

1-1-2000

## QR-Submanifolds and Almost Contact 3-Structure

RIFAT GÜNEŞ

BAYRAM ŞAHİN

SADIK KELEŞ

Follow this and additional works at: <https://journals.tubitak.gov.tr/math>



Part of the [Mathematics Commons](#)

---

### Recommended Citation

GÜNEŞ, RIFAT; ŞAHİN, BAYRAM; and KELEŞ, SADIK (2000) "QR-Submanifolds and Almost Contact 3-Structure," *Turkish Journal of Mathematics*: Vol. 24: No. 3, Article 3. Available at: <https://journals.tubitak.gov.tr/math/vol24/iss3/3>

This Article is brought to you for free and open access by TÜBİTAK Academic Journals. It has been accepted for inclusion in Turkish Journal of Mathematics by an authorized editor of TÜBİTAK Academic Journals. For more information, please contact [academic.publications@tubitak.gov.tr](mailto:academic.publications@tubitak.gov.tr).

## QR-Submanifolds and Almost Contact 3-Structure

*Rıfat Güneş, Bayram Şahin and Sadık Keleş*

### Abstract

In this paper, QR-submanifolds of quaternion Kaehlerian manifolds with  $\dim \nu^\perp = 1$  has been considered. It is shown that each QR-submanifold of quaternion Kaehlerian manifold with  $\dim \nu^\perp = 1$  is a manifold with an almost contact 3-structure. We apply geometric theory of almost contact 3-structure to the classification of QR-submanifolds (resp. Real hypersurfaces) of quaternion Kaehler manifolds (resp.  $IR^{4m}$ ,  $m > 1$ ). Some results on integrability of an invariant distribution of a QR-submanifold and on the immersions of its leaves are also obtained.

**Key Words:** Quaternion Kaehler Manifold, QR-Submanifold, Almost Contact 3-Structure

### 1. Introduction

The geometry of QR-submanifolds of a quaternion Kaehlerian manifolds was firstly reported by Bejancu[1]. Among all submanifolds of a quaternion Kaehlerian manifold, QR-submanifolds have been intensively studied from different points of view by many authors [1],[2],[4].

In case of  $\dim \nu^\perp = 1$ , the study of QR-submanifolds has a significant importance. We show that QR-submanifolds of quaternion Kaehler manifolds with  $\dim \nu^\perp = 1$  admit an almost contact 3-structure.

### 2. Preliminaries

Let  $\overline{M}$  be a  $4n$ - dimensional manifold and  $g$  be a Riemannian metric on  $\overline{M}$ . Then  $\overline{M}$  is said to be quaternion Kaehlerian manifold (see, [1]) if there exists a 3-dimensional vector bundle of tensors of type(1.1) with local basis of almost Hermitian structures

*AMS Subject Classification:* 53C15,53C40

$J_1, J_2, J_3$  satisfying

$$J_1 \circ J_2 = -J_2 \circ J_1 = J_3 \quad (\text{II.1})$$

and

$$\bar{\nabla}_X J_a = \sum Q_{ab}(X) J_b, a = 1, 2, 3 \quad (\text{II.2})$$

for all vector fields  $X$  tangent to  $\bar{M}$ , where  $\bar{\nabla}$  is the Levi-Civita connection determined by  $g$  on  $\bar{M}$  and  $Q_{ab}$  are certain 1-forms locally defined on  $\bar{M}$  such that  $Q_{ab} + Q_{ba} = 0$ .

Let  $M$  be  $(4m + 3)$ -dimensional differentiable manifold and  $(\phi_a, \xi_a, \eta_a)$  be three almost contact structures on  $M$  i.e. We have

$$\begin{aligned} \phi_a^2(X) &= -X + \eta_a(X)\xi_a, \phi_a(\xi_a) = 0 \\ \eta_a(\xi_a) &= 1, \eta_a \circ \phi_a = 0 \end{aligned} \quad (\text{II.3})$$

where  $X$  tangent to  $M$ . Suppose that the almost contact structures satisfy the following conditions

$$\begin{aligned} \eta_a(\xi_b) &= 0, a \neq b, \phi_a(\xi_b) = -\phi_b(\xi_a) = \xi_c \\ \eta_a \circ \phi_b &= -\eta_b \circ \phi_a = \eta_c \\ (\phi_a \circ \phi_b)(X) - \xi_a(\eta_b(X)) &= (\phi_b \circ \phi_a)(X) - \xi_b(\eta_a(X)) = \phi_c(X) \end{aligned} \quad (\text{II.4})$$

for any cyclic permutation  $(a, b, c)$  of  $(1, 2, 3)$ . Then, we say that  $M$  is endowed with an almost contact 3-structure. If  $M$  is a Riemannian manifold, then we can choose a Riemannian metric  $g$  on  $M$  such that we have

$$\begin{aligned} (\phi_a X, \phi_a Y) &= g(X, Y) - \eta_a(X)\eta_a(Y) \\ \eta_a(X) &= g(X, \xi_a) \end{aligned} \quad (\text{II.5})$$

for any  $X, Y \in \chi(M)$ . In this case we say that  $(\phi_a, \xi_a, \eta_a), a = 1, 2, 3$  define an almost contact metric structure (See [5]). Taking account of ( II.3) and (II.5), we obtain

$$g(\phi_a X, Y) + g(X, \phi_a Y) = 0, a = 1, 2, 3$$

for any  $X, Y$  tangent to  $M$ .

Let  $M$  be an  $m$ -dimensional Riemannian manifold isometrically immersed in  $\overline{M}$ . We denote by  $TM$  (resp.  $TM^\perp$ ) the tangent (resp. normal) bundle to  $M$ . Then we say that  $M$  is a quaternion-real submanifold (QR-Submanifold) if there exists a vector bundle  $v$  of the normal bundle such that we have

$$J_a(v_x) = v_x \tag{II.6}$$

and

$$J_a(v_x^\perp) \subset T_M(x) \tag{II.7}$$

for  $x \in M$  and  $a = 1, 2, 3$ , where  $v^\perp$  is the complementary orthogonal bundle to  $v$  in  $TM^\perp$ . Let  $M$  be a QR-submanifold of  $\overline{M}$ . For sake of shortness we use  $D_{ax}$  for  $D_{ax} = J_a(v_x^\perp)$ ,  $a = 1, 2, 3$ . We consider  $D_{1x} \oplus D_{2x} \oplus D_{3x} = D_x^\perp$  and  $3s$ -dimensional distribution  $D^\perp : x \longrightarrow D_x^\perp$  globally defined on  $M$ . Where  $s = \dim v_x^\perp$ . Also we have, for each  $x \in M$ ,

$$J_a(D_{ax}) = v_x^\perp, J_a(D_{bx}) = D_{cx}, \tag{II.8}$$

where  $(a, b, c)$  is a cyclic permutation of  $(1, 2, 3)$ . Next, we denote by  $D$  the complementary orthogonal distribution to  $D^\perp$  in  $TM$ , we see that  $D$  is invariant with respect to the action of  $J_a$ . i.e. we have

$$J_a(D_x) = D_x \tag{II.9}$$

for any  $x \in M$ .  $D$  is called quaternion distribution . Also note that  $D_{1x}, D_{2x}, D_{3x}$  are mutually orthogonal vector spaces of  $T_M(x)$  (see [1]).

In [3] D.E.Blair introduced the concept cosymplectic structure as it follows. An almost contact metric structure  $(\phi, \xi, \eta, g)$  is a cosymplectic structure if and only if

$$(\nabla_X \phi)Y = 0, (\nabla_X \eta)Y = 0,$$

where  $\nabla$  is Levi-civita connection on  $M$ .

**Definition 2.1** *An almost 3-contact structure  $(\phi_a, \xi_a, \eta_a)$  is*

a) a 3-cosymplectic structure if

$$(\nabla_X \phi_a)Y = 0, \quad (\nabla_X \eta_a)Y = 0 \quad (\text{II.10})$$

b) a 3-Sasakian structure if

$$(\nabla_X \phi_a)Y = \eta_a(Y)X - g(X, Y)\xi_a \quad (\text{II.11})$$

c) a nearly 3-Sasakian structure if

$$(\nabla_X \phi_a)X = \eta_a(X)X - g(X, X)\xi_a \quad (\text{II.12})$$

where  $\nabla$  denotes the Levi-Civita connection and  $X, Y, Z$  are arbitrary vector fields on  $M$ .

For  $Y \in \chi(M)$ , we decompose as follows

$$J_a Y = P_a Y + F_a Y, \quad a = 1, 2, 3 \quad (\text{II.13})$$

where  $P_a Y$  and  $F_a Y$  the tangential and normal parts of  $J_a Y$ , respectively.

The formula Gauss and Weingarten are given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y) \quad (\text{II.14})$$

and

$$\bar{\nabla}_X V = -A_V X + \nabla_X^\perp V \quad (\text{II.15})$$

for any vector fields  $X, Y$  tangent to  $M$  and any vector field  $V$  normal to  $M$ . Where  $\nabla$  is the induced Riemann connection in  $M$ ,  $h$  is the second fundamental form,  $A_V$  is fundamental tensor of Weingarten with respect to the normal section  $V$  and  $\nabla^\perp$  normal connection on  $M$ . Moreover we have the relation

$$g(A_V X, Y) = g(h(X, Y), V) \quad (\text{II.16})$$

### 3. QR-Submanifolds with $\dim v^\perp = 1$

Let  $M$  be a QR-submanifold of a quaternion Kaehlerian manifold  $\bar{M}$  such that the dimension  $v^\perp$  is equal to one. In this case  $v^\perp$  is generated by unit vector field, say  $N$ .

We shall show in the sequel that  $N$  is precisely determined with one of the almost contact 3-structure. Let  $-J_a(N) = \xi_a$ ,  $a = 1, 2, 3$  and hence the distributions  $D_a$  are generated by the vector fields  $\xi_a$ . It is natural expect that a QR-submanifold of quaternion Kaehlerian manifold with  $\dim v^\perp = 1$  is almost contact 3-structure, we describe it as follows;

**Proposition 3.2** *Let  $\overline{M}$  be a quaternion Kaehlerian manifold and  $M$  be a QR-submanifold of  $\overline{M}$ . Then  $M$  is a manifold with almost contact 3-structure.*

**Proof.** For any  $X \in \Gamma(TM)$ , put

$$\phi_a X = P_a X, F_a(X) = \eta_a(X)N. \quad (\text{III.1})$$

Then  $g(F_a X, N) = \eta_a(X)$  and

$$\eta_a(X) = g(X, \xi_a). \quad (\text{III.2})$$

Moreover  $g(P_a X, P_a Y) = g(J_a X, J_a Y) - g(F_a X, F_a Y)$  implies

$$g(\phi_a X, \phi_a Y) = g(X, Y) - \eta_a(X)\eta_a(Y) \quad (\text{III.3})$$

and

$$J_a^2 X = J_a P_a X + J_a F_a X$$

or

$$-X = P_a^2 X + \eta_a(X)J_a N.$$

Hence

$$\phi_a^2 X = -X + \eta_a(X)\xi_a. \quad (\text{III.4})$$

On the other hand, from (III.2) and (III.4) we obtain  $\phi_a(\xi_a) = 0$  and  $\eta_a(\xi_a) = 1$ , respectively. Moreover

$$(\eta_a \circ \phi_a) X = \eta_a(\phi_a X) = g(\phi_a X, \xi_a) = 0$$

for any  $X \in \Gamma(TM)$ . From (III.1) and (III.2)

$$\eta_a(\xi_b) = 0$$

and

$$\begin{aligned} \phi_a(\xi_b) &= J_a(\xi_b) - F_a(\xi_b) \\ &= -J_a(J_b N) \\ &= -J_c N = \xi_c. \end{aligned}$$

By using (II.13) and (III.4), we obtain

$$\begin{aligned} (\eta_a \circ \phi_b)(X) &= \eta_a(\phi_b X) \\ &= g(\phi_b X, \xi_a) \\ &= \eta_c(X) \end{aligned}$$

and

$$\begin{aligned} (\phi_a \circ \phi_b)(X) - \xi_a(\eta_b(X)) &= (\phi_b \circ \phi_a)(X) - \xi_b(\eta_a(X)) \\ &= (\phi_b(\phi_a(X)) - \xi_b(\eta_a(X))) \\ &= \phi_c(X). \end{aligned}$$

This shows that on a QR-submanifold of a quaternion Kaehlerian manifold  $\dim v^\perp = 1$  there exists a natural almost contact 3-structure. i.e. tensor field  $\phi_a$  of type (1,1), 1-form  $\eta_a$  and unit vector field  $\xi_a$  satisfy (II.3), (II.4) and (II.5).

From now on will denote by  $M$  a QR-submanifold with  $\dim v^\perp = 1$ .

**Theorem 3.3** *Let  $M$  be a QR-submanifold of a quaternion Kaehlerian manifold. If for any  $X, Y \in \Gamma(TM)$ ,  $h(X, Y)$  has no component in  $\Gamma(v^\perp)$  and  $D_a, a = 1, 2, 3$  are parallel in  $M$ , then  $M$  is a manifold with cosymplectic 3-structure.*

**Proof.** For any  $X, Y \in \Gamma(TM)$ , from (II.2) we have

$$\bar{\nabla}_X J_a Y = Q_{ab}(X) J_b Y + Q_{ac}(X) J_c Y + J_a \bar{\nabla}_X Y.$$

Taking account of (II.13),(II.14),(II.15) and (III.1) we obtain

$$\begin{aligned}
 (\nabla_X \phi_a) Y + (\nabla_X \eta_a(Y)) N + h(X, \phi_a Y) \\
 - \eta_a(Y) A_N X + \eta_a(Y) \nabla_X^\perp N = Q_{ab}(X) \phi_b Y + Q_{ab}(X) \eta_b(Y) N \\
 + Q_{ac}(X) \phi_c Y + Q_{ac}(X) \eta_c(Y) N \\
 + B_a h(X, Y) + C_a h(X, Y).
 \end{aligned}$$

Comparing the tangential and normal parts both of sides of this equation, we have

$$\begin{aligned}
 (\nabla_X \phi_a) Y = \eta_a(Y) A_N X + Q_{ab}(X) \phi_b Y + Q_{ac}(X) \phi_c Y \\
 + B_a h(X, Y)
 \end{aligned} \tag{III.5}$$

and

$$\begin{aligned}
 (\nabla_X \eta_a(Y)) N = -h(X, \phi_a Y) + \eta_a(Y) \nabla_X^\perp N \\
 + Q_{ab}(X) \eta_b(Y) N + Q_{ac}(X) \eta_c(Y) N \\
 + C_a h(X, Y).
 \end{aligned} \tag{III.6}$$

Now, we decompose  $h(X, Y)$  into components  $h^1(X, Y)$  and  $h^2(X, Y)$  in  $v^\perp$  and  $v$  respectively as

$$h(X, Y) = h^1(X, Y) + h^2(X, Y)$$

where we can put  $h^1(X, Y) = \alpha(X, Y) N$  for some scalar valued bilinear function  $\alpha$ . Thus (III.5) and (III.6) gives

$$\begin{aligned}
 (\nabla_X \phi_a) Y = \eta_a(Y) A_N X + Q_{ab}(X) \phi_b Y + Q_{ac}(X) \phi_c Y \\
 - \alpha(X, Y) \xi_a
 \end{aligned} \tag{III.7}$$

and

$$(\nabla_X \eta_a)(Y) = -\alpha(X, \phi_a Y) + Q_{ab}(X) \eta_b(Y) + Q_{ac}(X) \eta_c(Y). \tag{III.8}$$

On the other hand, since  $\xi_a = -J_a N$  we have



$$\bar{\nabla}_X \xi_a = -(\bar{\nabla}_X J_a)N - J_a \bar{\nabla}_X N$$

for any  $X \in \Gamma(TM)$ . Thus by using (II.2),(II.8),(II.15) and taking tangential parts we obtain

$$\nabla_X \xi_a = Q_{ab}(X)\xi_b + Q_{ac}(X)\xi_c + \phi_a A_N X$$

for any  $X \in \Gamma(TM)$ . Thus we have

$$g(\nabla_X \xi_a, \xi_b) = Q_{ab}(X) + g(\phi_a A_N X, \xi_b).$$

From (II.4) and (II.16) we derive

$$\eta_b(\nabla_X \xi_a) + \alpha(X, \xi_c) = Q_{ab}(X) \quad (\text{III.9})$$

Similarly, we get

$$\eta_c(\nabla_X \xi_a) - \alpha(X, \xi_b) = Q_{ac}(X) \quad (\text{III.10})$$

Combining (III.7) and (III.8) with (III.9) and (III.10) we have

$$\begin{aligned} (\nabla_X \phi_a) Y &= \eta_a(Y) A_N X + (\eta_b(\nabla_X \xi_a) + \alpha(X, \xi_c)) \phi_b Y \\ &\quad + (\eta_c(\nabla_X \xi_a) - \alpha(X, \xi_b)) \phi_c Y - \alpha(X, Y) \xi_a \end{aligned} \quad (\text{III.11})$$

and

$$\begin{aligned} (\nabla_X \eta_a)(Y) &= -\alpha(X, \phi_a Y) + (\eta_b(\nabla_X \xi_a) + \alpha(X, \xi_c)) \eta_b(Y) + \\ &\quad (\eta_c(\nabla_X \xi_a) - \alpha(X, \xi_b)) \eta_c(Y) \end{aligned} \quad (\text{III.12})$$

for any  $X, Y \in \Gamma(TM)$ . From (II.16) and (III.11) we get

$$\begin{aligned} g((\nabla_X \phi_a) Y, Z) &= \eta_a(Y) \alpha(X, Z) + \eta_b(\nabla_X \xi_a) g(\phi_b Y, Z) \\ &\quad + \alpha(X, \xi_c) g(\phi_b Y, Z) + (\eta_c(\nabla_X \xi_a) - \alpha(X, \xi_b)) g(\phi_c Y, Z) \\ &\quad - \alpha(X, Y) \eta_a(Z). \end{aligned} \quad (\text{III.13})$$

Thus if  $h(X, Y)$  has no components in  $\Gamma(v^\perp)$  and  $D_a, a = 1, 2, 3$  are parallel in  $M$ , then from (III.12) and (III.13), we get  $(\nabla_X \phi_a)Y = 0$  and  $(\nabla_X \eta_a)(Y) = 0$  that is  $M$  is a manifold with cosymplectic 3-structure.

As immediate consequence of theorem we have the following;

**Corollary 3.4** *Let  $M$  be real hypersurface of quaternion Kaehler manifold  $\overline{M}$ . If  $M$  is totally geodesic and  $\xi_a$  are parallel in  $M$  then  $M$  is a manifold with cosymplectic 3-structure.*

**Corollary 3.5** *Let  $M$  be real hypersurface in  $IR^{4m}(m > 1)$ . Then  $M$  is totally geodesic if and only if  $M$  is a manifold with cosymplectic 3-structure.*

**Proof.** Since  $\overline{M} = IR^{4m}(m > 1)$  we have  $\overline{\nabla}_X J_a = 0$ . Thus from (III.13) we get

$$g((\nabla_X \phi_a)Y, Z) = \eta_a(Y)\alpha(X, Z) - \alpha(X, Y)\eta_a(Z) \quad (III.14)$$

for any  $X, Y, Z \in \Gamma(TM)$ . Let  $M$  be a totally geodesic real hypersurface of  $IR^{4m}(m > 1)$  then from (III.14) we have

$$g((\nabla_X \phi_a)Y, Z) = 0$$

and from (III.8) we obtain

$$(\nabla_X \eta_a)(Y) = 0$$

for any  $X, Y \in \Gamma(TM)$ . Thus  $M$  is a manifold with cosymplectic 3-structure.

Conversely, if  $M$  is a manifold with cosymplectic 3-structure, from (III.14) we get

$$\eta_a(Y)A_N X = \alpha(X, Y)\xi_a$$

or

$$\eta_1(Y)A_N X = \alpha(X, Y)\xi_1$$

$$\eta_2(Y)A_N X = \alpha(X, Y)\xi_2$$

$$\eta_3(Y)A_N X = \alpha(X, Y)\xi_3$$

since  $\xi_1, \xi_2, \xi_3$  are linearly independent we get  $\alpha(X, Y) = 0$ . Thus proof is complete.

From (III.14) we have the following corollary.

**Corollary 3.6** *Let  $M$  be real hypersurface in  $IR^{4m}(m > 1)$ . Then  $M$  is a manifold with Sasakian 3-structure if and only if  $\alpha(X, Y) = g(X, Y)$  for any  $X, Y \in \Gamma(TM)$ .*

**Corollary 3.7** *Let  $M$  be totally umbilical real hypersurface in  $IR^{4m}(m > 1)$ . Then  $M$  is a manifold with nearly Sasakian 3-structure.*

**Proof.** For any  $X, Y \in \Gamma(TM)$ , from (III.7) we get

$$\begin{aligned} g((\nabla_X \phi_a) X, Y) &= \eta_a(X)g(h(X, Y), N) - \alpha(X, X)\eta_a(Y) \\ &= \eta_a(X)g(X, Y) - g(X, X)g(Y, \xi_a). \end{aligned}$$

Thus  $M$  is a manifold with nearly Sasakian 3-structure.

**Theorem 3.8** *Let  $M$  be a QR-submanifold of quaternion Kaehler manifold such that  $(\nabla_X \phi_a)Y = 0, X, Y \in \Gamma(D)$ . Then the quaternion distribution is involutive.*

**Proof.** By using (III.6), we obtain

$$\begin{aligned} g([X, Y], \xi_a) &= g(\nabla_X Y, \xi_a) - g(\nabla_Y X, \xi_a) \\ &= Xg(Y, \xi_a) - g(Y, \nabla_X \xi_a) - Yg(X, \xi_a) + g(X, \nabla_Y \xi_a) \\ &= g(X, \nabla_Y \xi_a) - g(Y, \nabla_X \xi_a) \\ &= -2g(Y, \nabla_X \xi_a) \\ &= 2g(\nabla_X Y, \xi_a) \end{aligned}$$

for any  $X, Y \in \Gamma(D)$  and  $\xi_a \in \Gamma(D^\perp)$ . Since  $D$  is invariant under  $\phi_a$  there exists nonzero vector field  $Z \in \Gamma(D)$  such that  $Y = \phi_a Z$ . Thus we have

$$\begin{aligned} g([X, Y], \xi_a) &= 2g(\nabla_X \phi_a Z, \xi_a) \\ &= 2g((\nabla_X \phi_a)Z + \phi_a(\nabla_X Z), \xi_a) \\ &= 2g(\phi_a(\nabla_X Z), \xi_a) \end{aligned}$$

here, by using (II.3) we obtain  $g([X, Y], \xi_a) = 0$  i.e.  $[X, Y] \in \Gamma(D)$ .

**Corollary 3.9** *Let  $M$  be a QR-submanifold of quaternion Kaehler manifold. If  $D$  is involutive, then each leaf of  $D$  is totally geodesic in  $M$ .*

**Proof.** The proof is obvious from proof of the Theorem 3.7.

**Theorem 3.10** *Let  $M$  be a QR-submanifold of quaternion Kaehler manifold. Then we have*

$$PA_N X = -X \iff P \nabla_X \xi_a = -\phi_a X$$

for any  $X \in \Gamma(D)$ . Where  $P$  is the projection morphism of  $TM$  to the quaternion distribution  $D$ .

**Proof.** From (II.2) we have

$$\begin{aligned} -\overline{\nabla}_X J_a N - h(X, J_a N) &= \nabla_X \xi_a \\ -(\overline{\nabla}_X J_a)N - J_a(\overline{\nabla}_X N) - h(X, J_a N) &= \nabla_X \xi_a. \end{aligned}$$

By using (II.2) and (II.15) we obtain

$$-Q_{ab}(X)J_b N - Q_{ac}(X)J_c N - J_a(A_N X + \nabla^\perp_X N - h(X, \xi_a)) = \nabla_X \xi_a.$$

Here, (II.1) and [1] we have

$$\begin{aligned} -Q_{ab}(X)J_b N - Q_{ac}(X)J_c N + J_a P A_N X \\ + \eta_b(A_n X)\xi_c - \eta_c(A_n X)\xi_b - \eta_a(A_n X)N \\ - B_a \nabla^\perp_X N - C_a \nabla^\perp_X N \\ - h(X, \xi_a) = P \nabla_X \xi_a + \eta_a(\nabla_X \xi_a)\xi_a \\ + \eta_b(\nabla_X \xi_a)\xi_b + \eta_c(\nabla_X \xi_a)\xi_c \end{aligned}$$

thus we obtain  $J_a P A_N X = P \nabla_X \xi_a$ . Hence we get assertion of theorem.

### Acknowledgement

We are thankful to the referee for his(her)valuable comments on the paper.

### References

- [1] Bejancu, A. QR-Submanifolds of Quaternion Kaehler Manifolds, Chinese Journal of Math. Vol:14 No:42 (1986)
- [2] Bejancu, A. Generic Submanifolds of Manifolds with Sasakian 3-Structure, Math. Rep. ToyamaUniv. 75-101 (1985)
- [3] Blair, D. E. Contact Manifolds in Riemannian Geometry, Lecture Notes in Math. 509. Springer-Verlag (1976)

GÜNEŞ, ŞAHİN, KELEŞ

- [4] Khan, K. A. CR-Submanifolds and Almost Contact Structure, Math. J. Toyama Univ. Vol:16, 29-37 (1993)
- [5] Kuo, Y. Y. On Almost Contact 3-Structure, Tohoku Math. J. 22, 325-332 (1970)

Rıfat GÜNEŞ, Bayram ŞAHİN and Sadık KELEŞ  
Department of Mathematics,  
Faculty of Science and Arts,  
Inonu University,  
44100 Malatya-TURKEY  
e-mail: bsahin@inonu.edu.tr

Received 28.01.1999