = \lim_n f(x_{2n}) = 0$ and $\widetilde{f}(\eta) = \lim_n f(x_{2n+1}) = 1$ which is a contradiction since (x_{2n}) and (x_{2n+1}) both converges to η. □

We state the following lemma that will be used later on, and its proof is straightforward.

Lemma 2.2 Let $\xi \in \mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^{+}\backslash\mathbf{R}$ and (x_n) be a sequence in $\mathbf{R}$ such that $x_{n+1} - x_n \geq 1$ and $\xi \in \mathrm{cl}_{\widetilde{\mathbf{R}}}\{x_n\}$. Then $U = \{A \subseteq \mathbf{N} : \xi \in \mathrm{cl}_{\widetilde{\mathbf{R}}}\{x_n\}_{n\in A}\}$ is an ultrafilter on $\mathbf{N}$.

Remark 2.3 We can easily proved that if $\xi \in \mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^{-}\backslash\mathbf{R}$ and if (x_n) is a sequence in $\mathbf{R}$ such that $x_n - x_{n+1} \geq 1$ and $\xi \in \mathrm{cl}_{\widetilde{\mathbf{R}}}\{x_n\}$, then $U = \{A \subseteq \mathbf{N} : \xi \in \mathrm{cl}_{\widetilde{\mathbf{R}}}\{x_n\}_{n\in A}\}$ is an ultrafilter on $\mathbf{N}$.

Lemma 2.3 For each $m \in \mathbf{N}$, there is a unique point $\xi_m \in \mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^+\backslash\mathbf{R}$ satisfying $\xi_m \in \bigcap\limits_{A\in U} \mathrm{cl}_{\widetilde{\mathbf{R}}}\{x_n + \frac{1}{m}\}_{n\in A}$.

Proof. Clearly, for each $m \in \mathbf{N}$ $\{\{x_n + \frac{1}{m}\}_{n\in A}, A \in U\}$ has near finite intersection property and so such near ultrafilter exists. Now we will show that such near ultrafilter is unique. To see this, suppose that $\eta, \zeta \in \bigcap\limits_{A\in U} \mathrm{cl}_{\widetilde{\mathbf{R}}}\{x_n + \frac{1}{m}\}_{n\in A}$. If $\eta \neq \zeta$, there exists $Y \in \eta$ and $Z \in \zeta$ and $W \in \mathcal{B}$ such that $(Y + W)\bigcap(Z + W) = \emptyset$. Now, for any $A \in U$, $\{x_n + \frac{1}{m}\}_{n\in A} \in \eta$ and $\{x_n + \frac{1}{m}\}_{n\in A} \in \zeta$. We claim that $A = \{n \in \mathbf{N} : x_n + \frac{1}{m} \in Y + W\} \in U$ for all $W \in \mathcal{B}$. To see this suppose that $A \notin U$, then there is a set $B \in U$ such that $A\bigcap B = \emptyset$. Since $\{x_n + \frac{1}{m}\}_{n\in B} \in \eta$ and $Y \in \eta$, for all $W \in \mathcal{B}$ $(\{x_n + \frac{1}{m}\}_{n\in B})\bigcap(Y + W) \neq \emptyset$ which implies that $x_{n_0} + \frac{1}{m} \in Y + W$ for some $n_0 \in B$. Hence, $n_0 \in A$ and so $A\bigcap B \neq \emptyset$ and it is a contradiction. Hence, $A \in U$. Similarly, $C = \{n \in \mathbf{N} : x_n + \frac{1}{m} \in Z + W\} \in U$. Therefore, there exists $n \in A\bigcap C$, and so $x_n + \frac{1}{m} \in Y + W$ and $x_n + \frac{1}{m} \in Z + W$ which implies that $(Y + W)\bigcap(Z + W) \neq \emptyset$, it is a contradiction. Therefore, for all Y$\in \eta$ and $Z \in \zeta$ and $W \in \mathcal{B}$, $(Y + W)\bigcap(Z + W) \neq \emptyset$ and so $\xi = \zeta$. □

Remark 2.4 We can easily prove that for each $m \in \mathbf{N}$, there is a unique point $\xi_m \in \mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^-\backslash\mathbf{R}$ satisfying $\xi_m \in \bigcap\limits_{A\in U} \mathrm{cl}_{\widetilde{\mathbf{R}}}\{x_n - \frac{1}{m}\}_{n\in A}$.

Theorem 2.5 *Every point ξ in $\widetilde{\mathbf{R}}\backslash\mathbf{R}$ is a limit point of a countable subset of $\widetilde{\mathbf{R}}$ which does not contain ξ.*

Proof. We may suppose that $\xi \in \mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^+\backslash\mathbf{R}$, the case $\xi \in \mathbf{cl}_{\widetilde{\mathbf{R}}}\mathbf{R}^-\backslash\mathbf{R}$ can be proved similarly. To see this, we will show that $\xi \in \mathrm{cl}_{\widetilde{\mathbf{R}}}\{\xi_n\}$. Let $C_{(Y+W_1)}$ be a basic neighborhood of ξ. We will show that there exists $m_0 \in \mathbf{N}$ such that for every $W \in \mathcal{B}$ and $A \in U$, $(\{x_n + \frac{1}{m_0}\}_{n\in A})\bigcap((Y + W_1) + W) \neq \emptyset$. It will follow that for each fixed $m \geq m_0$, $Y + W$ will be in ξ_m. Therefore, $\xi_m \in C_{(Y+W)}$ for each $m \geq m_0$.

Suppose that there is a set $A \in U$ and $W \in \mathcal{B}$ such that $(\{x_n + \frac{1}{m}\}_{n\in A})\bigcap((Y + W_1) + W) = \emptyset$. Then $(\{x_n + \frac{1}{m}\}_{n\in A})\bigcap(Y + W_1) = \emptyset$. Let W_2 be symmetric such that $W_2 + W_2 \subset W_1$. Then $(\{x_n + \frac{1}{m}\}_{n\in A})\bigcap(Y + W_2) = \emptyset$ which implies that $(\{x_n\}_{n\in A})\bigcap(Y + W_2 - \frac{1}{m}) = \emptyset$. Let m_0 be the smallest integer such that $\frac{1}{m_0} \in W_2$, then $W_2 - \frac{1}{m_0} = W_3$ is in $\mathcal{B}$ and $(\{x_n\}_{n\in A})\bigcap(Y + W_3) = \emptyset$ which is a contradiction since $Y \in \xi$ and $\{x_n\}_{n\in A} \in \xi$ for all $A \in U$. □

We remind the reader that a point of a topological space is called a P-point if every G_δ-set containing the point is a neighborhood of the point. Since no P-point can be a non-trivial limit of any sequence (see [5]), we have the following Corollary.

Corollary 2.2 $\widetilde{\mathbf{R}}\backslash\mathbf{R}$ has no P-point.

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