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Abstract

The aim of this paper is to introduce the class of OFR-sets as the sets that are the intersection of an open set and a feeble-regular set. Several classes of matroids are studied via the new concept. New decompositions of strong maps are provided.

Key Words: Feeble-matroid, Strong map, Hesitant map, ORF-set, OFF-set, OFR-set.

1. Introduction

For an introduction on matroids see [3, 4, 7, 8, 9]. In particular, a *matroid* M is an ordered pair $(E,\mathcal{O})$ such that $\mathcal{O}$ is a collection of subsets, called *open sets* of M, of a finite set E, called the *ground set* of M, such that $\varnothing$ is an open set, unions of open sets are open and if O_1 and O_2 are open sets and $x \in O_1 \cap O_2$, there exists an open set O_3 such that

$$(O_1 \cup O_2) - (O_1 \cap O_2) \subseteq O_3 \subseteq (O_1 \cup O_2) - \{x\}.$$

An equivalent way of defining a matroid M, is that M is an ordered pair $(E, \mathfrak{F}_M)$ such that $\mathfrak{F}_M$ is a collection of subsets, called *flats* or *closed sets* of M, of a finite set E such that $E \in \mathfrak{F}_M$, intersections of flats are flats and if $F \in \mathfrak{F}_M$ and $\{F_1, F_2, ..., F_k\}$ is the set of minimal members of $\mathfrak{F}_M$ (with respect to inclusion) that properly contain F, then $F_1 \cup F_2 \cup ... \cup F_k = E$. The *closure* of a subset $A \subseteq E$ will be denoted by $\bar{A}$. Clearly $\bar{A}$ is the smallest flat containing A and $x \in \bar{A}$ if and only if for every open set O in M that contains x, $O \cap A \neq \varnothing$, see Oxley [4]. A is a *spanning set* of M if $\bar{A} = E$. Let $M_1 = (E_1,\mathcal{F}_1)$ and $M_2 = (E_2,\mathcal{F}_2)$ be matroids. A *strong map* f from M_1 to M_2 is a map $f : E_1 \rightarrow E_2$ such that the inverse image of any flat of M_2 is a flat of M_1. We abbreviate

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this as $f : M_1 \to M_2$. Clearly, f is strong if and only if the inverse image of any open set in M_2 is open in M_1. A set $U \subseteq E$ is called a *feeble-open set (=FO-set)* in M if there exists an open set $O \in \mathcal{O}$ such that $O \subseteq U \subseteq \bar{O}$, see Al-Hawary [1]. A subset $A \subseteq E$ is *feeble-flat (=FF-set)* if its complement is an FO-set. *Feeble-closure* $\underline{A}$ of A can be defined in a manner analogous to the closure $\bar{A}$ of A. The *inner* of A is the set

$$\overset{o}{A}=: \{x \in A \mid \exists O \in \mathcal{O},\ x \in O \subseteq A\}.$$

Clearly A is a FO-set if and only if $A \subseteq \overline{\overset{o}{A}}$ and A is a FF-set if and only if $\overset{o}{A}=\overset{o}{\bar{A}}$. *Feeble-inner* A_o of A and *feeble-spanning* set can be defined an analogous manner to the inner and spanning set notions, respectively. A is called a *local-flat (=LF-set)* if A is open in $M|\bar{A}$ or equivalently if $A = O \cap F$, where O is open and F is a flat. A is called *regular-open (=RO-set)* if $A = \overset{o}{\bar{A}}$. Complements of RO-sets are called *regular-flats (=RF-set).* Clearly A is a RF-set if and only if A is a flat and $A = \overline{\overset{o}{A}}$. A is called a *ORF-set* (resp. a *OFF-set*) if $A = O \cap C$, where O is open and C is a RF-set (resp. FF-set). Clearly every ORF-set is a LF-set and every LF-set is a OFF-set. A is called a *feeble-preopen (=FP-set)* if $A \subseteq \overline{\overset{o}{A}}$. The *feeble-closure* of A is the intersection of all FF supersets of A.

The concepts of ORF-sets, LF-sets and OFF-sets play an important role when strong maps are decomposed. A map $f : M_1 \to M_2$ is *feeble-strong (=FS)* if the inverse image of any open set in M_2 is feeble-open set in M_1. f is called $\hat{A}$*-strong* if for every open set O in M_2, the set $f^{-1}(O) \in \hat{A}$, where $\hat{A}$ is a collection of subsets of E_1. Most of the definitions of maps used through this paper are consequences of the definition of $\hat{A} - strong$ map.

The aim of this paper is to introduce the classes of ORF-sets and OFF-sets and a class of sets very closely related to these classes, in fact properly placed between them, called *OFR-sets.* Under consideration are sets that can be represented as the intersection of an open set and a feeble-regular set. A subset A of the ground set of a matroid $M = (E,\mathcal{O})$ is called *feeble-regular set(=FR-set)* if it is both FO-set and FF-set.

Theorem 1 [1]*Let $M = (E,\mathcal{O})$ be a matroid and $A \subseteq E$. Then $(\bar{A})^o \subseteq (\underline{A})_o$*

Theorem 2 *If a subset A of the ground set of a matroid $M = (E,\mathcal{O})$ is a FR-set, then there exists a RO-set O such that $O \subseteq A \subseteq \bar{O}$.*

Proof. Let A be a FR-set and let $O = A^o$. As A is a FO-set, $A \subseteq \overset{o}{\bar{O}}$ or $O \subseteq \overset{o}{\bar{O}}$. On the other hand, by Theorem 1, $(\bar{O})^o \subseteq (\underline{O})_o = (\underline{A^o})_o = (A^o)_o = O_o \subseteq O$. Thus $O = \overset{o}{\bar{O}}$ and hence O is a RO-set such that $O \subseteq A \subseteq \bar{O}$. □

In this paper, the connection of OFR-sets to the other classes of "generalized open" sets is investigated as well as several characterizations of matroids via OFR-sets are given. The concept of OFR-strong maps is also introduced. New decompositions of strong maps and decompositions of OFR-strong maps are produced at the end of the paper.

2. OFR-sets

Definition 1 *A subset A of the ground set of a matroid M is called a* OFR-set *if $A = O \cap B$, where O is open and B is FR. The collection of all OFR-sets of M will be denoted by $OFR(M)$.*

Since RF-sets are FR-sets and since FR-sets are FF-sets, then the following implications are obvious.

$$ORF - set \Rightarrow OFR - set \Rightarrow OFF - set.$$

None of them of course is reversible as the following examples show:

Example 1 *Let $E = \{a, b, c, d\}$ and let $\mathcal{O} = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$. Set $A = \{a, b\}$. It is easily observed that A is a OFR-set but not a ORF-set.*

Example 2 *Let $E = \{a, b, c, d\}$ and let $\mathcal{O} = \{\varnothing, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. Set $A = \{c\}$. It is easily observed that $A = \{b, c\} \cap \{c, d\}$ is a OFF-set but not a OFR-set.*

Next, the relation between ORF-sets and OFF-sets is shown but first consider the following lemma.

Lemma 1 *The feeble-closure of every FP-set is a FR-set.*

Proof. Let A be such that $A \subseteq \overline{\overset{o}{A}}$ and $C = \bigcap\{B : A \subseteq B = \underline{B}\}$. Then $\underline{C} \subseteq \bigcap\{B : A \subseteq B = \underline{B}\} = C$ and as $C \subseteq \underline{C}$, C is a FF-set. Since $\overline{\overset{o}{A}}$ is a flat, it is a FF-set and hence it equals its feeble-closure and as it contains A,

$$C \subseteq \overline{\overset{o}{A}}. \tag{2.1}$$

As $\bar{A}$ is the smallest flat containing A and as every flat is a FF-set, $\bar{A} \subseteq C$. This together with 2.1 implies that $C \subseteq \overline{C^o}$ and hence C is a FO-set. Therefore, C is a FR-set. □

Theorem 3 *Let M be a matroid in which every OFR-set is a FO-set. Then for a subset A of the ground set of M the following are equivalent:*

(1) A is a OFR-set.

(2) A is an FO- and OFF-set.

(3) A is an FP- and OFF-set.

Proof. (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (1) Since A is a OFF-set, then there exists an open set O such that $A = O \cap \underline{A}$. By Lemma 1, $\underline{A}$ is a FR-set, since by (3) A is an FP-set. Thus A is a OFR-set. □

Definition 2 *A matroid M is called* maximal *if every spanning set is open and every OFR-set is a FO-set.*

Let $FP(M)$ denote the collection of all FP-sets in M. Then we obtain the following result.

Theorem 4 *If $M = (E, \mathcal{O})$ is a maximal matroid, then every subset of E is a OFF-set.*

Proof. Let $A \subseteq E$. Since every submatroid of a maximal matroid is maximal, then $M|\bar{A}$ is maximal. Since $\underline{A}$ is a spanning in $M|\bar{A}$, A is open in $M|\bar{A}$. Thus $A = O \cap \bar{A}$ where O is an open set in M and $\bar{A}$ is a FF-set in M. Thus A is a OFF-set. □

Corollary 1 *If $M = (E, \mathcal{O})$ is a maximal matroid, then $OFR(M) = FP(M)$.*

Proof. Since M is maximal, then by Theorem 4, every (FP-) set is a OFF-set. Thus by Theorem 3, every FP subset of E is a OFR-set. On the other hand, every OFR-set is a FP-set. □

Lemma 2 *In a matroid $M = (E, \mathcal{O})$, a LF-set that is also a FO-set is a ORF-set.*

Proof. Let $A \subseteq E$ be both LF- and FO-set. Then $A \subseteq \overline{\overset{o}{A}}$ and $A = O \cap \bar{A}$ for some open set O. Thus $\bar{A} = \overline{\overset{o}{A}}$ and so A is a RF-set. Hence A is a ORF-set. □

The class of LF-sets is also properly placed between the class of ORF- and OFF-sets but the concepts of OFR-sets and LF-sets are independent from each other: First, the set $A = \{a\}$ is a LF-set that is not a OFR-set in the matroid of Example 2, hence not every LF-set is a OFR-set. Second, if every OFR-set would be a LF-set, then again it must be a ORF-set but as shown before not all OFR-sets are ORF-sets.

Let $M = (E, \mathcal{O})$ be a matroid and $A \subseteq E$. Define

$$Fr(A) := \{e \in E : O \cap A \neq \varnothing \, and \, O \cap E \backslash A \neq \varnothing, \forall O \in \mathcal{O}\}.$$

Then A is called an CIB if and only if $\overset{o}{\overline{Fr(A)}} = \varnothing$. The following result shows that the defined property coincides with the class of FF-sets.

Theorem 5 *Let $M = (E, \mathcal{O})$ be a matroid in which $E \in \mathcal{O}$. Then the following are equivalent:*

(1) $\overset{o}{\bar{A}} = \overset{o}{A}$.

(2) A is a FF-set.

(3) $E \backslash A$ is a FP-set and A is a OFF-set.

(4) $E \backslash A$ is a FP-set and A is a CIB-set.

Proof. (1) $\Rightarrow$ (2) Since $\overset{o}{\bar{A}} = \overset{o}{A} \subseteq A$, then $E \backslash A \subseteq \overline{(E \backslash A)^o}$. Thus $E \backslash A$ is a FO-set, hence A is a FF-set.

(2) $\Rightarrow$ (3) Every FF-set is trivially a FP-set. Since $A = E \cap A$, where E is open and A is FF-set, then A is a OFF-set.

(3) $\Rightarrow$ (4) Clearly the intersection of two CIB-sets is a CIB-set. Since a OFF-set is an intersection of an (FO-set) open set and a FF-set, it is enough to show that every FO-set and every FF-set is a CIB-set. If A is a FO-set, then for some open set O we have $O \subseteq A \subseteq \bar{O}$. Since $Fr(A) = \bar{A} \cap \overline{E \backslash A} = \bar{O} \cap \overline{E \backslash A} \subseteq \bar{O} \cap \overline{E \backslash O} = Fr(O)$, clearly $\overset{o}{\overline{Fr(A)}} = \varnothing$ as $\overset{o}{\overline{Fr(O)}} = \varnothing$. In fact, it is obvious that every open set is CIB. Thus FO-(and hence every FF-) is a CIB-set.

(4) $\Rightarrow$ (1) Since A is a CIB-set, $B = E \backslash A$ is also a CIB-set. It is easy to see that from the identity

$$(Fr(B))^o = \overset{o}{\bar{B}} \cap \overset{o}{\overline{E \backslash B}} = \overset{o}{\bar{B}} \cap E \backslash \overline{\overset{o}{B}} = \overset{o}{\bar{B}} \backslash \overline{\overset{o}{B}},$$

it follows that $\overset{o}{\bar{B}} \subseteq \overline{\overset{o}{B}}$. Since B is a FP-set, $B \subseteq \overline{\overset{o}{\bar{B}}}$. Thus $B \subseteq \overline{\overset{o}{B}}$ or equivalently $\bar{B} = \overline{\overset{o}{B}}$. Since $B = E \backslash A$, $\overset{o}{\bar{A}} = \overset{o}{A}$. □

Theorem 6 *Let $M = (E, \mathcal{O})$ be a matroid in which every OFR-set is a FO-set and $E \in \mathcal{O}$. Then for a subset A of the ground set of M the following are equivalent:*

(1) A is a FR-set.

(2) A is a FF-set and a OFR-set.

(3) A is a OFR-set and $E \backslash A$ is a FP-set.

Proof. (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (1) By Theorem 5 A is a FF-set, since $E \backslash A$ is a FP-set and A is a OFF-set. On the other hand A is a FO-set, since it is a OFR-set. Thus A is a FR-set, being both FO- and FF-set. ∎

Definition 3 *A subset A of the ground set of a matroid M is called an* inner-flat (=IF-set) *if A^o is a flat in $M|A$. If $A \subseteq \overset{o}{\overline{A}}$, then A is called* prespanning.

Lemma 3 *In a matroid $M = (E, \mathcal{O})$, if a subset $A \subseteq E$ is a prespanning set and a OFF-set, then A is open.*

Proof. Since A is a OFF-set, we have $A = O \cap S$ where O is open and $\overset{o}{\bar{S}} = \overset{o}{S}$. Because A is prespanning, we have

$$A \subseteq \overset{o}{\bar{A}} = \overset{o}{\overline{(O \cap S)}} \subseteq (\bar{O} \cap \bar{S})^o \subseteq \overset{o}{\bar{O}} \cap \overset{o}{\bar{S}} = \overset{o}{\bar{O}} \cap \overset{o}{S} .$$

Hence

$$A = O \cap S = (O \cap S) \cap O \subseteq (\overset{o}{\bar{O}} \cap \overset{o}{S}) \cap O = (\overset{o}{\bar{O}} \cap O) \cap \overset{o}{S} = O \cap \overset{o}{S} .$$

Notice $A = O \cap S \supseteq O \cap \overset{o}{S}$, we have $A = O \cap \overset{o}{S}$. ∎

Theorem 7 *Let $M = (E, \mathcal{O})$ be a matroid in which every OFR-set is a FO-set and $E \in \mathcal{O}$. Then for a subset A of the ground set of M the following are equivalent:*

(1) A is open.

(2) A is a OFR-set and A is either prespanning or a IF-set.

Proof. (1) $\Rightarrow$ (2) is obvious.

(2) $\Rightarrow$ (1) If A is prespanning, then since A is also a OFF-set, it follows by Lemma 3 that A is open. If A is a IF-set, then A is again open, since A is also a FO-set. ∎

3. Some peculiar matroids

A matroid $M = (E, \mathcal{O})$ is called *extremally disconnected (=ED)* if every open subset has open closure or equivalently if every RF subset of E is open.

Theorem 8 *Let $M = (E, \mathcal{O})$ be an ED matroid in which every OFR-set is a FO-set. If A is a OFR-set, then A is prespanning.*

Proof. Let A be a OFR-set. Then A is a FO-set and so $A \subseteq \overline{A^o}$. As A^o is open, by assumption, $\overline{A^o}$ is open and hence $\overline{A^o} = \overset{o}{\overline{A^o}}$. Thus $A \subseteq \overset{o}{\overline{A^o}} \subseteq \overset{o}{\overline{A}}$. Therefore, A is prespanning. ∎

Theorem 9 *For a matroid $M = (E, \mathcal{O})$, in which every OFR-set is a FO-set and $E \in \mathcal{O}$, the following are equivalent:*

(1) M is ED.

(2) $\mathcal{O} = OFR(M)$.

(3) Every OFR-set is open.

Proof. (1) $\Rightarrow$ (2) Let A be a OFR-set. By Theorem 8, it follows that A is prespanning, since M is ED. Moreover, A is a OFF-set and since it is prespanning, it follows from Theorem 3 that $A \in \mathcal{O}$. Hence $OFR(M) \subseteq \mathcal{O}$. On the other hand it is obvious that $\mathcal{O} \subseteq OFR(M)$.

(2) $\Rightarrow$ (3) is obvious.

(3) $\Rightarrow$ (1) Let A be a RF-set. Then A is a OFR-set. By (3) A is open. So, M is ED. ∎

Theorem 10 *For a matroid $M = (E, \mathcal{O})$, in which every OFR-set is a FO-set and $E \in \mathcal{O}$, the following are equivalent:*

(1) M is maximal.

(2) Every prespanning set is a OFR-set.

(3) Every spanning set is a OFR-set.

Proof. (1) $\Rightarrow$ (2) Let A be a prespanning set. By (1), A is open, since in a maximal matroid every prespanning set is open. Hence A is a OFR-set.

(2) $\Rightarrow$ (3) Every spanning set is a prespanning set.

(3) $\Rightarrow$ (1) Let A be a spanning set. By (3), A is a OFR-set. Hence A is both prespanning and a OFF-set. It follows by Theorem 8 that A is open. Thus M is maximal. ∎

A matroid is called a *partition matroid (=PM)* if every open set is a flat.

Theorem 11 *If $M = (E, \mathcal{O})$ is a PM, every OFR-set is a flat.*

Proof. Let A be a OFR-set. Then A is a OFF-set and hence $A = O \cap B$, where O is open and B is a FF-set. By assumption, O is a flat. On the other hand $\bar{B}$ is open by assumption and thus $\overset{o}{\bar{B}} \subseteq B \subseteq \bar{B}$ implies $B = \overset{o}{B} = \bar{B}$ and thus B is a flat. Thus A is a flat being the intersection of two flats. ∎

Theorem 12 *For a matroid $M = (E, \mathcal{O})$ in which $E \in \mathcal{O}$, the following are equivalent:*

(1) $M \cong U_{1,n}$ for some positive integer $n \geq 1$.

(2) The only OFF-sets in M are the trivial ones.

(3) The only ORF-sets in M are the trivial ones.

Proof. (1) ⇒ (2) If A is a OFF-set, then $A = O \cap B$, where O is open and B is FF-set ($B^o = \overline{B^o}$). If $A \neq \varnothing$, then $O \neq \varnothing$ and by (1) $O = E$. Thus $A = B$ and so $A^o = (\bar{A})^o = E^o = E$. Hence $A = E$.

(2) ⇒ (3) Every ORF-set is a OFF-set.

(3) ⇒ (1) Since every open set is a ORF-set, by (3) the only open sets in M are the trivial ones. ∎

Corollary 2 *For a matroid $M = (E, \mathcal{O})$ in which $E \in \mathcal{O}$, the following are equivalent:*

(1) $M \cong U_{1,n}$ for some positive integer $n \geq 1$.

(2) $OFR(M) = \{\varnothing, E\}$.

Theorem 13 *For a matroid $M = (E, \mathcal{O})$, in which every OFR-set is a FO-set and $E \in \mathcal{O}$, the following are equivalent:*

(1) M is free.

(2) Every subset of E is a OFR-set.

(3) Every singleton is a OFR-set.

Proof. (1) ⇒ (2) and (2) ⇒ (3) are obvious.

(3) ⇒ (1) Let $e \in E$. By (3), $\{e\}$ is a OFR-set and hence a FO-set. Then $\{e\}$ must contain a non-void open subset. Since the only possibility is $\{e\}$ itself, then each singleton is open or equivalently M is free. ∎

A matroid $M(E,\mathcal{O})$ is called *hyperconnected* if every open set is a spanning set. $M=(E,\mathcal{O})$ is called *feeble-connected (=FC)* if E cannot be expressed as the disjoint union of two non-void FO-sets.

Theorem 14 *For a matroid $M=(E,\mathcal{O})$, in which every OFR-set is a FO-set and $E\in\mathcal{O}$, the following are equivalent:*
(1) M is hyperconnected.
(2) Every OFR-set is spanning.

Proof. (1) $\Rightarrow$ (2) Let A be a OFR-set. Then A is a FO-set and hence there exists an open set O such that $O\subseteq A\subseteq\bar{O}$. By (1), O is spanning. Hence its superset A is also spanning.

(2) $\Rightarrow$ (1) Every open set is a OFR-set and hence by (2) spanning. ∎

Theorem 15 *For a matroid $M=(E,\mathcal{O})$, in which every OFR-set is a FO-set and $E\in\mathcal{O}$, the following are equivalent:*
(1) M is FC.
(2) E is not the union of two disjoint non-void OFR-sets.

Proof. (1) $\Rightarrow$ (2) If E is the union of two disjoint non-void OFR-sets, then M is not FC, since OFR-sets are FO-sets.

(2) $\Rightarrow$ (1) If M is not FC, then M has a non-trivial FO-set A with FO complement. Since both A and $B=E\backslash A$ are FR-sets, then A and B are OFR-sets. So E is the union of two disjoint non-void OFR-sets, contradictory to (2). ∎

4. OFR-strong maps

Decompositions of continuous maps have been studied by several authors, see for example [2, 5, 6]. In this section, we study several decompositions of strong maps.

Definition 4 *A map $f: M_1=(E_1,\mathcal{O}_1)\rightarrow M_2=(E_2,\mathcal{O}_2)$ is called* ORF-strong *(resp.* OFF-strong, OFR-strong*) if the preimage of every open set in M_2 is a ORF-set (resp. OFF-set, OFR-set) of M_1. f is* hesitant *if $f(\underline{A})\subseteq f(A)$, for every subset $A\subseteq E_1$, see Al-Hawary [1].*

All through this section, we only consider matroids in which the ground sets are open and every OFR-set is a FO-set. It is easily observed that f is hesitant if and only if the inverse image of every subset of E_2 is a FR-set in M_1. The last four following theorems are consequences of results from the beginning of this paper, therefore their proofs are omitted. Theorem 16 gives the relations between OFR-strong maps and other forms of "generalized strong maps". Note that none of the implications in Theorem 16 is reversible. Theorem 17 gives a decomposition of OFR-strong maps, while Theorem 18 gives a decomposition of OFR-strong maps and Theorem 19 gives a decomposition of strong dual to OFR-strong.

Theorem 16 *(1) Every ORF-strong map is OFR-strong.*

(2) Every hesitant map is OFR-strong.

(3) Every OFR-strong map is OFF-strong.

(4) Every OFR-strong map is feeble-strong.

Theorem 17 *For a map $f : M_1 \to M_2$, the following are equivalent:*

(1) f is OFR-strong.

(2) f is feeble-strong and OFF-strong.

(3) f is FP-strong and OFF-strong.

Theorem 18 *For a map $f : M_1 \to M_2$, the following are equivalent:*

(1) f is ORF-strong.

(2) f is FP-strong and prespanning-strong.

Theorem 19 *For a map $f : M_1 \to M_2$, the following are equivalent:*

(1) f is strong.

(2) f is OFR-strong and either prestrong or IF-strong.

References

[1] Al-Hawary, T.A.: *Feeble-Matroids,* to appear in Italian J. Pure Appl. Math., no. 16.

[2] Ganster, M. and Reilly, I.: *A Decomposition of Continuity,* Acta Math. Hung. **56(3-4)**, 299-301, 1990.

[3] Kung, J.: A Source Book in Matroid Theory. Birkhäuser Boston Inc. Basel 1986.

[4] Oxley, J.O.: Matroid Theory. Oxford University Press. Oxford 1992.

[5] Tong, J.: *A Decomposition of Continuity,* Acta Math. Hung. **48(1-2)**, 11-15, 1986.

[6] Tong, J.: *On Decomposition of Continuity in Topological Spaces,* Acta Math. Hung. **54(1-2)**, 51-55, 1989.

[7] Truemper, K.: Matroid Decomposition, New York, Academic Press INC., 1992.

[8] White, W.: ed., Matroid Applications, Cambridge, Cambridge University Press, 1992.

[9] White, W.: ed., Theory of Matroids. New York, Cambridge University Press., 1986.

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