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## Renewable energy integration in smart grids-multicriteria assessment using the fuzzy analytical hierarchy process

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**Abstract:** Unlike the traditional way of efficiency assessment of renewable energy sources integration, the smart grid concept is introducing new goals and objectives regarding increased use of renewable electricity sources, grid security, energy conservation, energy efficiency, and deregulated energy market. Possible benefits brought by renewable sources integration are evaluated by the degree of the approach to the ideal smart grid. In this paper, fuzzy analytical hierarchy process methodology for the integration efficiency has been proposed, taking into account the presence of multiple criteria of both qualitative and quantitative nature, different performance indicators, and the uncertain environment of the smart grid. The methodology has been illustrated on the choice of the size and location of a distributed generator in the radial distribution feeder.

**Key words:** Distributed generation, fuzzy analytical hierarchy process, smart grid

### 1. Introduction

Renewable energy decision-making can be viewed as a multiple criteria decision-making (MCDM) problem with correlating criteria and alternatives. This task should take into consideration several conflicting aspects due to the increasing complexity of social, technological, environmental, and economic factors [1,2]. The traditional single criteria decision-making approaches cannot handle the complexity of current systems and this problem [3,4]. An overview of the state of the art models and methods applied to the problem, analyzing and classifying current and future research trends in this field, is given in [5,6].

The application area of MCDM in renewable energy has generally been divided into five categories [5,7,8]:

- 1) renewable energy planning and policy, referring to the assessment of a feasible energy plan and the diffusion of different renewable energy option;
- 2) evaluation and assessment, referring to the assessment of different alternative energies or energy technologies;
- 3) technology and project selection, including the site selection, technology selection, and decision support in renewable energy harnessing projects;

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- 4) environmental, concerned with alternative technologies from an environmental perspective and climate issues; and
- 5) operational, referring to the optimal distributed generation outputs to satisfy all the criteria and constraints imposed by the distribution network.

With the development of smart grid architecture, the perspective of renewable sources assessment has changed, introducing new goals and objectives. “Smart grid” generally refers to an electricity network that can intelligently integrate the actions of all users connected to it in order to efficiently deliver sustainable, economic, and secure electricity supplies. These systems are made possible by two-way communication technology and computer processing that has been used for decades in other industries. According to [9] the main objectives of smart grids are: increased use of renewable electricity sources, grid security, energy conservation and energy efficiency, and deregulated energy market. Therefore, the strategy for sustainable, competitive, and safe energy primarily implies competitiveness, use of different energy sources, sustainability, innovation, and technological improvement [10], while possible benefits brought by renewable sources integration have to be evaluated by the degree of the approach to the ideal smart grid.

In the smart grid context, three main assessment frameworks have been introduced. The EC Task Force for smart grids [11,12] has introduced the characteristics of the ideal smart grids (services) and the outcomes of the implementation of the ideal smart grid (benefits). A measure of the contribution of projects to the ideal smart grid is quantified in terms of benefits, via a set of key performance indicators (KPIs). The European Electricity Grid Initiative [13] has divided the ideal smart grid system into thematic areas (clusters) and is currently mapping these projects into clusters. In the United States, the ideal characteristics and a set of metrics to measure progress toward ideal smart grids have been defined [14]: build metrics that describe attributes that are built in support of a smart grid (e.g., percentage of substations using automation) and value or impact metrics that describe the value that may derive from achieving a smart grid (e.g., percentage of energy consumed to generate electricity that is not lost, or quantity of electricity delivered to consumers compared to electricity generated expressed as a percentage).

Due to this proliferation of both quantitative and qualitative criteria, and many uncertainties related to the smart grid operation environment, the paper proposes a new algorithm for the assessment of renewable energy integration in the smart grid, which uses the fuzzy analytical hierarchy process (AHP) method for multicriteria decision-making.

The original AHP was developed by Saaty in the late 1970s [15]. In this method, human judgments are represented as crisp values. However, in many practical cases the human preference model is uncertain and decision makers cannot assign crisp values to comparison judgments. In these cases it is useful to implement the fuzzy AHP method. The fuzzy AHP method is designed to improve decision support for uncertain valuations and priorities. The methodology presented in this paper uses data and preferences of experts evaluated under a fuzzy set environment [16]. The use of fuzzy set theory allows the decision makers to incorporate unquantifiable information, incomplete information, nonobtainable information, and partially ignorant information into decision model [17].

Many authors have used the fuzzy AHP method for solving problems in different areas: to solve multicriteria problems involving qualitative data [18,19], water management [20–22], evaluation of naval tactical missile systems [23], hazardous waste management [24], prioritization of human capital measurement indicators [25], shipping asset management [26], and occupational safety management [27,28]. There are numerous cases for

employing fuzzy AHP in the sector of renewable energy, focusing on prioritizing energy technologies [4,29,30], econometric analysis of renewable technology efficiency [31], or allocation of renewable resources [32,33]. In this paper the fuzzy AHP method is used for ranking and selection of smart grid projects, precisely because of the many uncertain and nontangible benefits and criteria involved in smart grid projects' assessment.

Starting from a general set of smart grid performance indicators, we established a new assessment framework for the evaluation of integration of renewable sources in a smart grid. In the second stage, based on fuzzy matching of alternatives, the methodology proposed in this paper determines the optimal set of activities concerning renewable energy. Validating this methodology on a test network, we proved that the method is highly successful in the evaluation of alternatives in the presence of heterogeneous criteria.

After a brief overview of key performance indicators for smart grid evaluation, the fuzzy AHP methodology is presented. The methodology is illustrated on the choice of four different alternatives (of different size, location, and technology) of distributed generator insertion in the IEEE 33-bus test radial distribution feeder. Finally, the conclusions about the adequacy of the proposed methodology and directions for further research are presented.

## 2. Smart grid assessment frameworks

The implementation of a smart grid is useful to achieve strategic policy goals, such as the smooth integration of renewable energy sources, a more secure and sustainable electricity supply, and full inclusion of consumers in the electricity market. For utilities, a better understanding of the electrical grid's status at a second-by-second level allows the grid to be operated at much tighter tolerances, resulting in greater efficiencies and reliability.

Steering the smart grid transition is a challenging, long-term task, which requires balancing energy policy goals, environmental constraints, and market profitability. From this perspective, a first approach in smart grid assessment is to evaluate to what extent renewable energy projects are contributing to progresses toward the "ideal smart grid" and its expected outcomes (e.g., sustainability, efficiency, consumer inclusion), which are directly linked with the policy goals that have triggered this transition. This first approach is conducted via the definition of suitable metrics and key performance. A second complementary approach is to assess the profitability of renewable energy solutions and investments integrated into the smart grid through an appropriate multicriteria decision analysis methodology. Both steps will be explained in following sections.

### 2.1. Smart grid evaluation metrics

The characteristics of the ideal smart grids and defined metrics to measure progress and outcomes resulting from the implementation of these projects were defined in [12–14,34]. The ideal smart grid has been defined in terms of "characteristics" in the United States and in terms of "services" in the European Union, including:

- Enabling the network to integrate users with new requirements;
- Enabling and encouraging stronger and more direct involvement of consumers in their energy usage and management;
- Improving market functioning and customer service;
- Enhancing efficiency in day-to-day grid operation;
- Enabling better planning of future network investment;
- Ensuring network security, system control, and quality of supply.

For each service, a number of corresponding smart grid functionalities has been defined. To measure progress toward the ideal grid, Built/Value metrics in the United States and Benefits/KPIs in Europe are used.

The EC Smart Grid Task Force [12] has identified a list of benefits deriving from the implementation of a smart grid:

- Increased sustainability;
- Adequate capacity of transmission and distribution grids for ‘collecting’ and bringing electricity to the consumers;
- Adequate grid connection and access for all kinds of grid users;
- Satisfactory levels of security and quality of supply;
- Enhanced efficiency and better service in electricity supply and grid operation;
- Effective support of transnational electricity markets by load flow control to alleviate loop flows and increased interconnection capacities;
- Coordinated grid development through common European, regional, and local grid planning to optimize transmission grid infrastructure;
- Enhanced consumer awareness and participation in the market by new players;
- Enabling consumers to make informed decisions related to their energy to meet the EU energy efficiency targets;
- Creating a market mechanism for new energy services such as energy efficiency or energy consulting for customers;
- Consumer bills are either reduced or upward pressure on them is mitigated.

Each benefit is expressed via a set of KPIs including both quantitative and qualitative indicators. For illustration, the first benefit, increased sustainability, is valued by the quantified reduction of carbon emissions, environmental impact of electricity grid infrastructure, and quantified reduction of accidents and risk associated with generation technologies. The complete list of indicators can be found in [12]. Smart grid services and benefits are strongly linked to the policy goals that are driving the smart grid deployment (sustainability, competitiveness, and security of supply), and, consequently, they can be considered as useful indicators to evaluate the contribution of projects toward the achievement of these policy goals. A clearly defined framework can concretize where exactly the project contributed to a smart electricity grid.

As far as renewable energy projects are concerned, in order to get a thorough understanding of the status of smart grid development and starting from the list of main services and corresponding benefits, an adapted list of main criteria can be defined, including:

- Technology, covering all aspects of advanced services and new requirements imposed to the distribution and transmission network;
- Cost reduction, including the optimized asset utilization, enhanced efficiency, and better planning of future investment;

- Customer satisfaction, encompassing different options of customer choice, new energy services, and market participation;
- Environmental impact reduction.

After the first level of benefits was defined, the second set of performance indicators on the base level of renewable integration efficiency assessment was chosen out of the complete indicator list. The indicators that can be measured are the quantified reduction of carbon emissions, voltage quality performance of electricity grids (e.g., voltage dips, voltage and frequency deviations, and the level of losses in distribution networks (absolute or percentage)). If new projects are evaluated, the net present value of the investment can be added. Qualitative indicators are the evaluated environmental impact and societal benefits of the project.

The mixture of quantitative and qualitative indicators is one of the major reasons for introducing the MCDM techniques. Another reason is the shortcomings of cost-benefit analysis, which will be explained in the next section.

## 2.2. Multicriteria assessment model

The implementation of the smart grid should be market-driven. Another necessary approach in smart grid assessment is therefore to assess the costs, the benefits, and the beneficiaries of different solutions. A comprehensive methodology for cost-benefit analysis of these projects was defined [35], while the European Commission has adapted and expanded the DOE/EPRI methodology to fit the European context [36,37].

However, the traditional cost-benefit analysis approach does not catch all the effects involved in development policies, where intangible aspects are not secondary, but are dominating [38]. The main disadvantage of cost-benefit analysis is the translation of all the effects into a common numerical and single aggregate measure. Therefore, multiple criteria analysis seems to be better in measuring intangibles and soft impacts than cost-benefit; it uses more than one criterion introducing qualitative aspects in the analysis.

As explained in Section 2.1, a multicriteria model is developed based on the list of both quantitative and qualitative indicators, defined in Sections 2.2.1 and 2.2.2.

### 2.2.1. Quantitative indicators

A set of four quantitative indicators is used in this methodology:

### 2.2.2. Quantified reduction of carbon emissions

For every alternative, this indicator is measured by the kilograms of CO<sub>2</sub> emission per produced kilowatt-hour of electrical energy. The impact of renewable sources is taken as the reduction of the emission produced by the conventional energy source.

- Total voltage deviation is calculated using Eq. (1):

$$V_D = \sqrt{\sum_{k=1}^{NB} (V_k - V_{ref})^2}, \quad (1)$$

where  $V_k$  is the magnitude of voltage at bus  $k$ ,  $V_{ref}$  is the magnitude of the slack bus voltage, and  $NB$  is the total number of nodes in the network.

- The active power losses are calculated as:

$$P_{loss} = \sum_{j=1}^{NL} i_j^2 R_j, \tag{2}$$

where  $R_j$  and  $i_j$  are resistance and actual current of the  $j$ th line, respectively, and  $NL$  is the total number of lines.

- Net present value (NPV) is used to determine the present value of an investment by the discounted sum of all cash flows received from the project. The formula for the discounted sum of all cash flows can be rewritten as:

$$NPV = \sum_{i=1}^n C_i (1 + d)^i - C_0, \tag{3}$$

where  $C_0$  is initial investment,  $C_i$  is cash flow,  $d$  is discount rate, and  $n$  is time period.

### 2.2.3. Qualitative indicators

Both indicators that cannot be exactly measured,

- Environmental impact of electricity grid infrastructure and
- Societal benefit of a proposed infrastructure investment,

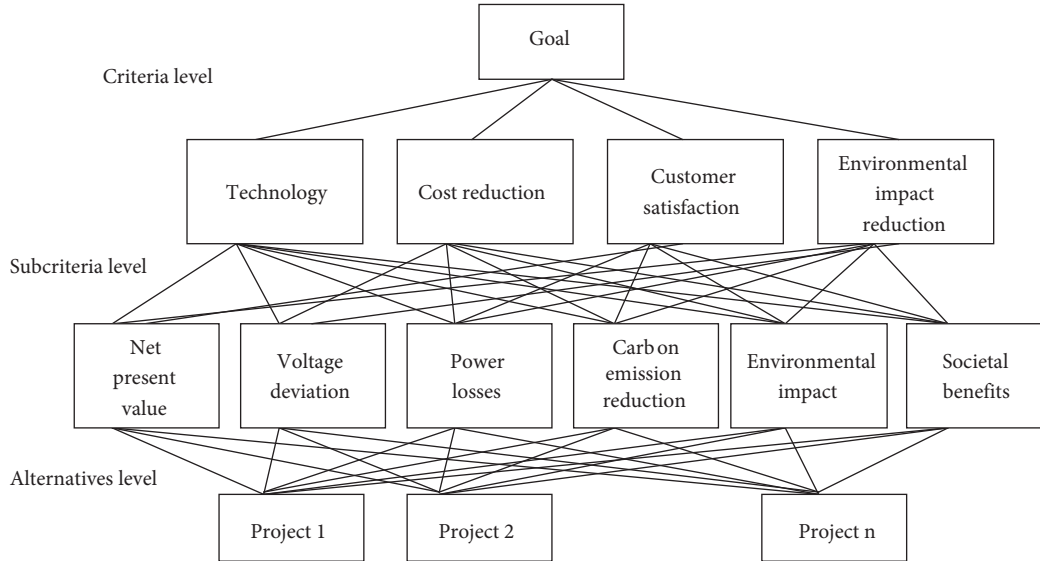
are evaluated through ordinal comparison. In this approach, we adopt the five-grade verbal scale for the assessment of these indicators, which can be composed from opinion polls results, expert judgments, or other integrated approaches. The description of the scale is given in Table 1.

**Table 1.** Description of qualitative indicators.

Grade	Environmental impact	Societal benefits
Minor	Negligible land and material requirements for producing necessary power. No substantial environmental impact.	Unreasonable to expect any changes in local economy or enhancement in market services.
Low	No visual or noise problems caused by the operation of plant. Small land and material requirements.	New jobs created with great risk to retainment as a result of new renewable energy source.
Moderate	Limited visual or noise problems, with some disruption to habitat. No impact to the wildlife.	New market mechanism for new energy services such as energy efficiency or energy consulting for customers.
High	Increased emission pollutants, with impact to the wildlife and landscape.	Improving market functioning and customer service, new jobs created and retained as a result of new renewable energy source.
Very high	Large emission pollutants, land and material requirements, other life-cycle steps contributing significantly to the total environmental impact.	More direct involvement of consumers in their energy usage and management, new jobs created and retained as a result of new renewable energy source.

All indicators (quantitative and qualitative) influence all of four main criteria to different extents determined by the decision maker. For instance, reduced voltage deviation and stable voltage profile in the network

will enable the usage of advanced technologies and services; they will reduce the costs of low power quality, increasing customer satisfaction. The scheme of hierarchical levels and interdependencies of criteria, subcriteria, and alternatives is represented in Figure 1.



**Figure 1.** Hierarchical levels and interdependencies of criteria, subcriteria, and alternatives.

### 3. Smart grid evaluation method

In this paper the fuzzy AHP method is used for the evaluation of integration of renewable sources in the smart grid. The mathematical basis for the fuzzy AHP method is based on fuzzy sets, fuzzy numbers, and fuzzy arithmetic.

#### 3.1. Fuzzy sets, triangular fuzzy numbers, and fuzzy arithmetic

Zadeh defines a fuzzy set  $A$  by degree of membership  $\mu_A(x)$  over a universe of discourse  $X$  as [39]:

$$\mu_A(x) : X \rightarrow [0, 1]. \tag{4}$$

A fuzzy number is a convex and normalized fuzzy set  $A = \{(x, \mu_A(x)), x \in R\}$ . A triangular fuzzy number can be denoted as  $M = (a, b, c)$ , and the membership function is:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & x \in [a, b] \\ \frac{c-x}{c-b}, & x \in [b, c] \\ 0, & otherwise \end{cases}, \tag{5}$$

where  $a \leq b \leq c$ ,  $a$  and  $c$  stand for the lower and upper value of the support of  $M$  respectively, and  $b$  is the modal value. When  $a = b = c$ , it is a crisp number.

Fuzzy arithmetic is based on Zadeh’s extension principle. If  $f : X \rightarrow Y$  is a function, and  $A$  is a fuzzy set in  $X$ , then  $f(A)$  is defined as:

$$\mu_{f(A)}(y) = \sup_{x \in X, f(x)=y} \mu_A(x), \tag{6}$$

where  $y \in Y$ .



Basis laws for triangular fuzzy number  $M = (a, b, c)$ ,  $a > 0$ , are:

$$M^{-1} = (a, b, c)^{-1} = \left(\frac{1}{c}, \frac{1}{b}, \frac{1}{a}\right); \tag{7}$$

$$M^n = (a, b, c)^n = (a^n, b^n, c^n), \quad n \in N; \tag{8}$$

$$M^{1/n} = (a, b, c)^{1/n} = (a^{1/n}, b^{1/n}, c^{1/n}), \quad n \in N. \tag{9}$$

The main laws for operations for two triangular fuzzy numbers  $M_1(a_1, b_1, c_1)$  and  $M_2(a_2, b_2, c_2)$  are:

1. Fuzzy number addition:

$$M_1 \oplus M_2 = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2). \tag{10}$$

2. Fuzzy number subtraction:

$$M_1 \ominus M_2 = (a_1, b_1, c_1) \ominus (a_2, b_2, c_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2). \tag{11}$$

3. Fuzzy number multiplication:

$$M_1 \otimes M_2 = (a_1, b_1, c_1) \otimes (a_2, b_2, c_2) = (a_1 \cdot a_2, b_1 \cdot b_2, c_1 \cdot c_2), \quad a_1, a_2 > 0. \tag{12}$$

4. Fuzzy number division:

$$M_1 \oslash M_2 = (a_1, b_1, c_1) \oslash (a_2, b_2, c_2) = \left(\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}\right), \quad a_1, a_2 > 0. \tag{13}$$

### 3.2. Fuzzy AHP method

The fuzzy AHP method involves the following steps:

- Step 1.** The overall goal (objective) is identified and clearly defined;
- Step 2.** The criteria, subcriteria, and alternatives are identified;
- Step 3.** The hierarchical structure is formed;
- Step 4.** Pairwise comparison is made using Saaty’s fuzzified evaluation scale;
- Step 5.** The priority weighting vectors are evaluated using the row geometric mean method (RGMM);
- Step 6.** Consistency of the judgments is checked by the geometric consistency index (GCI);
- Step 7.** The defuzzification and the final ranking of alternatives are defined.

The seven-step algorithm of fuzzy AHP applied to the problem of evaluation of renewable sources encompasses the following steps:

1. Goal identification. The goal is to evaluate the efficiency of the renewable energy plant integration in the smart grid context.
2. Identification of criteria, subcriteria, and alternatives. Criteria for smart grid projects selection are: technology, costs reduction, customer satisfaction, encompassing different options of customer choice, and environmental impact reduction. Subcriteria are the KPIs, as explained in Sections 2.2.1 and 2.2.2. Finally, the different renewable integration projects are identified as alternatives.

3. Hierarchical structure formation. The fuzzy AHP method presents a problem in the form of hierarchy: the first level represents the goal; the second level considers relevant criteria (four identified criteria); the third level considers relevant subcriteria (six identified KPIs); and the fourth level defines renewable alternatives (four alternatives).
4. Pairwise comparison. Pairs of elements at each level are compared according to their relative contribution to the elements at the hierarchical level above, using Saaty’s fuzzified scale, as shown in Table 2.

**Table 2.** Crisp and fuzzified Saaty scale for pairwise comparisons [22].

Crisp values (x)	Judgment description	Fuzzy values
1	Equal importance	(1, 1, 1+δ)
3	Weak dominance	(3-δ, 3, 3+δ)
5	Strong dominance	(5-δ, 5, 5+δ)
7	Demonstrated dominance	(7-δ, 7, 7+δ)
9	Absolute dominance	(9-δ, 9, 9)
2, 4, 6, 8	Intermediate values	(x-1, x, x+1)

In this paper fuzzification is implemented by triangular fuzzy numbers, and the value of fuzzy distance of 2 is used, as recommended in [22], because the most consistent results can be expected.

Pairwise comparisons at each level, starting from the top of the hierarchy, are presented in the square matrix form  $A = [a_{ij}]_{i,j=1,\overline{n}}$ , where  $a_{ij}$  is the fuzzy value about the relative importance of criteria/subcriteria/alternative  $i$  over criteria/subcriteria/ alternative  $j$ ,  $a_{ij} = 1$  for  $i = j$  and  $a_{ij} \cdot a_{ji} = 1$  for  $i \neq j$ .

5. Priority weights vectors evaluation. The priority weighting vectors on each level are evaluated using the RGMM. The ranking procedure starts with the determination of the criteria weighting vector:

$$W_c = (w_{c1}, w_{c2}, w_{c3}, w_{c4})^T, \tag{14}$$

where  $w_{ci}$  is the fuzzy weight of the  $i$ th criterion:

$$w_{ci} = \frac{\left(\prod_{j=1}^4 a_{ij}\right)^{\frac{1}{4}}}{\sum_{i=1}^4 \left(\prod_{j=1}^4 a_{ij}\right)^{\frac{1}{4}}}, \quad i = \overline{1,4}. \tag{15}$$

Subcriteria weighting vectors are defined by pairwise comparison of performance indicators according to each criterion. Appropriate elements of these vectors are calculated as follows:

$$w_{sci}^p = \frac{\left(\prod_{j=1}^6 a_{ij}\right)^{\frac{1}{6}}}{\sum_{i=1}^6 \left(\prod_{j=1}^6 a_{ij}\right)^{\frac{1}{6}}}, \quad i = \overline{1,6}, \quad p = \overline{1,4}, \tag{16}$$

where  $w_{sci}^p$  represents the fuzzy weight of the  $i$ th performance indicator with respect to the  $p$ th criterion. The final subcriteria weighting vector is obtained by multiplying the matrix of the subcriteria weights according to

all criteria ( $W_1$ ) and the matrix of the criteria weights ( $W_c$ ):

$$W_{sc} = W_1 \otimes W_c. \tag{17}$$

Finally, the projects are compared according to the each performance indicator. Proper weights of projects, i.e. alternatives with respect to the individual performance indicator, are determined as follows:

$$w_{ai}^r = \frac{\left(\prod_{j=1}^4 a_{ij}\right)^{\frac{1}{4}}}{\sum_{i=1}^4 \left(\prod_{j=1}^4 a_{ij}\right)^{\frac{1}{4}}}, \quad i = \overline{1,4}, \quad r = \overline{1,6}, \tag{18}$$

where  $w_{ai}^r$  represents the fuzzy weight of the  $i$ th project with respect to the  $r$ th performance indicator. Final projects weights are obtained by multiplying the matrix of the projects weights according to all alternatives ( $W_2$ ) and the matrix of subcriteria weights:

$$W_a = W_2 \otimes W_{sc} = (w_{a1}, w_{a2}, w_{a3}, w_{a4})^T. \tag{19}$$

6. Consistency control. Consistency in this case means that the decision procedure is producing coherent judgments in specifying the pairwise comparison of the criteria, subcriteria, or alternatives. When the RGMM is employed as the prioritization procedure, the GCI is used for consistency control [16,40,41]. For an  $n \times n$  judgement matrix the GCI is computed as follows:

$$GCI = \frac{2}{(n-1)(n-2)} \sum_{i < j} \log^2 e_{ij}, \tag{20}$$

where  $e_{ij} = a_{ij}w_j/w_i$  is the error obtained when the ratio  $\omega_i/\omega_j$  is approximated by  $a_{ij}$ ,  $i, j = \overline{1, n}$  ( $a_{ij}, w_i, w_j$  are defuzzification values, i.e. crisp values). For this measure, the thresholds associated with the 10% level of inconsistency suggested by Saaty are: GCI = 0.31 for  $n = 3$ , GCI = 0.35 for  $n = 4$ , GCI = 0.37 for  $n > 4$  [42,43].

7. Defuzzification and the final ranking of alternatives. In this paper, triangular fuzzy numbers are ranked by applying the mean value method. For the given triangular fuzzy number  $M = (a, b, c)$ , the mean value method for defuzzification is a defined crisp number value as follows:

$$m = \frac{a + b + c}{3}. \tag{21}$$

The highest rank has the alternative with the highest value of  $m$ .

#### 4. Results and discussion

The proposed methodology is illustrated on the choice of the technology, size, and location of one distributed renewable generator. Four possible alternatives are evaluated on the IEEE radial 33-bus test feeder (Figure 2; see Appendix for data for the test system, on the journal’s website) with parameters including the nominal active power ( $P_{nom}$ ), the node the generator is connected to (Bus  $N^\circ$ ), type of renewable source (RS), and expected annual energy production of generator (W), as represented in Table 3.

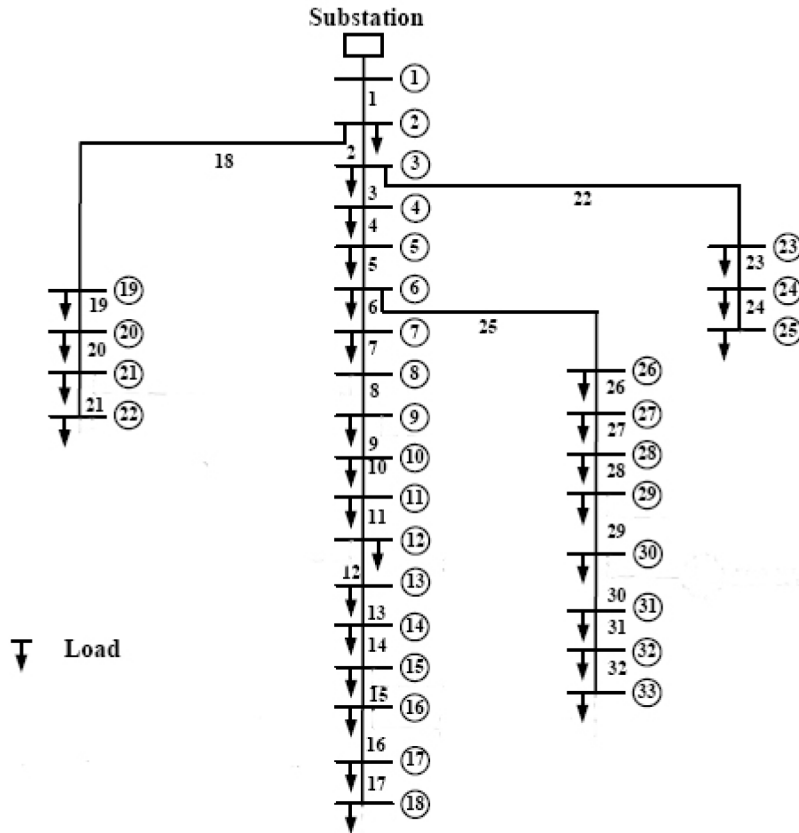


Figure 2. IEEE 33-bus radial distribution test feeder.

Table 3. Scenarios for projects.

Project	$P_{nom}$	Bus N°	RS	W (GWh)
Project 1	1.8 MW	6	Wind	5.2
Project 2	1 MW	10	Biomass	7.0
Project 3	2 MW	17	Hydro	4.0
Project 4	1 MW	17	Biomass	5.00

Values for both qualitative and quantitative indicators, as explained in Sections 2.2.1 and 2.2.2, are represented in Table 4.

Experts first perform pairwise comparison of the following criteria: technology ( $C_1$ ), costs reduction ( $C_2$ ), customer satisfaction ( $C_3$ ), and environmental impact reduction ( $C_4$ ). The results of the comparison, fuzzy weights, crisp weights, and ranks of criteria are shown in Table 5.

Table 4. Quantitative and qualitative values of indicators for projects.

Project	NPV (M€)	$V_D$ (%)	$P_{loss}$ (kW)	Reduction of $CO_2$ (t/year)	Environmental impact	Societal benefits
Project 1	4.2	29.65	156.4	5148	Moderate	High
Project 2	5.1	30.5	176.8	6930	Moderate	Moderate
Project 3	2.7	22.0	265.3	3960	Low	Very high
Project 4	3.8	26.3	190	4950	Very low	Moderate

**Table 5.** The pair wise comparison, fuzzy weights, crisp weights, and ranks of criteria.

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	Fuzzy weights
C <sub>1</sub>	$\tilde{1}$	$\tilde{3}$	$\tilde{5}^{-1}$	$\tilde{3}^{-1}$	(0.0496, 0.1178, 0.3249)
C <sub>2</sub>	$\tilde{3}^{-1}$	$\tilde{1}$	$\tilde{7}^{-1}$	$\tilde{5}^{-1}$	(0.0286, 0.0550, 0.1453)
C <sub>3</sub>	$\tilde{5}$	$\tilde{7}$	$\tilde{1}$	$\tilde{3}$	(0.2374, 0.5638, 1.2048)
C <sub>4</sub>	$\tilde{3}$	$\tilde{5}$	$\tilde{3}^{-1}$	$\tilde{1}$	(0.1062, 0.2634, 0.6956)
GCI = 0.1773					

Then the experts compare the following key performance indicators in relation to every criterion: net present value of investment (SC<sub>1</sub>), voltage deviation (SC<sub>2</sub>), power losses (SC<sub>3</sub>), emission reduction (SC<sub>4</sub>), environmental impact (SC<sub>5</sub>), and societal benefits (SC<sub>6</sub>). The results are presented in Tables 6–9.

**Table 6.** The pairwise comparison matrix of subcriteria in relation to the technology.

	SC <sub>1</sub>	SC <sub>2</sub>	SC <sub>3</sub>	SC <sub>4</sub>	SC <sub>5</sub>	SC <sub>6</sub>	Fuzzy weights
SC <sub>1</sub>	$\tilde{1}$	$\tilde{7}^{-1}$	$\tilde{5}^{-1}$	$\tilde{3}^{-1}$	$\tilde{7}^{-1}$	$\tilde{9}^{-1}$	(0.0134, 0.0259, 0.0637)
SC <sub>2</sub>	$\tilde{7}$	$\tilde{1}$	$\tilde{3}$	$\tilde{5}$	$\tilde{1}$	$\tilde{3}^{-1}$	(0.0871, 0.2026, 0.5664)
SC <sub>3</sub>	$\tilde{5}$	$\tilde{3}^{-1}$	$\tilde{1}$	$\tilde{3}$	$\tilde{3}^{-1}$	$\tilde{7}^{-1}$	(0.0353, 0.0882, 0.2501)
SC <sub>4</sub>	$\tilde{3}$	$\tilde{5}^{-1}$	$\tilde{3}^{-1}$	$\tilde{1}$	$\tilde{3}^{-1}$	$\tilde{7}^{-1}$	(0.0213, 0.0516, 0.1506)
SC <sub>5</sub>	$\tilde{7}$	$\tilde{1}$	$\tilde{3}$	$\tilde{3}$	$\tilde{1}$	$\tilde{3}^{-1}$	(0.0725, 0.1861, 0.5355)
SC <sub>6</sub>	$\tilde{9}$	$\tilde{3}$	$\tilde{7}$	$\tilde{7}$	$\tilde{3}$	$\tilde{1}$	(0.1715, 0.4457, 0.9276)
GCI = 0.1870							

**Table 7.** The pairwise comparison matrix of subcriteria in relation to the costs reduction.

	SC <sub>1</sub>	SC <sub>2</sub>	SC <sub>3</sub>	SC <sub>4</sub>	SC <sub>5</sub>	SC <sub>6</sub>	Fuzzy weights
SC <sub>1</sub>	$\tilde{1}$	$\tilde{7}$	$\tilde{5}$	$\tilde{3}$	$\tilde{9}$	$\tilde{3}$	(0.1653, 0.4446, 0.9011)
SC <sub>2</sub>	$\tilde{7}^{-1}$	$\tilde{1}$	$\tilde{5}^{-1}$	$\tilde{3}^{-1}$	$\tilde{3}$	$\tilde{5}^{-1}$	(0.0211, 0.0500, 0.1270)
SC <sub>3</sub>	$\tilde{5}^{-1}$	$\tilde{5}$	$\tilde{1}$	$\tilde{1}$	$\tilde{5}$	$\tilde{1}$	(0.0794, 0.1546, 0.4208)
SC <sub>4</sub>	$\tilde{3}^{-1}$	$\tilde{3}$	$\tilde{1}$	$\tilde{1}$	$\tilde{5}$	$\tilde{3}^{-1}$	(0.0505, 0.1182, 0.3313)
SC <sub>5</sub>	$\tilde{9}^{-1}$	$\tilde{3}^{-1}$	$\tilde{5}^{-1}$	$\tilde{5}^{-1}$	$\tilde{1}$	$\tilde{5}^{-1}$	(0.0153, 0.0305, 0.0765)
SC <sub>6</sub>	$\tilde{3}^{-1}$	$\tilde{5}$	$\tilde{1}$	$\tilde{3}$	$\tilde{5}$	$\tilde{1}$	(0.0839, 0.2021, 0.5503)
GCI = 0.2189							

**Table 8.** The pairwise comparison matrix of subcriteria in relation to the customer satisfaction.

	SC <sub>1</sub>	SC <sub>2</sub>	SC <sub>3</sub>	SC <sub>4</sub>	SC <sub>5</sub>	SC <sub>6</sub>	Fuzzy weights
SC <sub>1</sub>	$\tilde{1}$	$\tilde{3}$	$\tilde{7}$	$\tilde{5}$	$\tilde{3}$	$\tilde{3}^{-1}$	(0.0902, 0.2475, 0.6291)
SC <sub>2</sub>	$\tilde{3}^{-1}$	$\tilde{1}$	$\tilde{5}$	$\tilde{3}$	$\tilde{1}$	$\tilde{5}^{-1}$	(0.0499, 0.1139, 0.3335)
SC <sub>3</sub>	$\tilde{7}^{-1}$	$\tilde{5}^{-1}$	$\tilde{1}$	$\tilde{3}^{-1}$	$\tilde{5}^{-1}$	$\tilde{9}^{-1}$	(0.0144, 0.0278, 0.0707)
SC <sub>4</sub>	$\tilde{5}^{-1}$	$\tilde{3}^{-1}$	$\tilde{3}$	$\tilde{1}$	$\tilde{3}^{-1}$	$\tilde{5}^{-1}$	(0.0230, 0.0555, 0.1672)
SC <sub>5</sub>	$\tilde{3}^{-1}$	$\tilde{1}$	$\tilde{5}$	$\tilde{3}$	$\tilde{1}$	$\tilde{5}$	(0.0499, 0.1139, 0.3335)
SC <sub>6</sub>	$\tilde{3}$	$\tilde{5}$	$\tilde{9}$	$\tilde{5}$	$\tilde{5}^{-1}$	$\tilde{1}$	(0.1800, 0.4413, 0.9202)
GCI = 0.2002							

**Table 9.** The pairwise comparison matrix of subcriteria in relation to the environmental impact reduction.

	SC <sub>1</sub>	SC <sub>2</sub>	SC <sub>3</sub>	SC <sub>4</sub>	SC <sub>5</sub>	SC <sub>6</sub>	Fuzzy weights
SC <sub>1</sub>	$\tilde{1}$	$\tilde{3}^{-1}$	$\tilde{5}^{-1}$	$\tilde{9}^{-1}$	$\tilde{9}^{-1}$	$\tilde{5}^{-1}$	(0.0140,0.0251, 0.0522)
SC <sub>2</sub>	$\tilde{3}$	$\tilde{1}$	$\tilde{3}^{-1}$	$\tilde{7}^{-1}$	$\tilde{7}^{-1}$	$\tilde{3}^{-1}$	(0.0205,0.0467, 0.1101)
SC <sub>3</sub>	$\tilde{5}$	$\tilde{3}$	$\tilde{1}$	$\tilde{5}^{-1}$	$\tilde{5}^{-1}$	$\tilde{1}$	(0.0458,0.0985, 0.2168)
SC <sub>4</sub>	$\tilde{9}$	$\tilde{7}$	$\tilde{5}$	$\tilde{1}$	$\tilde{1}$	$\tilde{5}$	(0.1904,0.3657, 0.6880)
SC <sub>5</sub>	$\tilde{9}$	$\tilde{7}$	$\tilde{5}$	$\tilde{1}$	$\tilde{1}$	$\tilde{5}$	(0.1904,0.3657, 0.6880)
SC <sub>6</sub>	$\tilde{5}$	$\tilde{3}$	$\tilde{1}$	$\tilde{5}^{-1}$	$\tilde{5}^{-1}$	$\tilde{1}$	(0.0458,0.0985, 0.2168)
GCI = 0.2002							

The final fuzzy weights of the KPIs, according to Eq. (18) and Tables 5–9, are:

$$W_{sc} = \begin{bmatrix} (0.0283, 0.1737, 0.9459) \\ (0.0189, 0.1031, 0.6809) \\ (0.0123, 0.0605, 0.3784) \\ (0.0282, 0.1402, 0.7771) \\ (0.0361, 0.1841, 1.0655) \\ (0.0585, 0.3384, 1.6408) \end{bmatrix}. \tag{22}$$

At the end, four smart grid projects (Project 1 [A<sub>1</sub>], Project 2 [A<sub>2</sub>], Project 3 [A<sub>3</sub>], and Project 4 [A<sub>4</sub>]) are compared in relation to the KPIs presented in Tables 3 and 4 as presented in Table 10.

**Table 10.** The pairwise comparison of alternatives in relation to KPIs (BS - basis of comparison; EI - equal importance; WD - weak dominance; SD - strong dominance).

	SC <sub>1</sub>	SC <sub>2</sub>	SC <sub>3</sub>	SC <sub>4</sub>	SC <sub>5</sub>	SC <sub>6</sub>
A <sub>1</sub>	WD	EI	SD	WD	BC	WD
A <sub>2</sub>	BC	BC	WD	SD	EI	BC
A <sub>3</sub>	SD	SD	BC	BC	WD	SD
A <sub>4</sub>	WD	WD	WD	WD	SD	EI

The final fuzzy weights for smart grid projects, according to Eq. (19) and the results of pairwise comparison of alternatives in relation to all KPIs calculated from values given in Table 10, are:

$$W_a = \begin{bmatrix} (0.0140, 0.2067, 3.3081) \\ (0.0121, 0.1589, 2.5160) \\ (0.0251, 0.3982, 5.0638) \\ (0.0176, 0.2363, 3.7016) \end{bmatrix}. \tag{23}$$

Based on the previous results, we can conclude the following:

1. The most important criterion for the selection of a project according to the efficiency of the renewable energy plant integration in the smart grid context is customer satisfaction, followed by the environmental impact reduction, selection technology, and cost reduction (Table 5).
2. In relation to technology, the best-ranked performance indicator is societal benefits; in relation to costs reduction, net present value of investment; in relation to customer satisfaction, societal benefits; and in

relation to environmental impact reduction, carbon emission reduction and environmental impact level (Tables 6–9).

3. The final ranking of the KPIs according to all criteria is: 1) societal benefits; 2) environmental impact; 3) net present value of investment; 4) carbon emission reduction; 5) voltage deviation; 6) power losses (Eq. (22)).
4. Project 1 is dominant in relation to power losses; Project 2 in relation to carbon emission reduction; Project 3 in relation to net present value of investment, voltage deviation, and the societal benefits (however, it is the worst in relation to power losses and carbon emission reduction); and Project 4 in relation to environmental impact.
5. The final rank of the projects indicates that the highest rank is that of Project 3, followed by Project 4 and Project 1; the lowest priority is that of Project 2 (Eq. (23)). This means that for implementation of the smart grid Project 3 should be selected.

## 5. Conclusion

The new approach in the assessment of renewable energy projects efficiency is to evaluate to what extent these projects are contributing to progress toward the “ideal smart grid” and its expected outcomes (e.g., sustainability, efficiency, consumer inclusion). In this paper, the fuzzy AHP method is used to improve decision support for uncertain valuations and priorities. Starting from a general set of smart grid performance indicators, a new assessment framework for the evaluation of integration of renewable sources in the smart grid has been established. Based on fuzzy matching of alternatives, the methodology proposed in this paper determines the optimal allocation of renewable energy resources.

The proposed methodology is illustrated on the choice of the optimal size, location, and technology of renewable resources planned for integration in the existing distribution network. Using four main criteria and six subcriteria derived from the adopted set of smart grid benefits, we proved that the method is highly successful in the evaluation of alternatives in the presence of heterogeneous criteria. This method allows decision makers to incorporate unquantifiable information, incomplete information, nonobtainable information, and partially ignorant information into the decision model.

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**Appendix.** Data for 33 bus test system (substation voltage = 12.66 kV, MVA base = 10 MVA).

Line number	Sending bus	Receiving bus	Resistance ( $\Omega$ )	Resistance ( $\Omega$ )	Load at receiving end bus	
					Real power (kVAr)	Reactive power (kVAr)
1	1	2	0.0922	0.0477	100	6
2	2	3	0.4930	0.2511	90	40
3	3	4	0.3660	0.1864	120	80
4	4	5	0.3811	0.1941	60	30
5	5	6	0.8190	0.7070	60	20
6	6	7	0.1872	0.6188	200	100
7	7	8	1.7114	1.2351	200	100
8	8	9	1.0300	0.7400	60	20
9	9	10	1.0400	0.7400	60	20
10	10	11	0.1966	0.0650	45	30
11	11	12	0.3744	0.1238	60	35
12	12	13	1.4680	1.1550	60	35
13	13	14	0.5416	0.7129	120	80
14	14	15	0.5910	0.5260	60	20
15	15	16	0.7463	0.5450	60	20
16	16	17	1.2890	1.720	60	20
17	17	18	0.7320	0.5740	90	40
18	2	19	0.1640	0.1565	90	40
19	19	20	1.5042	1.3554	90	40
20	20	21	0.4095	0.4784	90	40
21	21	22	0.7089	0.9373	90	40
22	3	23	0.4512	0.3083	90	50
23	23	24	0.8980	0.7091	420	200
24	24	25	0.8960	0.7011	420	200
25	6	26	0.2030	0.1034	60	25
26	26	27	0.2842	0.1447	60	25
27	27	28	1.0590	0.9337	60	20
28	28	29	0.8042	0.7006	120	70
29	29	30	0.5075	0.2585	200	600
30	30	31	0.9744	0.9630	150	70
31	31	32	0.3105	0.3619	210	100