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## QR-Submanifolds and Almost Contact 3-Structure

*Rıfat Güneş, Bayram Şahin and Sadık Keleş*

### Abstract

In this paper, QR-submanifolds of quaternion Kaehlerian manifolds with  $\dim \nu^\perp = 1$  has been considered. It is shown that each QR-submanifold of quaternion Kaehlerian manifold with  $\dim \nu^\perp = 1$  is a manifold with an almost contact 3-structure. We apply geometric theory of almost contact 3-structure to the classification of QR-submanifolds (resp. Real hypersurfaces) of quaternion Kaehler manifolds (resp.  $IR^{4m}$ ,  $m > 1$ ). Some results on integrability of an invariant distribution of a QR-submanifold and on the immersions of its leaves are also obtained.

**Key Words:** Quaternion Kaehler Manifold, QR-Submanifold, Almost Contact 3-Structure

### 1. Introduction

The geometry of QR-submanifolds of a quaternion Kaehlerian manifolds was firstly reported by Bejancu[1]. Among all submanifolds of a quaternion Kaehlerian manifold, QR-submanifolds have been intensively studied from different points of view by many authors [1],[2],[4].

In case of  $\dim \nu^\perp = 1$ , the study of QR-submanifolds has a significant importance. We show that QR-submanifolds of quaternion Kaehler manifolds with  $\dim \nu^\perp = 1$  admit an almost contact 3-structure.

### 2. Preliminaries

Let  $\overline{M}$  be a  $4n$ - dimensional manifold and  $g$  be a Riemannian metric on  $\overline{M}$ . Then  $\overline{M}$  is said to be quaternion Kaehlerian manifold (see, [1]) if there exists a 3-dimensional vector bundle of tensors of type(1.1) with local basis of almost Hermitian structures

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$J_1, J_2, J_3$  satisfying

$$J_1 \circ J_2 = -J_2 \circ J_1 = J_3 \quad (\text{II.1})$$

and

$$\bar{\nabla}_X J_a = \sum Q_{ab}(X) J_b, a = 1, 2, 3 \quad (\text{II.2})$$

for all vector fields  $X$  tangent to  $\bar{M}$ , where  $\bar{\nabla}$  is the Levi-Civita connection determined by  $g$  on  $\bar{M}$  and  $Q_{ab}$  are certain 1-forms locally defined on  $\bar{M}$  such that  $Q_{ab} + Q_{ba} = 0$ .

Let  $M$  be  $(4m + 3)$ -dimensional differentiable manifold and  $(\phi_a, \xi_a, \eta_a)$  be three almost contact structures on  $M$  i.e. We have

$$\begin{aligned} \phi_a^2(X) &= -X + \eta_a(X)\xi_a, \phi_a(\xi_a) = 0 \\ \eta_a(\xi_a) &= 1, \eta_a \circ \phi_a = 0 \end{aligned} \quad (\text{II.3})$$

where  $X$  tangent to  $M$ . Suppose that the almost contact structures satisfy the following conditions

$$\begin{aligned} \eta_a(\xi_b) &= 0, a \neq b, \phi_a(\xi_b) = -\phi_b(\xi_a) = \xi_c \\ \eta_a \circ \phi_b &= -\eta_b \circ \phi_a = \eta_c \\ (\phi_a \circ \phi_b)(X) - \xi_a(\eta_b(X)) &= (\phi_b \circ \phi_a)(X) - \xi_b(\eta_a(X)) = \phi_c(X) \end{aligned} \quad (\text{II.4})$$

for any cyclic permutation  $(a, b, c)$  of  $(1, 2, 3)$ . Then, we say that  $M$  is endowed with an almost contact 3-structure. If  $M$  is a Riemannian manifold, then we can choose a Riemannian metric  $g$  on  $M$  such that we have

$$\begin{aligned} (\phi_a X, \phi_a Y) &= g(X, Y) - \eta_a(X)\eta_a(Y) \\ \eta_a(X) &= g(X, \xi_a) \end{aligned} \quad (\text{II.5})$$

for any  $X, Y \in \chi(M)$ . In this case we say that  $(\phi_a, \xi_a, \eta_a), a = 1, 2, 3$  define an almost contact metric structure (See [5]). Taking account of ( II.3) and (II.5), we obtain

$$g(\phi_a X, Y) + g(X, \phi_a Y) = 0, a = 1, 2, 3$$

for any  $X, Y$  tangent to  $M$ .

Let  $M$  be an  $m$ -dimensional Riemannian manifold isometrically immersed in  $\overline{M}$ . We denote by  $TM$  (resp.  $TM^\perp$ ) the tangent (resp. normal) bundle to  $M$ . Then we say that  $M$  is a quaternion-real submanifold (QR-Submanifold) if there exists a vector bundle  $v$  of the normal bundle such that we have

$$J_a(v_x) = v_x \tag{II.6}$$

and

$$J_a(v_x^\perp) \subset T_M(x) \tag{II.7}$$

for  $x \in M$  and  $a = 1, 2, 3$ , where  $v^\perp$  is the complementary orthogonal bundle to  $v$  in  $TM^\perp$ . Let  $M$  be a QR-submanifold of  $\overline{M}$ . For sake of shortness we use  $D_{ax}$  for  $D_{ax} = J_a(v_x^\perp)$ ,  $a = 1, 2, 3$ . We consider  $D_{1x} \oplus D_{2x} \oplus D_{3x} = D_x^\perp$  and  $3s$ -dimensional distribution  $D^\perp : x \longrightarrow D_x^\perp$  globally defined on  $M$ . Where  $s = \dim v_x^\perp$ . Also we have, for each  $x \in M$ ,

$$J_a(D_{ax}) = v_x^\perp, J_a(D_{bx}) = D_{cx}, \tag{II.8}$$

where  $(a, b, c)$  is a cyclic permutation of  $(1, 2, 3)$ . Next, we denote by  $D$  the complementary orthogonal distribution to  $D^\perp$  in  $TM$ , we see that  $D$  is invariant with respect to the action of  $J_a$ . i.e. we have

$$J_a(D_x) = D_x \tag{II.9}$$

for any  $x \in M$ .  $D$  is called quaternion distribution . Also note that  $D_{1x}, D_{2x}, D_{3x}$  are mutually orthogonal vector spaces of  $T_M(x)$  (see [1]).

In [3] D.E.Blair introduced the concept cosymplectic structure as it follows. An almost contact metric structure  $(\phi, \xi, \eta, g)$  is a cosymplectic structure if and only if

$$(\nabla_X \phi)Y = 0, (\nabla_X \eta)Y = 0,$$

where  $\nabla$  is Levi-civita connection on  $M$ .

**Definition 2.1** *An almost 3-contact structure  $(\phi_a, \xi_a, \eta_a)$  is*

a) a 3-cosymplectic structure if

$$(\nabla_X \phi_a)Y = 0, \quad (\nabla_X \eta_a)Y = 0 \quad (\text{II.10})$$

b) a 3-Sasakian structure if

$$(\nabla_X \phi_a)Y = \eta_a(Y)X - g(X, Y)\xi_a \quad (\text{II.11})$$

c) a nearly 3-Sasakian structure if

$$(\nabla_X \phi_a)X = \eta_a(X)X - g(X, X)\xi_a \quad (\text{II.12})$$

where  $\nabla$  denotes the Levi-Civita connection and  $X, Y, Z$  are arbitrary vector fields on  $M$ .

For  $Y \in \chi(M)$ , we decompose as follows

$$J_a Y = P_a Y + F_a Y, \quad a = 1, 2, 3 \quad (\text{II.13})$$

where  $P_a Y$  and  $F_a Y$  the tangential and normal parts of  $J_a Y$ , respectively.

The formula Gauss and Weingarten are given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y) \quad (\text{II.14})$$

and

$$\bar{\nabla}_X V = -A_V X + \nabla_X^\perp V \quad (\text{II.15})$$

for any vector fields  $X, Y$  tangent to  $M$  and any vector field  $V$  normal to  $M$ . Where  $\nabla$  is the induced Riemann connection in  $M$ ,  $h$  is the second fundamental form,  $A_V$  is fundamental tensor of Weingarten with respect to the normal section  $V$  and  $\nabla^\perp$  normal connection on  $M$ . Moreover we have the relation

$$g(A_V X, Y) = g(h(X, Y), V) \quad (\text{II.16})$$

### 3. QR-Submanifolds with $\dim v^\perp = 1$

Let  $M$  be a QR-submanifold of a quaternion Kaehlerian manifold  $\bar{M}$  such that the dimension  $v^\perp$  is equal to one. In this case  $v^\perp$  is generated by unit vector field, say  $N$ .

We shall show in the sequel that  $N$  is precisely determined with one of the almost contact 3-structure. Let  $-J_a(N) = \xi_a$ ,  $a = 1, 2, 3$  and hence the distributions  $D_a$  are generated by the vector fields  $\xi_a$ . It is natural expect that a QR-submanifold of quaternion Kaehlerian manifold with  $\dim v^\perp = 1$  is almost contact 3-structure, we describe it as follows;

**Proposition 3.2** *Let  $\overline{M}$  be a quaternion Kaehlerian manifold and  $M$  be a QR-submanifold of  $\overline{M}$ . Then  $M$  is a manifold with almost contact 3-structure.*

**Proof.** For any  $X \in \Gamma(TM)$ , put

$$\phi_a X = P_a X, F_a(X) = \eta_a(X)N. \quad (\text{III.1})$$

Then  $g(F_a X, N) = \eta_a(X)$  and

$$\eta_a(X) = g(X, \xi_a). \quad (\text{III.2})$$

Moreover  $g(P_a X, P_a Y) = g(J_a X, J_a Y) - g(F_a X, F_a Y)$  implies

$$g(\phi_a X, \phi_a Y) = g(X, Y) - \eta_a(X)\eta_a(Y) \quad (\text{III.3})$$

and

$$J_a^2 X = J_a P_a X + J_a F_a X$$

or

$$-X = P_a^2 X + \eta_a(X)J_a N.$$

Hence

$$\phi_a^2 X = -X + \eta_a(X)\xi_a. \quad (\text{III.4})$$

On the other hand, from (III.2) and (III.4) we obtain  $\phi_a(\xi_a) = 0$  and  $\eta_a(\xi_a) = 1$ , respectively. Moreover

$$(\eta_a \circ \phi_a) X = \eta_a(\phi_a X) = g(\phi_a X, \xi_a) = 0$$

for any  $X \in \Gamma(TM)$ . From (III.1) and (III.2)

$$\eta_a(\xi_b) = 0$$

and

$$\begin{aligned} \phi_a(\xi_b) &= J_a(\xi_b) - F_a(\xi_b) \\ &= -J_a(J_b N) \\ &= -J_c N = \xi_c. \end{aligned}$$

By using (II.13) and (III.4), we obtain

$$\begin{aligned} (\eta_a \circ \phi_b)(X) &= \eta_a(\phi_b X) \\ &= g(\phi_b X, \xi_a) \\ &= \eta_c(X) \end{aligned}$$

and

$$\begin{aligned} (\phi_a \circ \phi_b)(X) - \xi_a(\eta_b(X)) &= (\phi_b \circ \phi_a)(X) - \xi_b(\eta_a(X)) \\ &= (\phi_b(\phi_a(X)) - \xi_b(\eta_a(X))) \\ &= \phi_c(X). \end{aligned}$$

This shows that on a QR-submanifold of a quaternion Kaehlerian manifold  $\dim v^\perp = 1$  there exists a natural almost contact 3-structure. i.e. tensor field  $\phi_a$  of type (1,1), 1-form  $\eta_a$  and unit vector field  $\xi_a$  satisfy (II.3), (II.4) and (II.5).

From now on will denote by  $M$  a QR-submanifold with  $\dim v^\perp = 1$ .

**Theorem 3.3** *Let  $M$  be a QR-submanifold of a quaternion Kaehlerian manifold. If for any  $X, Y \in \Gamma(TM)$ ,  $h(X, Y)$  has no component in  $\Gamma(v^\perp)$  and  $D_a, a = 1, 2, 3$  are parallel in  $M$ , then  $M$  is a manifold with cosymplectic 3-structure.*

**Proof.** For any  $X, Y \in \Gamma(TM)$ , from (II.2) we have

$$\bar{\nabla}_X J_a Y = Q_{ab}(X) J_b Y + Q_{ac}(X) J_c Y + J_a \bar{\nabla}_X Y.$$

Taking account of (II.13),(II.14),(II.15) and (III.1) we obtain

$$\begin{aligned}
 (\nabla_X \phi_a) Y + (\nabla_X \eta_a(Y)) N + h(X, \phi_a Y) \\
 - \eta_a(Y) A_N X + \eta_a(Y) \nabla_X^\perp N = Q_{ab}(X) \phi_b Y + Q_{ab}(X) \eta_b(Y) N \\
 + Q_{ac}(X) \phi_c Y + Q_{ac}(X) \eta_c(Y) N \\
 + B_a h(X, Y) + C_a h(X, Y).
 \end{aligned}$$

Comparing the tangential and normal parts both of sides of this equation, we have

$$\begin{aligned}
 (\nabla_X \phi_a) Y = \eta_a(Y) A_N X + Q_{ab}(X) \phi_b Y + Q_{ac}(X) \phi_c Y \\
 + B_a h(X, Y)
 \end{aligned} \tag{III.5}$$

and

$$\begin{aligned}
 (\nabla_X \eta_a(Y)) N = -h(X, \phi_a Y) + \eta_a(Y) \nabla_X^\perp N \\
 + Q_{ab}(X) \eta_b(Y) N + Q_{ac}(X) \eta_c(Y) N \\
 + C_a h(X, Y).
 \end{aligned} \tag{III.6}$$

Now, we decompose  $h(X, Y)$  into components  $h^1(X, Y)$  and  $h^2(X, Y)$  in  $v^\perp$  and  $v$  respectively as

$$h(X, Y) = h^1(X, Y) + h^2(X, Y)$$

where we can put  $h^1(X, Y) = \alpha(X, Y) N$  for some scalar valued bilinear function  $\alpha$ . Thus (III.5) and (III.6) gives

$$\begin{aligned}
 (\nabla_X \phi_a) Y = \eta_a(Y) A_N X + Q_{ab}(X) \phi_b Y + Q_{ac}(X) \phi_c Y \\
 - \alpha(X, Y) \xi_a
 \end{aligned} \tag{III.7}$$

and

$$(\nabla_X \eta_a)(Y) = -\alpha(X, \phi_a Y) + Q_{ab}(X) \eta_b(Y) + Q_{ac}(X) \eta_c(Y). \tag{III.8}$$

On the other hand, since  $\xi_a = -J_a N$  we have



$$\bar{\nabla}_X \xi_a = -(\bar{\nabla}_X J_a)N - J_a \bar{\nabla}_X N$$

for any  $X \in \Gamma(TM)$ . Thus by using (II.2),(II.8),(II.15) and taking tangential parts we obtain

$$\nabla_X \xi_a = Q_{ab}(X)\xi_b + Q_{ac}(X)\xi_c + \phi_a A_N X$$

for any  $X \in \Gamma(TM)$ . Thus we have

$$g(\nabla_X \xi_a, \xi_b) = Q_{ab}(X) + g(\phi_a A_N X, \xi_b).$$

From (II.4) and (II.16) we derive

$$\eta_b(\nabla_X \xi_a) + \alpha(X, \xi_c) = Q_{ab}(X) \quad (\text{III.9})$$

Similarly, we get

$$\eta_c(\nabla_X \xi_a) - \alpha(X, \xi_b) = Q_{ac}(X) \quad (\text{III.10})$$

Combining (III.7) and (III.8) with (III.9) and (III.10) we have

$$\begin{aligned} (\nabla_X \phi_a) Y &= \eta_a(Y) A_N X + (\eta_b(\nabla_X \xi_a) + \alpha(X, \xi_c)) \phi_b Y \\ &\quad + (\eta_c(\nabla_X \xi_a) - \alpha(X, \xi_b)) \phi_c Y - \alpha(X, Y) \xi_a \end{aligned} \quad (\text{III.11})$$

and

$$\begin{aligned} (\nabla_X \eta_a)(Y) &= -\alpha(X, \phi_a Y) + (\eta_b(\nabla_X \xi_a) + \alpha(X, \xi_c)) \eta_b(Y) + \\ &\quad (\eta_c(\nabla_X \xi_a) - \alpha(X, \xi_b)) \eta_c(Y) \end{aligned} \quad (\text{III.12})$$

for any  $X, Y \in \Gamma(TM)$ . From (II.16) and (III.11) we get

$$\begin{aligned} g((\nabla_X \phi_a) Y, Z) &= \eta_a(Y) \alpha(X, Z) + \eta_b(\nabla_X \xi_a) g(\phi_b Y, Z) \\ &\quad + \alpha(X, \xi_c) g(\phi_b Y, Z) + (\eta_c(\nabla_X \xi_a) - \alpha(X, \xi_b)) g(\phi_c Y, Z) \\ &\quad - \alpha(X, Y) \eta_a(Z). \end{aligned} \quad (\text{III.13})$$

Thus if  $h(X, Y)$  has no components in  $\Gamma(v^\perp)$  and  $D_a, a = 1, 2, 3$  are parallel in  $M$ , then from (III.12) and (III.13), we get  $(\nabla_X \phi_a)Y = 0$  and  $(\nabla_X \eta_a)(Y) = 0$  that is  $M$  is a manifold with cosymplectic 3-structure.

As immediate consequence of theorem we have the following;

**Corollary 3.4** *Let  $M$  be real hypersurface of quaternion Kaehler manifold  $\overline{M}$ . If  $M$  is totally geodesic and  $\xi_a$  are parallel in  $M$  then  $M$  is a manifold with cosymplectic 3-structure.*

**Corollary 3.5** *Let  $M$  be real hypersurface in  $IR^{4m}(m > 1)$ . Then  $M$  is totally geodesic if and only if  $M$  is a manifold with cosymplectic 3-structure.*

**Proof.** Since  $\overline{M} = IR^{4m}(m > 1)$  we have  $\overline{\nabla}_X J_a = 0$ . Thus from (III.13) we get

$$g((\nabla_X \phi_a)Y, Z) = \eta_a(Y)\alpha(X, Z) - \alpha(X, Y)\eta_a(Z) \quad (III.14)$$

for any  $X, Y, Z \in \Gamma(TM)$ . Let  $M$  be a totally geodesic real hypersurface of  $IR^{4m}(m > 1)$  then from (III.14) we have

$$g((\nabla_X \phi_a)Y, Z) = 0$$

and from (III.8) we obtain

$$(\nabla_X \eta_a)(Y) = 0$$

for any  $X, Y \in \Gamma(TM)$ . Thus  $M$  is a manifold with cosymplectic 3-structure.

Conversely, if  $M$  is a manifold with cosymplectic 3-structure, from (III.14) we get

$$\eta_a(Y)A_N X = \alpha(X, Y)\xi_a$$

or

$$\eta_1(Y)A_N X = \alpha(X, Y)\xi_1$$

$$\eta_2(Y)A_N X = \alpha(X, Y)\xi_2$$

$$\eta_3(Y)A_N X = \alpha(X, Y)\xi_3$$

since  $\xi_1, \xi_2, \xi_3$  are linearly independent we get  $\alpha(X, Y) = 0$ . Thus proof is complete.

From (III.14) we have the following corollary.

**Corollary 3.6** *Let  $M$  be real hypersurface in  $IR^{4m}(m > 1)$ . Then  $M$  is a manifold with Sasakian 3-structure if and only if  $\alpha(X, Y) = g(X, Y)$  for any  $X, Y \in \Gamma(TM)$ .*

**Corollary 3.7** *Let  $M$  be totally umbilical real hypersurface in  $IR^{4m}(m > 1)$ . Then  $M$  is a manifold with nearly Sasakian 3-structure.*

**Proof.** For any  $X, Y \in \Gamma(TM)$ , from (III.7) we get

$$\begin{aligned} g((\nabla_X \phi_a) X, Y) &= \eta_a(X)g(h(X, Y), N) - \alpha(X, X)\eta_a(Y) \\ &= \eta_a(X)g(X, Y) - g(X, X)g(Y, \xi_a). \end{aligned}$$

Thus  $M$  is a manifold with nearly Sasakian 3-structure.

**Theorem 3.8** *Let  $M$  be a QR-submanifold of quaternion Kaehler manifold such that  $(\nabla_X \phi_a)Y = 0, X, Y \in \Gamma(D)$ . Then the quaternion distribution is involutive.*

**Proof.** By using (III.6), we obtain

$$\begin{aligned} g([X, Y], \xi_a) &= g(\nabla_X Y, \xi_a) - g(\nabla_Y X, \xi_a) \\ &= Xg(Y, \xi_a) - g(Y, \nabla_X \xi_a) - Yg(X, \xi_a) + g(X, \nabla_Y \xi_a) \\ &= g(X, \nabla_Y \xi_a) - g(Y, \nabla_X \xi_a) \\ &= -2g(Y, \nabla_X \xi_a) \\ &= 2g(\nabla_X Y, \xi_a) \end{aligned}$$

for any  $X, Y \in \Gamma(D)$  and  $\xi_a \in \Gamma(D^\perp)$ . Since  $D$  is invariant under  $\phi_a$  there exists nonzero vector field  $Z \in \Gamma(D)$  such that  $Y = \phi_a Z$ . Thus we have

$$\begin{aligned} g([X, Y], \xi_a) &= 2g(\nabla_X \phi_a Z, \xi_a) \\ &= 2g((\nabla_X \phi_a)Z + \phi_a(\nabla_X Z), \xi_a) \\ &= 2g(\phi_a(\nabla_X Z), \xi_a) \end{aligned}$$

here, by using (II.3) we obtain  $g([X, Y], \xi_a) = 0$  i.e.  $[X, Y] \in \Gamma(D)$ .

**Corollary 3.9** *Let  $M$  be a QR-submanifold of quaternion Kaehler manifold. If  $D$  is involutive, then each leaf of  $D$  is totally geodesic in  $M$ .*

**Proof.** The proof is obvious from proof of the Theorem 3.7.

**Theorem 3.10** *Let  $M$  be a QR-submanifold of quaternion Kaehler manifold. Then we have*

$$PA_N X = -X \iff P \nabla_X \xi_a = -\phi_a X$$

for any  $X \in \Gamma(D)$ . Where  $P$  is the projection morphism of  $TM$  to the quaternion distribution  $D$ .

**Proof.** From (II.2) we have

$$\begin{aligned} -\overline{\nabla}_X J_a N - h(X, J_a N) &= \nabla_X \xi_a \\ -(\overline{\nabla}_X J_a)N - J_a(\overline{\nabla}_X N) - h(X, J_a N) &= \nabla_X \xi_a. \end{aligned}$$

By using (II.2) and (II.15) we obtain

$$-Q_{ab}(X)J_b N - Q_{ac}(X)J_c N - J_a(A_N X + \nabla^\perp_X N - h(X, \xi_a)) = \nabla_X \xi_a.$$

Here, (II.1) and [1] we have

$$\begin{aligned} -Q_{ab}(X)J_b N - Q_{ac}(X)J_c N + J_a P A_N X \\ + \eta_b(A_n X)\xi_c - \eta_c(A_n X)\xi_b - \eta_a(A_n X)N \\ - B_a \nabla^\perp_X N - C_a \nabla^\perp_X N \\ - h(X, \xi_a) = P \nabla_X \xi_a + \eta_a(\nabla_X \xi_a)\xi_a \\ + \eta_b(\nabla_X \xi_a)\xi_b + \eta_c(\nabla_X \xi_a)\xi_c \end{aligned}$$

thus we obtain  $J_a P A_N X = P \nabla_X \xi_a$ . Hence we get assertion of theorem.

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