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Some Characterization of Curves of Constant Breadth in E^n Space

Zülfiğar Akdoğan, Abdullah Mağden

Abstract

In this paper, the concepts concerning the space of constant breadth were extended to E^n -space. An approximate solution of the equation system which belongs to this curve was obtained. Using this solution vectorial expression of the curves of constant breadth was obtained. The relation $\int_0^{2\pi} \tilde{f}(s) ds = 0$ between the curvatures of curves of constant breadth in E^n was obtained.

Key Words and Phrases: Curvature, Constant Breadth, Integral Characterization of Curve

1. Introduction

Curves of constant breadth were introduced by L.Euler [3]. F. Reuleaux gave a method obtaining some curves of constant breadth and has found use in the kinematics of machinery [11]. Some authors have obtained the geometric properties of plane curves of constant breadth [2], [7].

W.Blascke defined the curve of constant breadth on the sphere [1] and M. Fujivara had obtained a problem to determine whether there exist "space curve of constant breadth" or not, and he defined "breadth" for space curves and obtained these curves on a surface of constant breadth [4]. Ö. Köse presented some concepts for space curves of constant breadth [8]. M. Sezer investigated differential equations characterizing space curves of constant breadth and gave a criterion for these curves [12]. A.R. Forsyth had given the

theory of curves in E^4 [5]. The curves of constant breadth were extended to the E^4 -space and some characterizations were obtained by [9].

Definition 1 *In E^n Euclidean space, if a normal hyperplane on a $X(s) = P$ point of a simple closed (C) curve meets (C) curve at single Q point other than P point, Q point is called opposite point of P .*

In E^n Euclidean space, if the distance between opposite points of simple closed C curve is constant, this curve is called the curve of constant breadth.

In this paper, this kind of curves were extended to E^n -space and some characterizations were obtained.

2. The Curves of Constant Breadth

Let $\vec{X} = \vec{X}(s)$ be a simple closed curve in E^n -space. These curves will be denoted by (C). The normal plane at every point P on the curve meets the curve at a single point Q other than P . We call the point Q the opposite point of P . We consider a curve having parallel tangents \vec{V}_1 and \vec{V}_1^* in opposite directions at the opposite points X and X^* of the curve. A simple closed curve of constant breadth having parallel tangents in opposite directions at opposite points can be represented by the equation

$$\vec{X}^*(s) = \vec{X}(s) + \sum_{i=1}^n m_i(s)V_i(s), \tag{1}$$

where \vec{X} and \vec{X}^* are opposite points and \vec{V}_i denote the Frenet-Serret frame in E^n -space. We have from equation (1)

$$\begin{aligned} \frac{dX^*}{ds} &= \frac{dX^*}{ds^*} \frac{ds^*}{ds} = V_1^* \frac{ds^*}{ds} \\ &= \left(1 + \frac{dm_1}{ds} - m_2k_1\right)V_1 + \left(\frac{dm_2}{ds} + m_1k_1 - m_3k_2\right)V_2 \\ &+ \left(\frac{dm_3}{ds} + m_2k_2 - m_4k_3\right)V_3 + \left(\frac{dm_4}{ds} + m_3k_3 - m_5k_4\right)V_4 + \dots \\ &+ \left(\frac{dm_{n-1}}{ds} + m_{n-2}k_{n-2} - m_nk_{n-1}\right)V_{n-1} + \left(\frac{dm_n}{ds} + m_{n-1}k_{n-1}\right)V_n, \end{aligned} \tag{2}$$

where k_i is the curvatures of the curve. Since $\vec{V}_1^* = -\vec{V}_1$, we obtain

$$\left. \begin{aligned}
 - \frac{ds^*}{ds} &= 1 + \frac{dm_1}{ds} - m_2 k_1 \\
 \frac{dm_2}{ds} + m_1 k_1 - m_3 k_2 &= 0 \\
 \frac{dm_3}{ds} + m_2 k_2 - m_4 k_3 &= 0 \\
 \frac{dm_4}{ds} + m_3 k_3 - m_5 k_4 &= 0 \\
 &\vdots \\
 \frac{dm_{n-1}}{ds} + m_{n-2} k_{n-2} - m_n k_{n-1} &= 0 \\
 \frac{dm_n}{ds} + m_{n-1} k_{n-1} &= 0.
 \end{aligned} \right\} \quad (3)$$

If we call ϕ as the angle between the tangent of the curve (C) at point $X(s)$ with a given fixed direction and consider $\frac{d\phi}{ds} = k_1$, we can rewrite equation (3) as

$$\left. \begin{aligned}
 \frac{dm_1}{d\phi} &= m_2 - f(\phi) \\
 \frac{dm_2}{d\phi} &= -m_1 + \rho k_2 m_3 \\
 \frac{dm_3}{d\phi} &= -\rho k_2 m_2 + \rho k_3 m_4 \\
 &\vdots \\
 \frac{dm_{n-1}}{d\phi} &= -\rho k_{n-2} m_{n-2} + \rho k_{n-1} m_n \\
 \frac{dm_n}{d\phi} &= -\rho k_{n-1} m_{n-1},
 \end{aligned} \right\} \quad (4)$$

where $f(\phi) = \rho + \rho^*$, $\rho = \frac{1}{k_1}$ and $\rho^* = \frac{1}{k_1^*}$ denote the radii of curvatures at X and X^* , respectively. If $\frac{dm_i}{d\phi} = m'_i$, then equation (4) can be written as

$$\left. \begin{aligned}
 m'_1 &= m_2 - f \\
 m'_2 &= -m_1 + \rho k_2 m_3 \\
 m'_3 &= -\rho k_2 m_2 + \rho k_3 m_4 \\
 &\vdots \\
 m'_{n-1} &= -\rho k_{n-2} m_{n-2} + \rho k_{n-1} m_n \\
 m'_n &= -\rho k_{n-1} m_{n-1}
 \end{aligned} \right\} \quad (5)$$

where the functions ρ and k_i are a function of ϕ and

$$\left. \begin{aligned} m_2 &= h_1^{-1}(m'_1 + h_0 f) \\ m_3 &= h_2^{-1}(m'_2 + h_1 m_1) \\ m_4 &= h_3^{-1}(m'_3 + h_2 m_2) \\ &\vdots \\ m_{n-1} &= h_{n-2}^{-1}(m'_{n-2} + h_{n-3} m_{n-3}) \\ m_n &= h_{n-1}^{-1}(m'_{n-1} + h_{n-2} m_{n-2}) \end{aligned} \right\} \quad (6)$$

where $h_0 = 1$ and $h_i^{-1} = \frac{1}{\rho k_i}$, ($i = 1, 2, 3, \dots, n-1$). From first equation of equation system (6) we obtain

$$m'_2 = h_1^{-1} m''_1 + (h_1^{-1})' m'_1 + (h_1^{-1} f)'$$

Here if $a_{22} = h_1^{-1}$, $a_{21} = (h_1^{-1})'$ and $A_1 = (h_1^{-1} f)'$, then

$$m'_2 = a_{22} m''_1 + a_{21} m'_1 + A_1 \quad (7)$$

having second derivative this will be

$$m''_2 = a_{22} m_1^{(3)} + (a'_{22} + a_{21}) m_1^{(2)} + a'_{21} m_1^{(1)} + A'_1. \quad (8)$$

If we put equation (8) in the derivation of second equation of equation system (6), we get the following equation

$$\begin{aligned} m'_3 &= h_2^{-1} a_{22} m_1^{(3)} + [(h_2^{-1} a_{22})' + h_2^{-1} a_{21}] m_1^{(2)} \\ &+ [(h_2^{-1} a_{21})' + h_2^{-1} h_1] m_1^{(1)} + h_2^{-1} h_1 m_1 + (h_2^{-1} A_1)'. \end{aligned} \quad (9)$$

After using abbreviations $a_{33} = h_2^{-1} a_{22}$, $a_{32} = (h_2^{-1} a_{22})' + h_2^{-1} a_{21}$, $a_{31} = (h_2^{-1} a_{21})' + h_2^{-1} h_1$, $a_{30} = a'_{20}$, $a_{20} = h_2^{-1} h_1$, $A_2 = (h_2^{-1} A_1)'$ the equation (9) is shortened as below

$$m'_3 = a_{33} m_1^{(3)} + a_{32} m_1^{(2)} + a_{31} m_1^{(1)} + a_{30} m_1 + A_2. \quad (10)$$

Second derivation of this equation will be

$$m''_3 = a_{33} m_1^{(4)} + (a'_{33} + a_{32}) m_1^{(3)} + (a'_{32} + a_{31}) m_1^{(2)} + (a'_{31} + a_{30}) m_1^{(1)} + a'_{30} m_1 + A'_2. \quad (11)$$

If we put equation (11) in the derivation of third equation of equation system (6), we get following equation

$$\begin{aligned}
 m'_4 &= h_3^{-1}a_{33}m_1^{(4)} + [(h_3^{-1}a_{33})' + h_3^{-1}a_{32}]m_1^{(3)} \\
 &+ [(h_3^{-1}a_{32})' + h_3^{-1}a_{31} + h_3^{-1}h_2a_{22}]m_1^{(2)} \\
 &+ [(h_3^{-1}a_{31})' + h_3^{-1}a_{30} + h_3^{-1}h_2a_{21} + h_3^{-1}h_2h_1^{-1}]m_1^{(1)} \\
 &+ (h_3^{-1}a_{30})'m_1 + (h_3^{-1}A_2)' + h_3^{-1}h_2A_1 + h_3^{-1}h_2h_1^{-1}f.
 \end{aligned} \tag{12}$$

Here if we do following abbreviations

$$\begin{aligned}
 a_{44} &= h_3^{-1}a_{33}, a_{43} = (h_3^{-1}a_{33})' + h_3^{-1}a_{32}, a_{42} = (h_3^{-1}a_{32})' + h_3^{-1}a_{31} + h_3^{-1}h_2a_{22} \\
 a_{41} &= (h_3^{-1}a_{31})' + h_3^{-1}a_{30} + h_3^{-1}h_2a_{21} + h_3^{-1}h_2h_1^{-1}, a_{40} = (h_3^{-1}a_{30})' \\
 A_3 &= (h_3^{-1}A_2)' + h_3^{-1}h_2A_1 + h_3^{-1}h_2h_1^{-1}f,
 \end{aligned}$$

the equation (12) is turned to

$$m'_4 = a_{44}m_1^{(4)} + a_{43}m_1^{(3)} + a_{42}m_1^{(2)} + a_{41}m_1^{(1)} + a_{40}m_1 + A_3. \tag{13}$$

If this process is going on like that, then derivation of m_n can be written by the power of derivation of m_1 as following

$$\begin{aligned}
 m'_n &= h_{n-1}^{-1}a_{(n-1)(n-1)}m_1^{(n)} + [(h_{n-1}^{-1}a_{(n-1)(n-1)})' \\
 &+ h_{n-1}^{-1}a_{(n-1)(n-2)}]m_1^{(n-1)} + \dots + [(h_{n-1}^{-1}a_{(n-1)0})' + \dots \\
 &+ (h_{n-1}^{-1}h_{n-2})'h_{n-3}^{-1}h_{n-4}h_{n-5}^{-1} \dots h_4h_3^{-1}a_{30}]m_1 + (h_{n-1}^{-1}A_{n-2})' \\
 &+ h_{n-1}^{-1}h_{n-2}A_{n-3} + (h_{n-1}^{-1}h_{n-2})'h_{n-3}^{-1}h_{n-4}h_{n-5}^{-1} \dots h_3^{-1}h_2h_1^{-1}A_0
 \end{aligned} \tag{14}$$

where $A_0 = f$. If a_0, a_1, \dots, a_n stand for coefficients of $m_1^{(n)}, m_1^{(n-1)}, \dots, m_1', m_1$ respectively, then the equation (14) will be

$$a_0m_1^{(n)} + a_1m_1^{(n-1)} + \dots + a_{n-1}m_1' + a_n m_1 = \tilde{f} \tag{15}$$

where

$$\begin{aligned}
 \tilde{f} &= -[(h_{n-1}^{-1}A_{n-2})' + h_{n-1}^{-1}h_{n-2}A_{n-3} + \dots \\
 &+ (h_{n-1}^{-1}h_{n-2})'h_{n-3}^{-1}h_{n-4}h_{n-5}^{-1} \dots h_5^{-1}h_4h_3^{-1}h_2h_1^{-1}A_0].
 \end{aligned}$$

If we choose $\tilde{a}_i = -\frac{a_i}{a_0}$, ($i = 1, 2, \dots, n$), then equation (15) will be written as

$$m_1^{(n)} = \tilde{a}_1m_1^{(n-1)} + \dots + \tilde{a}_nm_1 + \frac{\tilde{f}(\phi)}{a_0(\phi)}.$$

Having successively integration, we obtain

$$\begin{aligned}
 m_1^{(n-1)} &= \int_0^\phi (\tilde{a}_1(s)m_1^{(n-1)} + \cdots + \tilde{a}_n(s)m_1) ds + \int_0^\phi \frac{\tilde{f}(s)}{a_0(s)} ds \\
 m_1^{(n-2)} &= \int_0^\phi (\phi - s)(\tilde{a}_1(s)m_1^{(n-1)} + \cdots + \tilde{a}_n(s)m_1) ds + \int_0^\phi (\phi - s)\frac{\tilde{f}(s)}{a_0(s)} ds \\
 m_1^{(n-3)} &= \int_0^\phi \frac{(\phi - s)^2}{2!}(\tilde{a}_1(s)m_1^{(n-1)} + \cdots + \tilde{a}_n(s)m_1) ds + \int_0^\phi \frac{(\phi - s)^2}{2!}\frac{\tilde{f}(s)}{a_0(s)} ds \\
 &\vdots \\
 m_1 &= \int_0^\phi \frac{(\phi - s)^n}{n!}(\tilde{a}_1(s)m_1^{(n-1)} + \cdots + \tilde{a}_n(s)m_1) ds + \int_0^\phi \frac{(\phi - s)^n}{n!}\frac{\tilde{f}(s)}{a_0(s)} ds.
 \end{aligned}$$

Here it is imposible to get m_1 directly, but by using an approximate solution of system as [10], we get the following solution

$$m_{1,k} = \int_0^\phi \frac{(\phi - s)^n}{n!}(\tilde{a}_1(s)m_{1,k-1}^{(n-1)} + \cdots + \tilde{a}_n(s)m_{1,k-1}) ds + \int_0^\phi \frac{(\phi - s)^n}{n!}\frac{\tilde{f}(s)}{a_0(s)} ds.$$

and then

$$\begin{aligned}
 m'_{1,k} &= \int_0^\phi \frac{(\phi - s)^{n-1}}{(n-1)!}(\tilde{a}_1(s)m_{1,k-1}^{(n-1)} + \cdots + \tilde{a}_n(s)m_{1,k-1}) ds + \int_0^\phi \frac{(\phi - s)^{n-1}}{(n-1)!}\frac{\tilde{f}(s)}{a_0(s)} ds \\
 &\vdots \\
 m_{1,k}^{(n)} &= \tilde{a}_1(\phi)m_{1,k-1}^{(n-1)} + \cdots + \tilde{a}_n(\phi)m_{1,k-1} + \frac{\tilde{f}(\phi)}{a_0(\phi)}.
 \end{aligned}$$

We now calculate how $m_{1,k}$ approximate to m_1 . If $\varepsilon_{1,k} = \|m_1 - m_{1,k}\|$, then

$$\begin{aligned}
 \varepsilon_{1,k} &\leq \int_0^\phi \frac{(\phi - s)^n}{n!}[\|\tilde{a}_1\|_{\varepsilon_{1,k-1}^{(n-1)}} + \cdots + \|\tilde{a}_n\|_{\varepsilon_{1,k-1}}] ds \\
 &\vdots \\
 \varepsilon_{1,k}^{(n-1)} &\leq \int_0^\phi [\|\tilde{a}_1\|_{\varepsilon_{1,k-1}^{(n-1)}} + \cdots + \|\tilde{a}_n\|_{\varepsilon_{1,k-1}}] ds \\
 \varepsilon_{1,k}^{(n)} &\leq \|\tilde{a}_1\|_{\varepsilon_{1,k-1}^{(n-1)}} + \cdots + \|\tilde{a}_n\|_{\varepsilon_{1,k-1}}.
 \end{aligned}$$

If $L_1 = \max\{\|\tilde{a}_1\|, \dots, \|\tilde{a}_n\|\}$, then

$$\begin{aligned} \varepsilon_{1,k} &\leq L_1 \int_0^\phi \frac{(\phi-s)^n}{n!} (\sum_{j=0}^{n-1} \varepsilon_{1,k-1}^{(j)}) ds \\ \varepsilon'_{1,k} &\leq L_2 \int_0^\phi \frac{(\phi-s)^{n-1}}{(n-1)!} (\sum_{j=0}^{n-1} \varepsilon_{1,k-1}^{(j)}) ds \\ &\vdots \\ \varepsilon_{1,k}^{(n-1)} &\leq L_{n-1} \int_0^\phi (\sum_{j=0}^{n-1} \varepsilon_{1,k-1}^{(j)}) ds. \end{aligned}$$

We can sum up above equations side by side in order to obtain following equation

$$\sum_{j=0}^{n-1} \varepsilon_{1,k}^{(j)} \leq \int_0^\phi [L_1 \frac{(\phi-s)^n}{n!} + \dots + L_{n-1}] \sum_{j=0}^{n-1} \varepsilon_{1,k-1}^{(j)} ds.$$

In this case, if we say $\delta_k = \sum_{j=0}^{n-1} \varepsilon_{1,k}^{(j)}$, then we can write

$$\delta_k \leq \int_0^\phi [\sum_{j=0}^{n-1} L_j \frac{(\phi-s)^{n-j}}{(n-j)!} \delta_{k-1}] ds$$

and if

$$\sum_{j=0}^{n-1} L_j \frac{(\phi-s)^{n-j}}{(n-j)!} \leq L$$

then we can write

$$\delta_k \leq L \int_0^\phi \delta_{k-1} ds.$$

we now show approximate of δ_k . To do this

$$\begin{aligned} \delta_1 &\leq L\delta_0\phi \\ \delta_2 &\leq L \int_0^\phi \delta_1 ds \leq L^2\delta_0 \frac{\phi^2}{2!} \\ &\vdots \\ \delta_k &\leq \delta_0 \frac{(L\phi_0)^k}{k!} \quad (0 \leq \phi \leq \phi_0) \end{aligned}$$

thus

$$\sum_{j=0}^{n-1} \varepsilon_{1,k}^{(j)} \leq \delta_0 \frac{(L\phi_0)^k}{k!}.$$

Each term is smaller than $\delta_0 \frac{(L\phi_0)^k}{k!}$ since summation of all terms is smaller than $\delta_0 \frac{(L\phi_0)^k}{k!}$.

That is,

$$\varepsilon_{1,k}^{(j)} \leq \delta_0 \frac{(L\phi_0)^k}{k!}$$

and then

$$\begin{aligned} \|m_1 - m_{1,k}\| &\leq \delta_0 \frac{(L\phi_0)^k}{k!} \\ \|m'_1 - m'_{1,k}\| &\leq \delta_0 \frac{(L\phi_0)^k}{k!} \\ &\vdots \\ \|m_1^{(n-1)} - m_{1,k}\| &\leq \delta_0 \frac{(L\phi_0)^k}{k!} \end{aligned}$$

and

$$\varepsilon_{1,k}^{(n)} \leq L_n \delta_0 \frac{(L\phi_0)^k}{k!}$$

so

$$\|m_1^{(n)} - m_{1,k}^n\| \leq L_n \delta_0 \frac{(L\phi_0)^k}{k!}.$$

We take limit both side

$$\lim_{k \rightarrow \infty} \varepsilon_{i,k}^{(j)} \leq \lim_{k \rightarrow \infty} \delta_0 \frac{(L\phi_0)^k}{k!}$$

we get

$$\lim_{k \rightarrow \infty} m_{1,k}^{(j)} = m_1^{(j)} \quad (j = 0, 1, 2, \dots, n-1)$$

and

$$\lim_{k \rightarrow \infty} m_{1,k}^{(n)} = m_1^{(n)}.$$

We have that

$$\left. \begin{aligned} m_2 &= m'_1 + f \\ m_3 &= h_2^{-1}(m'_2 + h_1 m_1) \\ m_4 &= h_3^{-1}(m'_3 + h_2 m_2) \\ &\vdots \\ m_n &= h_{n-1}^{-1}(m'_{n-1} + h_{n-2} m_{n-2}) \end{aligned} \right\} \quad (16)$$

and then

$$\begin{aligned}\varepsilon_{2,k} &= \varepsilon'_{1,k} \\ \varepsilon_{3,k} &= k_2^{-1}\varepsilon'_{2,k} + k_1\varepsilon_{1,k} \\ \varepsilon_{4,k} &= k_3^{-1}\varepsilon'_{3,k} + k_2\varepsilon_{2,k} \\ &\vdots \\ \varepsilon_{n,k} &= k_{n-1}^{-1}\varepsilon'_{n-1,k} + k_{n-2}\varepsilon_{n-2,k}\end{aligned}$$

Therefore, we can find the value of m_2, m_3, \dots, m_n from equation (16) since the value of m_1 is known. Thus, the error of $\varepsilon_{2,k}, \varepsilon_{3,k}, \dots, \varepsilon_{n,k}$ can be found by depending on the error of $\varepsilon_{1,k}$. If $k \rightarrow \infty$, then \vec{X}^* can be presented by \vec{X} and its invariant. So the following theorem is proved.

Theorem 1 *Let $k_i(s)$, ($i = 1, 2, \dots, n; 0 \leq s \leq L$) be non-zero functions in the class Γ . Then $m_{i,k}$ ($k = 0, 1, 2, \dots$) which obtained from (4) limits to the unique solution of system as following*

$$\|m_1^{(i)} - m_{1,k}^{(i)}\| \leq L_i \delta_0 \frac{(L\phi_0)^k}{k!} \quad (i = 0, 1, \dots, n; k = 0, 1, \dots),$$

$$\|m_\ell^{(i)} - m_{\ell,k}^{(i)}\| \leq L_{\ell,i}^* \delta_0 \frac{(L_\ell^* \phi_0)^k}{k!} \quad (\ell = 2, 3, \dots, n),$$

where $L_i, L, L_{\ell,i}^*$ and L_ℓ^* are numbers which are known.

If the distance between the opposite points of (C) and (C^*) is constant, then

$$\|\alpha^* - \alpha\|^2 = m_1^2 + m_2^2 + \dots + m_n^2 = k^2, k \in \mathfrak{R}.$$

Hence, we write

$$m_1 \frac{dm_1}{d\phi} + m_2 \frac{dm_2}{d\phi} + \dots + m_n \frac{dm_n}{d\phi} = 0 \tag{17}$$

and then

$$m_1 \left(\frac{dm_1}{d\phi} - m_2 \right) = 0. \tag{18}$$

In this case, either $m_1 = 0$ or $\frac{dm_1}{d\phi} - m_2 = 0$ which means $f(\phi) = 0$. If $f(\phi) = 0$, then \vec{X}^* is a transition of \vec{X} with following constant vector

$$\ell = m_1 V_1 + m_2 V_2 + \cdots + m_n V_n. \quad (19)$$

If $m_1 = 0$, then from equation (14) we have

$$\begin{aligned} & (h_{n-1}^{-1} A_{n-2})' + h_{n-1}^{-1} h_{n-2} A_{n-3} + \cdots \\ & + (h_{n-1}^{-1} h_{n-2})' h_{n-3} h_{n-4} h_{n-5}^{-1} \cdots h_5^{-1} h_4 h_3^{-1} h_2 h_1^{-1} A_0 = 0. \end{aligned} \quad (20)$$

Thus, the function $f(\phi)$ satisfies the equation (20). $\vec{X}^*(0) = \vec{X}^*(2\pi)$ since the curve of constant breadth is closed. Accordingly, from (1) we can write

$$\begin{aligned} \alpha^*(0) &= \alpha(0) + \sum_{i=2}^n m_i(0) V_i(0) = \alpha^*(2\pi) \\ &= \alpha(2\pi) + \sum_{i=2}^n m_i(2\pi) V_i(2\pi) \end{aligned}$$

and finally $m_i(0) = m_i(2\pi)$.

Corollary 1 *In the equation*

$$m_{1,k} = \int_0^\phi \frac{(\phi-s)^n}{n!} (\tilde{a}_1(s) m_{1,k-1}^{(n-1)} + \cdots + \tilde{a}_n(s) m_{1,k-1}) ds + \int_0^\phi \frac{(\phi-s)^n}{n!} \frac{\tilde{f}(s)}{a_0(s)} ds$$

if we choose $m_{1,0} = 0$, then

$$m_{1,1} = \int_0^\phi \frac{(\phi-s)^n}{n!} \frac{\tilde{f}(s)}{a_0(s)} ds.$$

Assume that $m_{1,k}$ limits for $k = 1$ to m_1 . Then $m_{1,1} = 0 = m_1$ and then

$$\int_0^\phi \frac{(\phi-s)^n}{n!} \frac{\tilde{f}(s)}{a_0(s)} ds = 0. \quad (21)$$

Corollary 2 *If we choose $m_{1,1} = 0$, then*

$$m_{1,2} = \int_0^\phi \tilde{a}_n(s) ds + \int_0^\phi \frac{(\phi-s)^n}{n!} \frac{\tilde{f}(s)}{a_0(s)} ds.$$

Assume that $m_{1,k}$ limits for $k = 2$ to m_1 . Then $m_{1,2} = m_1 = 0$ and then

$$\int_0^\phi \tilde{a}_n(s) ds = - \int_0^\phi \frac{(\phi - s)^n}{n!} \frac{\tilde{f}(s)}{a_0(s)} ds.$$

By derivating

$$a_n(\phi) = a_0(\phi) \int_0^\phi \frac{(\phi - s)^{n-1}}{(n-1)!} \frac{\tilde{f}(s)}{a_0(s)} ds$$

Corollary 3 If we choose $m_1 = 0$ in equation (14), then

$$m'_n = -\tilde{f}$$

and

$$m_n = - \int_0^\phi \tilde{f} ds.$$

We have $m_n(0) = m_n(2\pi)$, so

$$\int_0^{2\pi} \tilde{f}(s) ds = 0.$$

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