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Abstract

In this article, the known characterization of the surjective linear isometries of the Bochner space $L^p(\mu, H)$, for a σ-finite measure μ and an arbitrary Hilbert space H, in terms of regular set isomorphisms of the σ-algebra involved and strongly measurable families of surjective isometries of H, is extended to arbitrary measures.

Introduction

Let $(\Omega, \mathcal{A}, \nu)$ be a positive[1] measure space. Following [1], we call a mapping ϕ of $\mathcal{A}$ into itself, defined modula null sets, a *regular set isomorphism* if $\phi(A') = \phi(\Omega) \setminus \phi(A)$ for all $A \in \mathcal{A}$, where A' denotes the complement of A, $\phi(\cup_n A_n) = \cup_n \phi(A_n)$ for any disjoint sequence $\{A_n\}$ in $\mathcal{A}$, and $\nu(\phi(A))$=0 if, and only if, $\nu(A) = 0$. Any such isomorphim induces a mapping which we shall call the *induced mapping* and denote by Φ, on the space of vector-valued measurable functions with values in a Banach space E characterized by $\Phi(\chi_A e) = \chi_{\phi(A)} e$, $A \in \mathcal{A}$, $e \in E$, where χ_A denotes the characteristic function of A. This process is described for scalar functions in [2, pp. 453-454].

For $1 \leq p \leq \infty$ we shall denote the Bochner space $L^p(\Omega, \mathcal{A}, \nu, E)$ by $L^p(\nu, E)$, and by $L^p(\nu)$ when E is the scalar field, if there is no chance of ambiguity about the underlying measurable space. For definitions and properties of these spaces we refer to [3]. (We recall that a measurable function F belongs to $L^\infty(\mu, E)$ if, and only if, there exists a number $a > 0$ such that the set $\{x \in \Omega : \| F(x) \| > a\}$ is locally null, i.e., its intersection with every set of finite measure is null.)

Let λ denote the Lebesgue measure on [0,1]. Banach [4] proved that for every linear isometry T of $L^p(\lambda), 1 \leq p < \infty, p \neq 2$, there exists a measurable function σ of [0,1] almost onto itself and a scalar measurable function h on [0,1] such that for $f \in L^p(\lambda)$

$$(Tf)(x) = h(x)f(\sigma(x)) \quad a.e. \text{ on } [0,1].$$

If ϕ is the regular set isomorphism defined by $\phi(A) = \sigma^{-1}(A)$ on the Borel algebra of [0,1], then the above representation becomes

$$(Tf)(x) = h(x)\Phi(f)(x) \quad a.e. \text{ on } [0,1]. \tag{0.1}$$

In [1], Lamperti proves that for any σ-finite measure space $(\Omega, \mathcal{A}, \mu)$ the linear isometries of $L^p(\mu)$ onto itself, $1 \leq p < \infty, p \neq 2$, are indeed of the above form (1) except that the isomorphism ϕ of the σ-algebra $\mathcal{A}$, need not be defined by a point mapping. Moreover, if the measure ν is defined by $\nu(A) = \mu[\phi^{-1}(A)], A \in \mathcal{A}$, then

$$\mid h(x) \mid^p = dv/d\mu \quad a.e. \text{ on } \Omega. \tag{0.2}$$

In [5], Cambern generalizes this result to the Bochner spaces. He proves that if $(\Omega, \mathcal{A}, \mu)$ is a σ-finite measure space and H is a separable Hilbert space, then for any linear isometry of $L^p(\mu, H)$ onto itself, $1 \leq p < \infty$, $p \neq 2$, in addition to the maps h and Φ in Lamperti's characterization now there also exists a weakly measurable operator-valued function U defined on X, where $U_x = U(x)$ is an isometry of H onto itself for almost all $x \in X$, such that for $F \in L^p(\mu, H)$,

$$T(F)(x) = U_x(h(x)\Phi(F)(x)) \quad a.e. \text{ on } X. \tag{0.3}$$

In [6], Greim and Jamison obtain the same representation for an arbitrary Hilbert space, but the measure is still σ-finite. In this article we shall show that even for an *arbitrary* measure space $(\Omega, \mathcal{A}, \mu)$ and an arbitrary Hilbert space H, the linear isometries of $L^p(\mu, H)$ have the same form in a somewhat different set-up.

The Results

Following [7], we shall call a Borel measure μ on an extremally disconnected locally compact Hausdorff space X, *perfect* if

(i) every nonempty clopen (closed and open) set has positive measure,

(ii) every nowhere dense Borel set has measure zero, and

(iii) every nonempty clopen set contains antoher clopen set with finite measure

In [8], Cengiz proves that an arbitrary measure space $(\Omega, \mathcal{A}, \nu)$ can be replaced by a perfect measure space $(X, \sum, \mu)$ which does not affect the spaces $L^p(\nu, E)$, that is, $L^p(\nu, E) \cong L^p(\mu, E)$ (isometric to), for all $1 \leq p < \infty$, but may enlarge $L^\infty(\nu, E)$. This new measure space also has the following additional properties:

(iv) X is the topological direct sum of a family $\{X_i : i \in I\}$ of extremally disconnected compact Hausdorff spaces, i.e., $X = \sum_i \oplus X_i$;

(v) the algebra $\sum$ contains the Borel algebra and a subset $A \subset X$ is measurable if, and only if, $A \cap X_i$ is measurable for all $i \in I$;

(vi) for each $i \in I$, the restriction of μ to the σ-algebra of X_i is a regular Borel measure on X_i;

(vii) each σ-finite measurable set is contained a.e. in the union of a countable subfamily of $\{X_i : i \in I\}$;

(viii) for each $A \in \sum$, $\mu(A) = \sum_i \mu(A \cap X_i)$;

(ix) $\mu(\bar{U}) = \mu(U)$ for every open set U, where $\bar{U}$ denotes the closure of U;

(x) each measurable set A is equivalent to a clopen set C, i.e., $\mu(A \Delta C) = \mu(A \setminus C) + \mu(C \setminus A) = 0$;

(xi) every locally null set is actually null; and

(xii) the measure μ is complete.

Hence, in the light of this discussion, we may and will assume that any measure space is a perfect one satisfying all the above conditions.

In the proof of the main result of this article we shall use the following theorem which is proved in a forthcoming paper [9].

Theorem 1. *Let H be an arbitrary Hilbert space, $(X, \sum, \mu)$ be an arbitrary perfect measure space, $1 \leq p < \infty$, and define q to be that extended real number such that $\frac{1}{p} + \frac{1}{q} = 1$. Then, for each $G \in L^q(\mu, H)$, the map*

$$F \to \int_X \langle F(.), G(.) \rangle d\mu, \quad F \in L^p(\mu, H)$$

is a bounded linear functional on $L^p(\mu, H)$ whose norm is $\| G \|_q$, and conversely every bounded functional on $L^p(\mu, H)$ is of this form, i.e., $L^p(\mu, H)^\star = L^q(\mu, H)$.

This theorem is proved in [8] for arbitrary μ but separable H.

Now we fix a measure space $(X, \sum, \mu)$ satisfying the conditions (i)-(xii) and prove the following theorem.

Theorem 2. *Let H be an arbitrary Hilbert space and let $1 \leq p < \infty$, $p \neq 1$. Then, for any linear isometry T of $L^p(\mu, H)$ onto itself there exists a regular set isomorphism ϕ of $\sum$ onto itself, defined modulo null sets, a measurable scalar function h on X and a locally strongly measurable operator-valued function U on X such that for each $x \in X, U_x = U(x)$ is a surjective linear isometry of H and for every $F \in L^p(\mu, H)$,*

$$T(F)(x) = U_x(h(x)\Phi(F)(x)) \;\; a.e. \;\; on X.$$

Moreover, $| \, h \, |^p = \frac{d(\mu o \phi^{-1})}{d\mu}$ on the ring of σ-finite measurable sets.

First we prove a chain of lemmas.

Throughout the rest of this article T will denote a fixed surjective linear isometry of $L^p(\mu, H)$.

For any $G \in L^q(\mu, H)$, where q is the extended real number such that $1/p + 1/q = 1$, the mapping

$$F \to \int_x < T^{-1}F, G > d\mu$$

is a bounded linear functional on $L^p(\mu, H)$, and therefore, by Theorem 1, there exists a function $G^\star$ in $L^q(\mu, H)$ such that

$$\int_x \langle T^{-1}F, G \rangle d\mu = \int_x \langle F, G^\star \rangle d\mu$$

for all $F \in L^p(\mu, H)$.

By substituting $T(F)$ for F in the above definition of $G^\star$ we get

$$\int_X \langle F, G \rangle d\mu = \int_X \langle T(F), G^\star \rangle d\mu$$

for all $F \in L^p(\mu, H)$.

For $G \in L^q(\mu, H)$ if we let Ψ_G denote the map on $L^p(\mu, H)$ defined by

$$\Psi_G(F) = \int_x \langle F, G \rangle d\mu,$$

then, we get $\Psi_{G^\star} = \Psi_G \circ T^{-1}$. Since T is a surjective isometry, it follows that $\| G^\star \|_q = \| G \|_q$.

We shall fix a measurable subset X_0 of X of finite measure and denote its characteristic function by χ.

When we refer to a function in $L^p(\mu, H)$ we shall mean a specific function, rather than an equivalence class. The support of a function F is the set $\{x : F(x) \neq 0\}$.

Lemma 1. *Let $e \in H, ||e|| = 1$, and let $E(x) = T(\chi e)(x) / \| T(\chi e)(x) \|$ for $x \in Y_0$ and $E(x) = 0$ elsewhere, where Y_0 is the support of $T(\chi e)$. Then*

$$(\chi e)^\star(x) = \| T(\chi e)(x) \|^{p-1} E(x),$$

a.e. on X if $p > 1$, and on Y_0 if $p = 1$. Consequently, $(\chi e)^\star$ and $T(\chi e)$ have a.e. the same support.

Proof. Let $F = \chi e$ and define $K(x) = F^\star(x) / \| F^\star(x) \|$ if $F^\star(x) \neq 0$ and $K(x) = 0$ otherwise.

Since $\langle T(F)(x), F^\star(x) \rangle = \| T(F)(x) \| \| F^\star(x) \| \langle E(x), K(x) \rangle, | \langle E(x), K(x) \rangle | \leq 1$ a.e., and $\| F \|_p = [\mu(X_0)]^{1/p}$, we have

$$\begin{aligned}
\mu(X_0) &= \int_X \langle F(.), F(.) \rangle d\mu = \int_X \langle T(F)(.), F^\star(.) \rangle d\mu \\
&= \int_X \| T(F)(.) \| \| F^\star(.) \| \langle E(.), K(.) \rangle d\mu \\
&\leq \int_X \| T(F)(.) \| \| F^\star(.) \| d\mu \\
&\leq \| T(F) \|_p \| F^\star \|_q = \mu(X_0).
\end{aligned}$$

Thus we have equality throughout.

For $p > 1$, from the equality

$$\int_X \| T(F)(.) \| \| F^\star(.) \| d\mu = \| T(F) \|_p \| F^\star \|_q$$

and a result in [10, p. 121], it follows that

$$\| T(F)(x) \|^p = (\| T(F) \|_p^p / \| F^\star \|_q^q) \| F^\star(x) \|^q = \| F^\star(x) \|^q$$

a.e. from which we obtain $\| F^\star(x) \| = \| T(F)(x) \|^{p-1}$ a.e., which in turn implies that the support of $K(x)$ equals Y_0 a.e.

It is easy to show that if $f(x) > 0$ and $| g(x) | \leq 1$ a.e. on a measurable set A and if fg and g have the same integral on A then $g(x) = 1$ a.e. on A. From this observation and the equality

$$\int_X \| T(F)(.) \| \| F^\star(.) \| \langle E(x), K(x) \rangle d\mu = \int_X \| T(F)(.) \| \| F^\star(.) \| d\mu$$

it follows that $\langle E(x), K(x) \rangle = 1$ a.e. on Y_0.

If $u, v \in H, \| u \| \leq 1, \| v \| \leq 1$ and $\langle u, v \rangle = 1$, then $u = v$. So, from the preceding result we conclude that $E(x) = K(x)$ a.e. on Y_0, and hence a.e. on X. Consequently,

$$\begin{aligned} F^\star(x) &= \| F^\star(x) \| K(x) = \| F^\star(x) \| E(x) \\ &= \| T(F)(x) \|^{p-1} E(x) \end{aligned}$$

a.e. on X, proving our lemma for $p > 1$.

Now let us assume that $p = 1$. Since $\| F^\star \|_\infty = \| F \|_\infty = 1$, from the equality

$$\int_X \| T(F)(.) \| \| F^\star(.) \| d\mu = \| T(F) \|_1$$

we obtain

$$\int_{Y0} \| T(F)(.) \| (1 - \| F^\star(.) \|) d\mu = 0$$

from which, since $\| F^\star(x) \| \leq 1$ a.e., it follows that $F^\star(x) = 1$ a.e. on Y_0. This completes the proof of our lemma. $\square$

Lemma 2. *Let $e_1, e_2 \in H, \| e_1 \| = \| e_2 \| = 1$ and $\langle e_1, e_2 \rangle = 0$. Let Y_i for $i = 1, 2$, be the support of $T(\chi e_i)$ and define $E_i(x) = T(\chi e_i)(x) / \| T(\chi e_i)(x) \|$ on Y_i and $E_i(x) = 0$ elsewhere. Then $\langle E_1(x), E_2(x) \rangle = 0$ a.e. on X.*

Proof. Let $F_i = T(\chi e_i)$, i=1, 2, and let $A \subset Y_i \cap Y_2$ be any measurable set of finite measure. Since

$$\chi e_1 = T^{-1}(F_1) = T^{-1}(\chi_A F_1) + T^{-1}(\chi_{(Y_1 \setminus A)} F_1),$$

and science T^{-1} maps functions of disjoint support info functions of disjoint support (see [5, p. 11]) the terms on the right have disjoint supports. Multiplying both sides by χ_B we get

$$\chi_B(\chi e_1) = \chi_B T^{-1}(\chi_A F_1) = T^{-1}(\chi_A F_1),$$

where B is the support of $T^{-1}(\chi_A F_1)$.

Since $\langle e_1, e_2 \rangle = 0$, by Lemma 1, we obtain

$$\begin{aligned} 0 &= \int_X \langle \chi_B \chi e_1, \chi e_2 \rangle d\mu = \int_X \langle T(\chi_B \chi e_1), (\chi e_2)^\star \rangle d\mu \\ &= \int_X \langle \chi_A F_1, (\chi e_2)^\star \rangle d\mu = \int_A \langle \| F_1(.) \| E_1(.), \| F_2(.) \|^{p-1} E_2(.) \rangle d\mu \\ &= \int_A \| F_1(.) \| \| F_2(.) \|^{p-1} \langle E_1(.), E_2(.) \rangle d\mu. \end{aligned}$$

Since this is true for every measurable subset A of $Y_1 \cap Y_2$ of finite measure and $\| F_1(x) \| \| F_2(x) \| > 0$ on $Y_1 \cap Y_2$, we conclude that $\langle E_1(x), E_2(x) \rangle = 0$ a.e. on $Y_1 \cap Y_2$, and hence, it is zero a.e. on X. This completes the proof of the lemma. $\square$

Lemma 3. *For any two nonzero vectors e_1, e_2 in $H, T(\chi e_1)$ and $T(\chi e_2)$ have a.e. the same support.*

Proof. Let u, v be two nonzero orthogonal vectors in $H, w = au + bv, a, b \in C$, and let Y_u, Y_v and Y_w denote the supports of $T(\chi u), T(\chi v)$ and $T(\chi w)$ respectively. We will show that $Y_w = Y_u = Y_v$ for any $a \neq 0, b \neq 0$. By Lemma 2, $\langle aT(\chi u)(x), bT(\chi v)(x)\rangle = 0$ a.e. on X. Since the sum of two orthogonal vectors is different from zero if, and only if at least one of them is different from zero, $T(\chi w)(x) = aT(\chi u)(x) + bT(\chi v)(x) \neq 0$ if, and only if, either $T(\chi u)(x) \neq 0$ or $T(\chi v)(x) \neq 0$, and therefore,

$$Y_w = Y_u \cup Y_v. \tag{0.4}$$

Now let $w_1 = au + v, w_2 = -au + v$ where $a = - \parallel v \parallel / \parallel u \parallel$. Since w_1, w_2 are orthogonal, by the preceding discussion we have $Y_{w_1} = Y_u \cup Y_v = Y_{w_2}$ and $Y_v = Y_{2v} = Y_{w_1} \cup Y_{w_2}$, and therefore $Y_v = Y_{w_1}$. Similarly, $Y_u = Y_{w_1}$. Now from (4) we get $Y_w = Y_u = Y_v$ proving our claim.

Thus, we have proved that for any two nonzero vectors w and z in the span of u, and v, the functions $T(\chi w)$ and $T(\chi z)$ have a.e. the same support which actually completes the proof of our lemma, for e_1 and e_2 are contained in the span of two orthogonal vectors.

□

Lemma 4. *Let $e \in H, e \neq 0$. Then T maps $L^p(X_0, H)$ onto $L^p(Y_0, H)$ where Y_0 is the support of $T(\chi e)$.*

Proof. Let $A \subset Y_0$ be a measurable set with finite measure, and let $F \in L^p(\mu, H)$ such that $T(F) = \chi_A e$. Now let S_1=supp $(F) \cap X_0', S_2$=supp $(F) \cap X_0, B_1 =$ supp $T(\chi_{S_1} F)$ and B_2=sups $T(\chi_{S_2} F)$. Then

$$\chi_A e = T(\chi_{S_1} F) + T(\chi_{S_2} F).$$

Since T maps functions of disjoint support into functions of disjoint support, $A = B_1 \cup B_2$ is an a.e. disjoint union. On the other hand, since $\chi_{S_1} F$ and χe have disjoint supports, B_1 is contained a. e. in the complement of Y_0. Thus, we conclude that B_1 is null which in turn implies that S_1 is null, i.e. supp$(F) \subset X_0$ a.e.

The above argument shows that $T(L^p(X_0, H))$ contains all simple functions in $L^p(Y_0, H)$ and consequently, all of $L^p(Y_0, H)$.

Now for the reverse inclusion let $B \subset X_0$ be a measurable set, and $z \in H, z \neq 0$. Then from $T(\chi z) = T(\chi_B z) + T(\chi_{(X_0 \setminus B)} z)$ we get supp $T(\chi_B z) \subset$ supp $T(\chi z) = Y_0$ (Lemma 3). Thus, $T(\varphi) \in L^p(Y_0, \mu, H)$ for every simple function $\varphi \in L^p(X_0, H)$, and consequently, $T(L^p(X_0, H)) \subset L^p(Y_0, H)$. Hence we have equality. □

Proof of the Theorem Now, to complete the proof of the theorem, for each $i \in I$ we let Y_i denote the support of $T(\chi_i e)$ where χ_i is the characteristic function of X_i, and e is any nonzero vector in H. By Lemma 3, Y_i does snot depend on e. Being the support of an integrable function each Y_i is σ-finite. We may and will assume that it is clopen. Since T maps functions of disjoint support into functions of disjoint support, Y_i's are mutually disjoint.

By Lemma 4, for each $i \in I, T$ maps $L^p(X_i, H)$ ont $L^p(Y_i, H)$. Therefore, by the Greim-Jamison theorem [6, p. 513] there exists a regular set isomorphism ϕ_i of the σ-algebra $\sum(X_i)$ onto the σ-algebra $\sum(Y_i)$, defined modulo null sets, a scalar measurable function h_i, and a strongly measurable operator-valued function $U^{(i)}$ on Y_i such that for each $y \in Y_i, U_y^{(i)} = U^{(i)}(y)$ is a surjective isometry of H, and for every $F \in L^p(X_i, \mu, H)$

$$T(F)(y) = U_y^{(i)}(h_i(y)\Phi_i(F)(y)) \text{ a.e. on } Y_i.$$

Let $Y = \bigcup_i Y_i$ and extend each h_i and $U^{(i)}$ to all Y by defining them to be zero in the complement of Y_i. Let $h = \sum_i h_i, U = \sum_i U^{(i)}$ and let ϕ be the map from the σ-algebra $\sum$ to the σ-algebra $\sum(Y)$ defined by

$$\phi(A) = \sum_i \phi_i(A \cap X_i), \quad A \in \sum.$$

Clearly ϕ is defined modulo null sets since each ϕ_i is defined so. Since a subset $B \subset Y$ belongs to $\sum(Y)$ if, and only if, for each $i, B \cap Y_i$ belongs to $\sum(Y_i)$, it follows that ϕ is surjective.

Let Φ denote the induced map. Then it can be shown easily that for each $F \in L^p(\mu, H)$,

$$T(F)(y) = U_y(h(y)\phi(F)(y)) \quad a.e. on Y.$$

Next we show that T maps $L^p(\mu, H)$ onto $L^p(Y, H)$ and that Y' is null. Let $G \in L^p(Y, H)$. Then $\operatorname{supp}(G) \subset \bigcup_k Y_{i_k}$ for some countable subfamily $\{Y_{i_k} : k = 1, 2, \ldots\}$ of $\{Y_i\}$. Therefore, $G = \sum_k G_k$ where $G_k = G\chi Y_{i_k}$. For each $k = 1, 2, \ldots,$ let $F_k \in L^p(X_{i_k}, H)$ such that $TF_k = G_k$ and let $F = \sum_k F_k$. Then, $F \in L^p(\mu, H)$ and $TF = G$.

To prove that Y' is null we let $B \subset Y', \mu(B) < \infty, e \in H, e \neq 0$. Then there exists $G \in L^p(Y, H)$ such that $T^{-1}(\chi Be) = T^{-1}(G)$, which implies that $G = \chi Be$ a.e.and since they have disjoint supports we conclude that B is null. Hence Y' is null.

Now we extend h and U to all of X by defining $h(x)$ =0 and $U_x = I$ (the identity map on H) for $x \in Y'$. Clearly U is locally strongly measurable on X, i.e., its restriction to each set of finite measure is strongly measurable, for any such set is contained a.e. in a countable union of X_i's. The measurability of h follows from the fact that a set $B \subset X$ is measurable if, and only if, $B \cap Y_j$ is measurable for all $j \in I$. To see this, fix $i \in I$. Then, since $\mu(X_i) < \infty$ and $X_i \cap Y_j, j \in I$, are disjoint clopen sets in X_i it follows that only countably many of these sets are nonempty, $X_i \cap Y_{jk} \neq \phi, k = 1, 2, \ldots$ say. Now assume that $B \cap Y_j$ is measurable for al $j \in I$. Then, since μ is complete, $X_i \cap Y' \cap B$ is measurable, and so, $X_i \cap B = [\bigcup_k (X_i \cap Y_{jk} \cap B)] \cup (X_i \cap Y' \cap B)$ is measurable for each $i \in I$. Hence B is measurable. This ends the proof of the theorem. □

Remark *A natural question is whether or not the characterization obtained for the linear isometries of the Bochner space $L^p(\mu, H)$ onto itself, $1 \leq p < \infty p \neq 2$, holds, should a Banach space replace H as the range space. In general, the answer is in the negative; however, Sourour [10] was able to replace H by a suitable Banach space.*

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