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Abstract

We argue that G_2 manifolds for M-theory admitting string theory Calabi-Yau duals are fibered by coassociative submanifolds. Dual theories are constructed using the moduli space of M-five-brane fibers as target space. Mirror symmetry and various string and M-theory dualities involving G_2 manifolds may be incorporated into this framework. To give some examples, we construct two non-compact manifolds with G_2 structures: one with a $K3$ fibration, and one with a torus fibration and a metric of G_2 holonomy. Kaluza-Klein reduction of the latter solution gives abelian BPS monopoles in $3+1$ dimensions.

1. Introduction

One of the main achievements in string theory during the last decade was the discovery of string dualities and relations among them. A particularly rich and interesting example of string duality is *mirror symmetry* between pairs of Calabi-Yau manifolds. A geometric framework for understanding this duality was proposed in [32], and involves constructing the mirror manifold by dualizing a torus fibration. This construction arose from the correspondence among nonperturbative states of dual theories. M-theory has united the disparate string theories and promises to reveal the nature of string dualities. In M-theory, the analogue of a Calabi-Yau manifold is a manifold with G_2 holonomy, simply by the counting of dimensions: what was $10 = 4 + 6$ for string theory is $11 = 4 + 7$ in M-theory. According to this simple formula, seven-dimensional G_2-holonomy manifolds are natural candidates for minimally supersymmetric (and phenomenologically interesting [3]) compactifications of M-theory to $3 + 1$ dimensions. If manifolds with G_2 holonomy are M-theory analogues of Calabi-Yau spaces, then what is the corresponding notion of mirror symmetry, and what is the geometry behind duality? Is there a fibration structure on G_2 manifolds relevant to this and possibly other string/M-theory dualities? These are the questions that one might naturally ask, and that we attempt to address in this paper.

We argue that, just as Calabi-Yau manifolds involved in mirror symmetry are fibered by special Lagrangian three-tori, in M-theory G_2-holonomy manifolds which admit string theory duals are fibered by coassociative four-manifolds. M-Theory on a seven-manifold X, with G_2 holonomy, leads to an effective field theory in four dimensions with $\mathcal{N} = 1$ supersymmetry. The same is true for the heterotic string theory on a Calabi-Yau manifold,

OV Z LO

$Y,$ [1]

$Z,$ fl

fl

[2] X, Y X, Z

X Y $Z,$

G_2

G_2

$C.$

C

Y $Z,$ $C.$

K

$T^3,$

K

K G_2

T^7

Remark 1. *j* G_2 *f* *b* *p* *p*

. *O* , *p* *f* G_2 *f* *per se*

. *F* *x* *p* *f* C *b* *f* *p* *f*

fib *f* *ff* G_2 *f* C *b*

K *z* K . *O* *xp* G_2 *f*

b *f* V *f* . *f* A ,

f G_2 *f* *fib* .

fib F M *f*

j G_2 *f* *p* *p* *f* G_2

.

Remark 2. *R* . *2* *p* *p*

. *W* *f* . *3* *4* *b* *p*

p *b* *p* . . *5*

x *b* .

---

[1] One requires $E \to Y$ to obey $p_1(E) = p_1(TY)$ and $c_1(E) = 0$ so that there is no anomaly, *i.e.* the heterotic theory contains no fivebranes.

[2] Kachru and Vafa first found heterotic-Type-IIA (Y, Z) pairs in [19]; some (X, Z) pairs are studied in [4, 5].

2. Fibrations from Brane Moduli Spaces

2.1. Fibrations from M-theory

G_2 $X.$

[3] G_2

$\times C,$ $\subset \mathbb{R}^4$ fl

$C \subset X$ $|_C$,

G_2 X

b

,

C

C

C

C

C

[4] C

β_+

C α ,

d $\alpha\beta_+$ $d\alpha$, α

b_2^+ C b_2^- C

---

[3] We recall that the field content of M-theory contains a three-form H with four-form field strength; it obeys $dH = \delta_D$, where D is the five-brane world-volume. This leads to a condition that the normal bundle have trivial Euler characteristic, which is true for the examples (T^4, $K3$) in this paper.

[4] One could also consider the Kaluza-Klein modes of the two-form, say, with only one leg along Σ. These modes, however, give rise to $U(1)$ gauge vector fields on Σ and, therefore, are non-dynamical.

OV Z LO

b_2^+ C

C b_2^+ C

p

H^2 C $\mathbb{R}$ $/H^2$ C $\mathbb{Z}$.

D H^3 D $\mathbb{R}$ $/H^3$ D $\mathbb{Z}$

b_2^+ C

b_2^+ C b_2^- C [5]

H^2 $C, \mathbb{Z}$.

C T^4 C K . b_2^+ C

X C C fl

$C \sim T^4$ b_2^+ b_2^- ,

$C \sim K$ b_2^+ , b_2^- ,

b_2^+ C C T^4 K ,

C

B

$C.$

[5]The metric is typically derived from the second-order variation of the five-brane action with respect to world-sheet time-dependence of the moduli fields, and the fermionic terms are derived from supersymmetry.

OV Z LO

C T^4,

G_2

H^2 C $\mathbb{R}$

G_2 C

fl fl

C T^4 C K

2.2. Torus Fibrations and Type-II/M-theory Duality

T^4

X

$C \sim T^4$

[6]

different

X,

mirror

dual

G_2

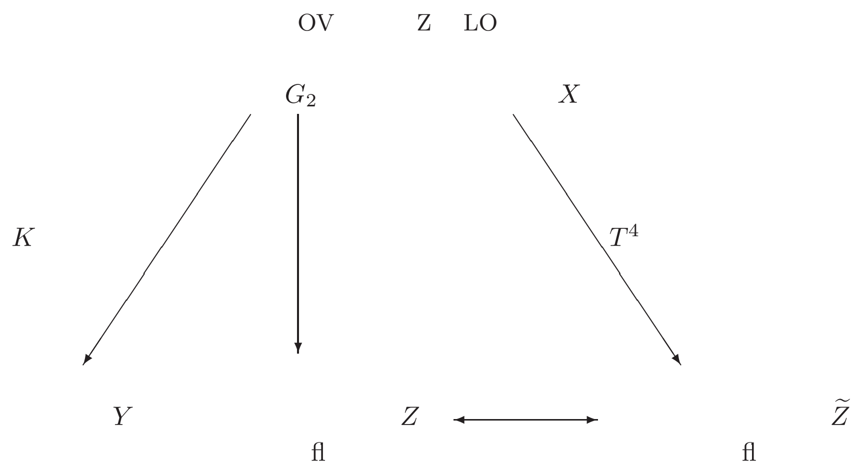
v

G_2 $\mathbb{R}^4/\mathbb{R}v$

$\mathbb{R}^4$, $\mathbb{R}^4$

G_2 $_0$ $\mathbb{R}^7$ $^2_+\mathbb{R}^4 \sim \mathbb{R}^4\ ^{\perp}$,

[6]If the fibration of T^4 over T^3 is not changing, then the dilaton is constant and $X \cong CY \times S^1$, so we recover $\mathcal{N} = 2$ supersymmetry.



Figu e 1. V X $_2$

$v,$ $w \in \mathbb{R}^4/\mathbb{R}v$ n

$_0$ v, w, n ,

[7]

G_2 $X,$

brane reduction is mirror to Kaluza-Klein reduction.

C

B

G_2

G_2 T^4

T^3 T^3 *or* T^4

G_2 b_2 $b_3,$

G_2

---

[7]For example, if we write

$$\Phi_0 = e_{125} + e_{345} + e_{136} - e_{246} + e_{147} + e_{237} + e_{567} \tag{2.2}$$

and $v = e_1$, then the duality pairs $e_2 \leftrightarrow e_5$, as $\iota_v \Phi_0 = e_{25} + e_{36} + e_{47}$, etc.

2.3. K Fibrations and Heterotic/M-theory Duality

Y

Y S^3

T^3 K

Y G_2

X K

X Y

X Y

X Y

f.

. .

b

G_2 X

C b_2 X

b_2 X $\leq$

G_2 K

$\chi/$

Y Y

G_2

X

$\mathcal{M}_{\mathrm{coassoc}}$

C.

Remark 3. N p f p fi

.

f p fi p p . I q f

p f p . W p fi G_2

q f f
z .

Remark 4. B C b p f p f b f [8] p G_2 p f f b p b f f p p f p (. . p b f / p). C b q $f\, T^*\mathcal{M}_{\text{coassoc}}$ b . G_2 f f q p x f p L fib . P p M f C b f f .

$\mathcal{M}_{\text{coassoc}}$ $\mathcal{M}_{\text{coassoc}}$

$$g\ V_1, V_2 \quad -\int_C \theta_1 \wedge \theta_2$$

θ_i $V_i \in T\mathcal{M}_{\text{coassoc}}|_C$.

$\mathcal{M}_{\text{coassoc}}$ Ω V_1, V_2, V_3 $T\mathcal{M}_{\text{coassoc}}|_C$, v_1, v_2, v_3 C X.

$$\Omega\ V_1, V_2, V_3\ |_C \quad \int_C \quad v_1, v_2, v_3\ dV,$$

G_2 $b_2^+\ C$,

$\mathcal{M}_{\text{coassoc}}$

2.4. Mirror Symmetry as Fourier-Mukai Transform

G_2 G_2 K G_2

K B

$$\pi \quad X \to B.$$

[8] This is pointed out in [27], for example.

OV Z LO

K

X
G_2 X
X X
p, q q B p
K X
$\mathcal{E}$ p, q
$\mathcal{E}_p$ K $q.$ $v\ \mathcal{E}_p$

$$v\ \mathcal{E}_p \quad ch\ \mathcal{E}_p\ \sqrt{A\ K\ }.$$

v , , $-$

K /
K
X
K X $b_2^+\ K$,
$q \in B$

X

v_{dual} , , .
K $p_1\ K$
v , , $-$
v , ,

$\mathcal{E}$ v r, l, s
$$dim\ M\ \mathcal{E} \quad l^2 - \ rs \quad .$$
v , , .

K T^4

T^4 $b_3^-\ T^4$
T^4 $b_1\ T^4$
fl

OV Z LO

G_2

R f B ?

K T^4

X B X

B

X G_2 b_1 X

fl U

3. A K Fibration

3.1. Idea and Basic Set-Up

G_2 K S^3,

,

$S^3 \setminus$.

G_2 K X [9]

B SO , $/SO$

g_B, SO , .

X $B \times K$

π $X \to B$

$p \in X$

π $T_V X$ TX,

connection,

$T_H X$ TX. TX TX $T_H X \oplus T_V X$,

P_H P_V

$b \in B$ K

K

$\mathcal{M}_{K3}$ SO , $/ SO$ $\times SO$.

SO , $\mathbb{Z}$

X ¨ K B $\mathcal{M}_{K3}$.

---

[9] This construction works for T^4 fibrations too, if in the following we simply replace $SO(3,19)$ and its maximal compact subgroup by $SO(3,3)$ with its corresponding subgroup.

τ $B \to \mathcal{M}_{K3}$. fl
K B τ,

3.2. The Fiberwise Metric and Connection

RicMet fl
K $RicMet \times K$ p, q
p q. $Diff$ $RicMet$
K $RicMet \times_{Diff} K$
fl K
$\mathcal{M}_{K3}$ $RicMet/Diff$.
fl K

X τ, $X \to B$
fl K π^{-1} b
τ b . X.
TX.
V X q,
$b \in B$. $U \subset B$ b π^{-1} U $\sim U \times K$.
$X \to B$,
fixed t γ t b γ $\pi_* V$.
g_t γ t
f_t K $\to K$

$$f_0 \quad , \quad \frac{d}{dt} f_t^* g_t \perp \quad .$$

f_t.
Γ t q Γ t γ t , f_t q ,
V

$$V_H \equiv \frac{d}{dt} \Gamma \; t \; |_{t=0}.$$

Lemma 5. *(3.11)* q f_t.

P f. f_t t ,
ξ. ξ $\xi_\mu dx^\mu$,
x^μ. η $\frac{d}{dt} g_t|_{t=0}$.

$$A_{\mu\nu} \equiv \frac{d}{dt} \; f_t^* g_t \;\; _{\mu\nu}|_{t=0} \quad \eta_{\mu\nu} \quad \nabla_\mu \xi_\nu \quad \nabla_\nu \xi_\nu.$$

$A_{\mu\nu}$

$$\nabla^\mu A_{\mu\nu} \quad .$$

$$\nabla^\mu \nabla_\mu \xi_\nu \quad \nabla^\mu \nabla_\nu \xi_\nu \quad -\nabla^\mu \eta_{\mu\nu} \equiv -B_\nu,$$

$$B_\nu \quad \nabla^\mu \eta_{\mu\nu}.$$

$$\nabla^\mu \nabla_\nu \xi_\mu \quad \nabla^\mu, \nabla_\nu \; \xi_\mu \quad \nabla_\nu \nabla^\mu \xi_\nu \quad R_\nu{}^\alpha \xi_\alpha \quad \nabla_\nu \nabla^\mu \xi_\nu \quad \nabla_\nu \nabla^\mu \xi_\nu,$$

fl

$$\nabla^\mu \nabla_\mu \xi_\nu \quad \nabla_\nu \nabla^\nu \xi_\nu \quad -B_\nu,$$

$$d^\dagger d\xi \quad dd^\dagger \xi \quad B.$$

K

ξ dh $d^\dagger k$ h $k,$ h k d $d^\dagger,$ B dH $d^\dagger K,$ H K $d^\dagger dd^\dagger k$ $dd^\dagger dh$ dH $d^\dagger K,$

$$k \quad \frac{}{dd^\dagger} K, \qquad h \quad \frac{}{d^\dagger d} H.$$

$\xi,$ $\frac{df_t}{dt}|_{t=0},$ ■ V_H $T_H \subset TX.$ TX $T_H \oplus T_V$ $U,$ T_V

Corollary 6. b b T_H fi $f X$ fi p TX $T_H \oplus T_V.$

3.3. Defining the Three-Form,

X $p \in X,$ b π p μ τ b , g_K^μ $\mu \in \mathcal{M}_{K3}.$ $U, V \in T_pX.$

$$g \; U, V \quad g_B \; \pi_* U, \pi_* V \; |_b \quad g_K^\mu \; P_V U, P_V V \; |_p$$

X P_V

B $\mathcal{M}_{K3}$ $G/H.$ H $G/H,$ G g $H_g \equiv gHg^{-1} \sim H.$ ghg^{-1} g gh g , g $gH.$ $\widetilde{\tau}$ SO , $\to SO$,

$$\widetilde{\tau} \; SO \quad \subset SO \quad \times SO$$

$$p_1 \circ \widetilde{\tau}$$

p_1 $\widetilde{\tau}$

$$\tau \quad B \to \mathcal{M}_{K3},$$

$c \in SO$, $TB.$ C $\widetilde{\tau}$ c $\in$ SO $\times SO$ $T\mathcal{M}_{K3}.$

$$\tau_* \circ c \quad C \circ \tau_*,$$

τ_*

SO [10]

G_2 X

$p \in X$ GL T_pX

0

0 $\mathbb{R}^7$

$\mathbb{H} \oplus \mathbb{H},$ e_i $\mathbb{H}$

α_i ${}^2\mathbb{H}$ $\mathbb{H}$ $e_i.$ α_1 $e_4 \wedge e_5$ $e_6 \wedge e_7$.

0 $e_1 \wedge e_2 \wedge e_3$ $\sum_{i=1}^3 e_i \wedge \alpha_i.$ G_2 ϕ g g u, v dV

$\iota_u\phi \wedge \iota_v\phi \wedge \phi,$ dV $\sqrt{\quad g}\, e_1 \wedge \ldots \wedge e_7.$

B

B

$$TB|_{[g]} \sim \mathfrak{g}/\mathfrak{h}_g, \quad \mathfrak{g} \quad so \; , \; , \mathfrak{h} \quad so \; ,$$

$\mathfrak{h}_g$ $g\mathfrak{h}g^{-1}$ ${}_g\mathfrak{h}$ g

$$\mathfrak{g}/\mathfrak{h}_g \sim \mathfrak{h}_g^\perp \sim \mathfrak{h}_g$$

$\mathfrak{g}$

$\mathfrak{g}$ so , $\sim sl$ $, \mathbb{C}$ $\sim \mathbb{C}^3,$

$\mathfrak{h}$ [11] $V \in T_{[g]}B$ $C_V \in$

so . C_V $\mathcal{M}_{K3}$

$\mathbb{R}^{3,19},$

H^2 K . SO $\subset SO$ $\times SO$

SO

V b

C_V $p_1 \circ \widetilde{\tau}$ so

π^{-1} b . θ_V p_1 $\widetilde{\tau}$ C_V ,

[10] This construction was meant to mimick the notion of holomorphicity for an an elliptic fibration, which can be written as the equivariant property $\tau_* \circ j = J \circ \tau_*$, where j and J are complex structures on the base of the fibration and the moduli of elliptic curves, respectively. We thus think of a complex structure as the $\pi/2$ element of an $S^1 = SO(2)$ action. In our case, $SO(3)$ is the relevant group.

[11] Explicitly, if at the identity coset we identify $so(3,1) = sl(2,\mathbb{C})$ with traceless, 2×2 complex matrices with the indefinite metric $\langle A, B\rangle = -\frac{1}{2}\mathrm{Tr}(AB)$, then $so(3) = su(2)$ are the anti-hermitian ones and the map $\mathfrak{g}/\mathfrak{h} \to \mathfrak{h}^\perp$ sends $A \mapsto (A - A^\dagger)/2$. The involution from $\mathfrak{h}^\perp \leftrightarrow \mathfrak{h}$ is just $A \mapsto iA^\dagger$, or $\sqrt{-1}$ times the Cartan involution. Put physically, the projection to $\mathfrak{h}^\perp$ eliminates rotational pieces of an infinitesimal Lorentz transformation, and then the correspondence $\mathfrak{h}^\perp \cong \mathfrak{h}$ associates to a pure boost in some direction a rotation in the same direction.

$\int \theta_V{}^2 /$ $|V|^2$. 12

$$\theta \quad T_b B \to H^2_+ \; \pi^{-1} \; b \quad ,$$

$$\mathfrak{h}_g \sim so \overset{p_1 \circ \widetilde{\tau}}{\sim} so \; H^2_+ \; K \quad \sim H^2_+ \; K \quad .$$

θ H^2_+

$p \in X$ $\pi \; p$ $g \in B$. $\{e_i\}$
$T^*_{[g]} B \sim \mathfrak{g}/\mathfrak{h}_g$, $\{\theta_i\}$ $\pi^{-1} \; b \; \sim K$.

$$Vol_B \quad \sum_i e_i \wedge \theta_i,$$

Vol_B B X

U, V, W $Vol_B \; \pi_* U, \pi_* V, \pi_* W$ $\theta_{\pi_* U} \; P_V V, P_V W$.
$\widetilde{\tau}$ e_i
θ_i

3.4. Explicit Formulas

$\eta_{\mu\nu}$

Lemma 7. η p .

P f. η
$\nabla^\mu \eta_{\mu\alpha}$.
$R^\mu{}_{\alpha\mu\beta}$

$$\nabla^\mu \nabla_\mu \eta_{\alpha\beta} \quad R_\alpha{}^\mu{}_\beta{}^\nu \eta_{\mu\nu} \quad \nabla_\alpha \nabla_\beta \eta^\mu{}_\mu \quad .$$

$g^{\alpha\beta}$
fl

$$\nabla^\mu \nabla_\mu \eta^\alpha{}_\alpha \quad .$$

[12]If we write an element C_V of $so(3)$ as $aX + bY + cZ$, then (a, b, c) defines the axis of rotation in three-space. After equating the three-plane $\mathbb{R}^3$ isometrically with $so(3)$, then θ_V is simply (a, b, c). That is, $End(E) \cong E$ for oriented, three-dimensional metric vector spaces.

$\triangle$ $\eta^{\alpha}{}_{\alpha}$ $\eta^{\alpha}{}_{\alpha}$ C, η
$\int C$ $C \cdot Vol$, η ■
S^a, a $,\ldots,$,
S^a i, j, k,
η A^a

$$A^a{}_{\mu\nu} \quad - \; S^a{}_{\mu\sigma}\eta^{\sigma}{}_{\nu} \quad \eta_{\mu\sigma}S^{a\,\sigma}{}_{\nu} \; .$$

A^a

$$\eta_{\mu\nu} \quad -\sum_a A^a{}_{\mu\sigma}S^{a\,\sigma}{}_{\nu}.$$

η [13]

W $\mathbb{R}^{3,19} \sim H^2$ K
, H^2_+ K . $\widetilde{\tau}$ SO , $\sim SO$ W .
$b \in B$ β W
S^a, A^a $r^a\beta$ a $,\ldots,$. V B
η, A^a,

$$\theta_V \quad \sum_a r^a S^a.$$

3.5. d /

τ
G_2,
d $\lambda *$.
G_2 G_2

G B,
$\widetilde{\tau}$ SO , $\mathbb{Z}$
SO , $\mathbb{Z}$

[13] This depends on some nice facts, including the following identity. Let A be a traceless, symmetric, four-by-four matrix acting on the quaternions $\mathbb{R}^4$. Let I, J, K be matrices representing multiplication by i, j, k. Then $A = IAI + JAJ + KAK$.

4. A Torus Fibration

4.1. Outline of Hitchin's Construction

G_2 SU
G_2
fl

G_2 G_2

Theorem 8. *: L M b 6 f* $\mathcal{A} \in H^3 M, \mathbb{R}$ $\mathcal{B} \in H^4 M, \mathbb{R}$ *b fix*
$\rho, \sigma \in \mathcal{A} \times \mathcal{B}$ *b b f f p p*
H *fl f f :*

$$H \quad V \; \rho \; - \; V \; \sigma \; .$$

H $V \; \rho$ $V \; \sigma$ *b f (fi b)* ϕ
: V $\int_M \phi$. *If f t* $t_0, \; \rho$ σ *fi f p b*
$\omega \wedge \rho$ $\phi \; \rho$ $\phi \; \sigma$ *(* σ $\omega^2/$ *) f*

$$dt \wedge \omega \quad \rho,$$

fi G_2 X $M \times \; a, b$ *f* a, b .

Defintion 9. *: L M b f f n* V TM.
f $\rho \in {}^pV^*$ *b f p b f ( )* $GL \; V$ ${}^pV^*$.
ρ $GL \; V$
ρ
ω $\omega^{n/2}$ /
M

$$\sigma \in \; {}^4V^* \sim \; {}^2V \otimes \; {}^6V^*.$$

$$\sigma^3 \in \; {}^6V \otimes \; {}^6V^* \; {}^3 \sim \; {}^6V^* \; {}^2$$

$$V \; \sigma \quad \int_M |\sigma^3|^{\frac{1}{2}}.$$

OV Z LO

$$V \ \rho \qquad \rho \in \ {}^3V^*$$

$$K_\rho \ V \to V \otimes \ {}^6V^*,$$

$v \in V \quad TM$

$$K \ v \qquad \imath \ v \ \rho \wedge \rho \in \ {}^5V^* \sim V \otimes \ {}^6V^*.$$

$$tr \ K^2 \ \in \ {}^6V^* \ {}^2.$$

$SL \ ,\mathbb{C}$ $\qquad tr \ K \ {}^2 <$

$$V \ \rho \qquad \int_M |\sqrt{-trK^2}|.$$

$$\mathcal{A} \times \mathcal{B} \sim \Omega^3_{exact} \ M \ \times \Omega^4_{exact}.$$

$$\omega \ \ \rho_1, \sigma_1 \ , \ \rho_2, \sigma_2 \qquad \langle \rho_1, \sigma_2 \rangle - \langle \rho_2, \sigma_1 \rangle,$$

$$\rho \qquad d\beta \in \Omega^p_{exact} \ M \qquad \sigma \qquad d\gamma \in \Omega^{n-p+1}_{exact} \ M$$

$$\langle \rho, \sigma \rangle \quad \int_M d\beta \wedge \gamma \qquad - \ \ {}^p \int_M \beta \wedge d\gamma.$$

fl

$dt \wedge \omega \quad \rho$

$$d \qquad , \quad d * \qquad .$$

G_2

$v, \ w \in W, \qquad W \quad TX$

$W \qquad {}^7W^*$

$$B_\Phi \quad --\imath \ v \quad \wedge \imath \ w \quad \wedge \quad .$$

$$K_\Phi \ \ W \to W^* \otimes \wedge^7 W^*. \qquad G_2$$

$$g_\Phi \ v, w \qquad B_\Phi \ v, w \qquad K_\Phi \ {}^{-\frac{1}{9}}.$$

4.2. Equations for the Metric

G_2

$M \quad S^3 \times T^3.$

S^3 SU M

G_2

ρ σ

SU

$$_1 \quad \psi d\theta \quad \psi \quad \theta d\phi,$$
$$_2 \quad - \quad \psi d\theta \quad \psi \quad \theta d\phi,$$
$$_3 \quad d\psi \quad \theta d\phi.$$

su

$$d \quad _a \quad -- \epsilon_{abc} \quad _b \wedge \quad _c.$$

α_i $H^1 \; T^3$

$$\alpha_{1,2,3} \in H^1 \; T^3 \quad \mathbb{R}^3.$$

$$T^3 \quad \mathbb{R}^3/\mathbb{Z}^3,$$

$\mathbb{R}^3$ $u_1 \; u_2 \quad u_3$

$$\alpha_i \quad du_i, \quad i \quad , \; , \; .$$

$\mathcal{A}$ $\mathcal{B}$ $H^3 \; M$ $H^4 \; M$

$$H^3 \; M \; \mathbb{R} \qquad \mathbb{R} \oplus \mathbb{R},$$
$$H^4 \; M \; \mathbb{R} \qquad \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R},$$

real $m \; n \; k_1 \; k_2 \quad k_3$
$m \quad n$
$\mathcal{A} \in H^3 \; M \; \mathbb{R}$ m T^3
n S^3
SU $\rho \in \mathcal{A}$

$$\rho \quad n \; _1 \; _2 \; _3 - m\alpha_1\alpha_2\alpha_3 \quad x_1 d \; _1\alpha_1 \quad x_2 d \; _2\alpha_2 \quad x_3 d \; _3\alpha_3 \; .$$

$x_i \; t$ t ρ
$m \quad n$ t
ρ

$$\sigma \quad k_1 \quad {}_1 \quad {}_2 \quad {}_3\alpha_1 \quad k_2 \quad {}_1 \quad {}_2 \quad {}_3\alpha_2 \quad k_3 \quad {}_1 \quad {}_2 \quad {}_3\alpha_3$$
$$y_1 \quad {}_2\alpha_2 \quad {}_3\alpha_3 \quad y_2 \quad {}_3\alpha_3 \quad {}_1\alpha_1 \quad y_3 \quad {}_1\alpha_1 \quad {}_2\alpha_2.$$

$${}_2\alpha_2 \quad {}_3\alpha_3 \quad d \quad {}_1 \ \alpha_2\alpha_3 \quad d \quad {}_1\alpha_2\alpha_3 \ ,$$

ρ σ

$k_{1,2,3}$

σ

$$V^2 \ \sigma \quad y_1y_2y_3.$$

k_1 k_2 k_3 SU σ

$$\sigma \quad y_1 d \quad {}_1\alpha_2\alpha_3 \quad y_2 d \quad {}_2\alpha_3\alpha_1 \quad y_3 d \quad {}_3\alpha_1\alpha_2 \ .$$

σ $\omega^2/$ ω

$\omega \wedge \rho$

$$\omega \quad \sqrt{\frac{y_2y_3}{y_1}} \ {}_1\alpha_1 \quad \sqrt{\frac{y_1y_3}{y_2}} \ {}_2\alpha_2 \quad \sqrt{\frac{y_1y_2}{y_3}} \ {}_3\alpha_3.$$

ω

$$\sigma \quad -\omega^2,$$

$$\omega \wedge \rho \quad .$$

fl

$x_i \ t$ $y_i \ t$

$$\langle \ {}_2\alpha_2 \ {}_3\alpha_3, d \ {}_1\alpha_1 \ \rangle \quad \int_{S^3\times T^3} {}_2\alpha_2 \ {}_3\alpha_3 \ {}_1\alpha_1 \qquad S^3 \qquad T^3 \ / \ .$$

$SU \quad \times SU$

$$dx_1 \wedge dy_1 \quad dx_2 \wedge dy_2 \quad dx_3 \wedge dy_3.$$

$V \ \rho$

$$V^2 \ \rho \quad -m^2n^2 - \ mx_1x_2x_3.$$

$V \ \rho$ $V \ \sigma$

$$y_1y_2y_3 > \ , \quad mx_1x_2x_3 < -m^2n^2.$$

$$H \quad V \rho - V \sigma$$
$$\sqrt{-m^2n^2 - mx_1x_2x_3} - \sqrt{y_1y_2y_3},$$

$i / j / k / i$, , , fl

$$y_i \quad \frac{mx_jx_k}{\sqrt{-m^2n^2 - mx_ix_jx_k}},$$
$$x_i \quad -\sqrt{\frac{y_jy_k}{y_i}}.$$

G_2 $a, b \times$ $S^3 \times T^3$

$$dt \wedge \left(\sqrt{\frac{y_2y_3}{y_1}} \;_1\alpha_1 \quad \sqrt{\frac{y_1y_3}{y_2}} \;_2\alpha_2 \quad \sqrt{\frac{y_1y_2}{y_3}} \;_3\alpha_3 \right)$$
$$n \;_1 \;_2 \;_3 - m\alpha_1\alpha_2\alpha_3 \quad x_1 d \;_1\alpha_1 \quad x_2 d \;_2\alpha_2 \quad x_3 d \;_3\alpha_3 .$$

n B SU

X $a, b \times SU$ $\times T^3$ $a, b \times T^3$

S^3 T^3 G_2

$$S^3 \quad \geq \quad |n|,$$
$$T^3 \quad \geq \quad |m|.$$

G_2

$$ds^2 \quad K_\Phi \;^{-1/9} \Big(\sqrt{y_1y_2y_3}\, dt^2 \quad \sqrt{\frac{y_2y_3}{y_1}} \; x_2x_3 \;_1^2 \quad mn \;_1\alpha_1 - mx_1\alpha_1^2$$
$$\sqrt{\frac{y_1y_3}{y_2}} \; x_1x_3 \;_2^2 \quad mn \;_2\alpha_2 - mx_2\alpha_2^2 \quad \sqrt{\frac{y_1y_2}{y_3}} \; x_1x_2 \;_3^2 \quad mn \;_3\alpha_3 - mx_3\alpha_3^2 \Big)$$

$$K_\Phi \quad -m^3 \; y_1y_2y_3 \;^{3/2} \Big(x_1x_2x_3 \quad -mn^2 \Big)^3 \quad y_1y_2y_3 \;^{9/2},$$

H

$K_\Phi \;^{-1/9}$ dt^2

$$ds^2 \quad dt^2 \quad \frac{}{y_1} \; x_2x_3 \;_1^2 \quad mn \;_1\alpha_1 - mx_1\alpha_1^2$$
$$\frac{}{y_2} \; x_1x_3 \;_2^2 \quad mn \;_2\alpha_2 - mx_2\alpha_2^2 \quad \frac{}{y_3} \; x_1x_2 \;_3^2 \quad mn \;_3\alpha_3 - mx_3\alpha_3^2 .$$

4.3. SU Symmetric Solution and Large Distance Asymptotics

G_2

m n

$$x_1 \quad x_2 \quad x_3, \quad y_1 \quad y_2 \quad y_3.$$

SU

SU

$f.$

$$x \qquad -\sqrt{y},$$

$$y \qquad \frac{mx^2}{\sqrt{-m^2n^2 - \ mx^3}}.$$

m

$x\ t$ $y\ t$

$$-\infty < \ x\ t \quad \leq -\left(\frac{mn^2}{\ }\right)^{1/3},$$

$$\leq \ y\ t \quad < \ \infty.$$

$H\ t$

$$-mx^3 \quad y^3 \quad -m^2n^2.$$

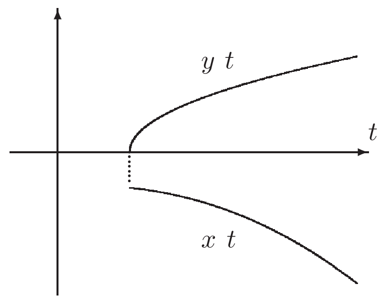
$$\frac{dy}{dt} \quad \frac{m}{y^{3/2}}\left(\frac{\ }{m}y^3 \quad -mn^2\right)^{2/3},$$

$$t \quad t_0 \quad -\left(\frac{7}{m^5n^4}\right)^{1/3} y^{5/2} F\left(\left[-,-\right],\left[-\right],-\frac{y^3}{m^2n^2}\right).$$

G_2

$$ds^2 \quad dt^2 \quad \frac{\ }{y}\sum_{j=1}^{3} x^2 \ {}^2_j \quad mn \ {}_j\alpha_j - mx\alpha_j^2 \ ,$$

$$SU \quad \times SU \quad \times U \quad {}^3.$$



Figu e 2. x t y t

x t y t

Solution with n and Large Distance Asymptotics

n

$$x \quad -\frac{m^{1/3}}{} \; t - t_0 \;^2, \quad y \quad \frac{m^{2/3}}{} \; t - t_0 \;^2.$$

t_0

$$ds^2 \quad dt^2 \quad -t^2 \quad {}^2_1 \quad {}^2_2 \quad {}^2_3 \quad m^{2/3} \; \alpha_1^2 \quad \alpha_2^2 \quad \alpha_3^2 \; .$$

/ $_i$

fl

$$\mathbb{R}^4 \times T^3.$$

m T^3

$$T^3 \; |_{t=\infty} \quad m,$$

SU

m fl
m

n y t

$x\ t$ $t \to \infty$ n^2

$$\frac{dy}{dt} \quad m^{1/3}\sqrt{y},$$

$pp \quad x$ SU n $\mathbb{R}^4 \times T^3$ $n\ /$ $\mathbb{R}^4 \times T^3$ $|n| \quad f.$ n

4.4. U Symmetric Solution and Short Distance Asymptotics

SU $x\ t$ $y\ t$ G_2

$$x \to -\Big(\frac{mn^2}{\quad}\Big)^{1/3}, \quad y \to \ ,$$

t $t \to$ n $M \quad S^3 \times T^3$ T^3 t

$$X \sim \mathbb{R}^4 \times T^3$$

$m\ /$ $n\ /$ S^3 T^3

$$ds^2 \quad dt^2 \quad t^{-2/5}\sum_{j=1}^{3} \quad {}_j \quad \alpha_j \ {}^2 \quad \ldots$$

$t \to$ G_2 t SU SU $U \quad \subset SU$

$$x_1 \quad x_2, \quad y_1 \quad y_2.$$

$$ds^2 \qquad dt^2 \quad \frac{x_1x_3}{y_1} \quad {}^2_1 \qquad {}^2_2 \quad \frac{mn}{y_1} \quad {}_1\alpha_1 - \quad {}_2\alpha_2 \ - \frac{mx_1}{y_1}\ \alpha_1^2 \quad \alpha_2^2$$
$$\frac{\quad}{y_3}\ x_1^2 \ {}^2_3 \quad mn \ \ {}_3\alpha_3 - mx_3\alpha_3^2 \ .$$

$$x_1 \quad -\sqrt{y_3}, \qquad y_1 \quad \frac{mx_1x_3}{y_1\sqrt{y_3}},$$
$$x_3 \quad -\frac{y_1}{\sqrt{y_3}}, \qquad y_3 \quad \frac{mx_1^2}{y_1\sqrt{y_3}}.$$

n

$$t \quad t_0 - x_1\Big(-\frac{B^2}{\beta m}\Big)^{1/4} F\left(\Big[-,-\Big],\Big[-\Big],-Bx_1^2/\beta\right),$$

$$\beta \quad x_3y_3 - x_1y_1$$

$B \leq$ B SU

β

$t \to$

x_i y_i t

S^3

$$S^3 \to S^2, \quad t \to \quad .$$

n

x_1 t y_1 t t x_3 t y_3 t

$\beta <$

$$y_3 \approx \frac{\beta}{x_3},$$

$$y_1 \approx \sqrt{-\frac{m}{\beta}}x_1x_3$$

x_1 x_3

$$x_1 \quad -A \quad \gamma t \;, \qquad y_1 \quad \frac{\sqrt{-m\beta} \quad \gamma t}{\gamma^2 A \quad ^2 \; \gamma t},$$
$$x_3 \quad \frac{\beta}{\gamma^2 A^2 \quad ^2 \; \gamma t}, \qquad y_3 \quad \gamma^2 A^2 \quad ^2 \; \gamma t \;,$$

OV Z LO

$$\gamma^2 \quad \sqrt{\frac{m}{|\beta|}}.$$

$|\beta| \gg |x_1 y_1|$

$^2 \ \gamma t \ \ll$

t

$$x_1 \approx -A\gamma t, \qquad y_1 \approx \frac{\sqrt{-m\beta}t}{\gamma A},$$
$$x_3 \approx \frac{\beta}{\gamma^2 A^2}, \qquad y_3 \approx \gamma^2 A^2.$$

$$ds^2 \approx dt^2 \quad \sqrt{\frac{|\beta|}{m}} \quad {}_1^2 \quad {}_2^2 \quad \frac{mA^2}{|\beta|} \alpha_1^2 \quad \alpha_2^2 \quad t^2 \ {}_3^2 \quad \frac{\beta^2}{A^4}\alpha_3^2.$$

$\gamma t \gg$ $\mathbb{R}^4/\mathbb{Z}_2 \times T^3$

X G_2

$$X \sim S^2 \times \mathbb{R}^2 \times T^3,$$

$\mathbb{R}^4/\mathbb{Z}_2 \times T^3$

$\sqrt{|\beta|}$

4.5. General Solution

$$H \quad \sqrt{-m^2 n^2 - \ m x_1 x_2 x_3} - \ \sqrt{y_1 y_2 y_3} \quad V \ \rho \ - \ V \ \sigma \ ,$$

H , $V \ \rho$ $V \ \sigma$,

$$\widetilde{H} \quad V \ \rho \ ^2 - \ V \ \sigma \ ^2 \quad -m^2 n^2 - \ m x_1 x_2 x_3 - \ y_1 y_2 y_3.$$

$d \ V \ \rho \ ^2 - \ V \ \sigma \ ^2 \quad V \ \rho \ dV \ \rho \ - \ dV \ \sigma$

$$\frac{dt}{d\widetilde{t}} \quad V \ \rho \quad V \ \sigma \quad \sqrt{y_1 y_2 y_3}.$$

$$\vec{x}, \vec{y} \ \mapsto \ M \cdot \vec{x}, M^{-1} \cdot \vec{y} \ , \qquad M$$

$$x_1 y_1 - x_3 y_3 \qquad x_2 y_2 - x_3 y_3.$$

$i \ / \ j \ / \ k \ / \ i.$

$$x_i \quad - \ y_j y_k \qquad y_i \quad m x_j x_k$$

OV Z LO

$\frac{d}{d\tilde{t}}$.

$$z_i \quad x_i y_i.$$

$\frac{d}{d\tilde{t}}$ $z_i - z_j$,

$$z \quad \equiv \quad z_1$$
$$z_2 \quad z \quad \alpha$$
$$z_3 \quad z \quad \beta.$$

$$X \equiv x_1 x_2 x_3 \qquad Y \equiv y_1 y_2 y_3.$$

$$m^2 n^2 \quad mX \quad Y \quad .$$

$$z \quad x_1 y_1 \quad x_1 y_1 \quad - \; Y \quad mX \quad -m^2 n^2 - \; Y.$$

$$Y \quad m \; x_2 x_3 y_2 y_3 \quad x_1 x_3 y_1 y_3 \quad x_1 x_2 y_1 y_2 \; ,$$

$$\ddot{z} \quad - \quad m \quad z^2 \quad z \; \alpha \quad \beta \quad \alpha\beta \; .$$

$$Y \quad - z \quad m^2 n^2 \; / \; ,$$

x_i y_i $x_1 \quad - \; y_2 y_3 \quad - \; Y/y_1 \quad - \quad Y/z \; x_1,$

$$x_1 \quad -A_1 \quad - \int Y/z \; d\tilde{t} \; ,$$

A_1 $y_1 \quad z/x_1.$ x_2, y_2, x_3, y_3

t,

$$\frac{dt}{d\tilde{t}} \quad \sqrt{Y}.$$

Equation for z

$$u \quad - \quad mz - \quad m\,\alpha \quad \beta$$

$$\ddot{u} \quad u^2 \quad D$$

$$D \quad m^{\ 2} \quad \alpha\beta - \ \alpha \quad \beta^{\ 2} \quad - \quad m^{\ 2}\ \alpha^2 - \alpha\beta \quad \beta^2 \ .$$

$u \quad f\ u\ .$

$\ddot{u} \quad f'u \quad ff',$

$$f^2\ ' \quad u^2 \quad D\ ,$$

$f^2 \quad \frac{2}{3}u^3 \quad Du \quad E.$ $\quad f \quad \pm\sqrt{\frac{2}{3}u^3 \quad Du \quad E}$ $\quad u \quad f\ u$

$$\tilde{t} \quad C \pm \int \frac{du}{\sqrt{\frac{2}{3}u^3 \quad Du \quad E}},$$

$$y^2 \quad -x^3 \quad Dx \quad E.$$

u $\tilde{t}$

$v \quad u$

$$g_2 \equiv -D/\ .$$

v

$$\ddot{v} \quad v^2 - g_2/$$

$$\tilde{t} \quad C \pm \int \frac{dv}{\sqrt{\ v^3 - g_2 v - g_3}}$$

C $\quad g_3$

$$v \quad \mathfrak{p}_\tau\ \tilde{t} \quad C\ ,$$

τ $\quad g_2$ $\quad g_3.$

$$-\tilde{t} \quad C$$

$\mathfrak{p}$

$$\mathfrak{p}'\ ^2 \quad \mathfrak{p}^3 - g_2\mathfrak{p} - g_3.$$

OV Z LO

$$\mathfrak{p}'\mathfrak{p}'' \quad \mathfrak{p}^2\mathfrak{p}' - g_2\mathfrak{p}',$$

$$\mathfrak{p}'' \quad \mathfrak{p}^2 - g_2/ \ .$$

g_2 g_3

g_2

$$v \quad {}^2\,\widetilde{t} \quad C \ ,$$

g_2 /

$$v \quad {}^2\,\widetilde{t} \quad C \ - \ / \ .$$

Asymptotics and Behavior Near the Poles

$\mathfrak{p}$

$\mathbb{R}^4 \times T^3$

$$v \approx \widetilde{t}^{-2}.$$

v $g_2/$

$$z \approx -\frac{u}{m} \approx -\frac{v}{m} \approx -\frac{}{m\widetilde{t}^2}.$$

$x_i\ \widetilde{t}$ $y_i\ \widetilde{t}$

$$x_i \sim y_i \sim \frac{}{\widetilde{t}},$$

$$Y \approx \frac{}{m\widetilde{t}^3}.$$

t $\widetilde{t}$

t

$$x_1 \quad -A_1t^2, \quad y_1 \quad \frac{m}{A_1}t^2,$$
$$x_2 \quad -A_2t^2, \quad y_2 \quad \frac{m}{A_2}t^2,$$
$$x_3 \quad -A_3t^2, \quad y_3 \quad \frac{m}{A_3}t^2,$$

OV Z LO

A_i

$x_i <$

$$A_1 A_2 A_3 \quad m.$$

A_i A_3

$$A_3 \quad \frac{m}{A_1 A_2}.$$

x_i $n \to$

$$y_i \quad \sqrt{-\frac{m x_j x_k}{x_i}}, \quad x_i \quad -\sqrt{\frac{y_j y_k}{y_i}}.$$

SU x_i y_i

t^2

G_2

$$ds^2 \quad dt^2 \quad -t^2 \quad {}^2_1 \quad {}^2_2 \quad {}^2_3 \quad A_1^2\alpha_1^2 \quad A_2^2\alpha_2^2 \quad A_3^2\alpha_3^2 \;.$$

$\mathbb{R}^4$ /

$$\frac{x_1 x_2}{t^2 y_3} \quad A_1 A_2 \Big(\frac{m}{A_3}\Big)^{-1} \quad \frac{A_1 A_2 A_3}{m} \quad -.$$

A

ﬂ

$$\mathbb{R}^4 \times T^3.$$

SU

A_1 A_2

$$T^3 \quad \int_{T^3} \sqrt{g} \quad {}^{3/2} A_1 A_2 A_3 \quad m,$$

$\mathbb{R}^4 \times T^3$

m n

t

OV Z LO

α β E A_1 A_2
A_1 A_2
T^3 m
n α β E

n $f.$
n β

5. Abelian BPS Monopoles from Torus Fibrations

τ
τ τ z

z $\tau \to \infty,$

G_2 S^3
K $f.$

X
τ

$$\mathbb{R}_\tau \times X.$$

X

[14]

[14]Kaluza-Klein reduction of certain $G2$ holonomy metrics to non-abelian monopoles has been discussed recently in [13].

T le 1. R $_2$ 4

G_2 X	
SU	
U	

SU U

t

τ t r

$\mathbb{R}_\tau \times X$

$$ds^2 \quad -d\tau^2 \quad dr^2 \quad \frac{}{y_1} \; x_2 x_3 \;\; {}^2_1 \quad mn \;\; {}_1\alpha_1 - m x_1 \alpha_1^2$$

$$\frac{}{y_2} \; x_1 x_3 \;\; {}^2_2 \quad mn \;\; {}_2\alpha_2 - m x_2 \alpha_2^2 \quad \frac{}{y_3} \; x_1 x_2 \;\; {}^2_3 \quad mn \;\; {}_3\alpha_3 - m x_3 \alpha_3^2 \; ,$$

x_i y_i r

$$x_i \quad \equiv \quad x_i \;\; r \;\; ,$$
$$y_i \quad \equiv \quad y_i \;\; r \;\; .$$

G_2 X SU $\times T^3$

U U $\subset SU$

ϕ $f.$

U T^3

$$\alpha_4 \equiv d\phi.$$

α_i i , , , T^4 $T^3 \times U$

T^4

$$ds^2 \quad ds^2_{1,3} \quad h_{ij} \;\; \alpha_i \quad A_i \;\; \alpha_j \quad A_j \;\; ,$$

$ds^2_{1,3}$

A_i i U

h_{ij} i U $-$ j U

4D Theory:

$$h \quad \begin{pmatrix} -\frac{mx_1}{y_1} & & & \frac{mn}{2y_1} \quad \psi \quad \theta \\ & -\frac{mx_2}{y_2} & & \frac{mn}{2y_2} \quad \psi \quad \theta \\ & & -\frac{mx_3}{y_3} & \frac{mn}{2y_3} \quad \theta \\ \frac{mn}{2y_1} \quad \psi \quad \theta & \frac{mn}{2y_2} \quad \psi \quad \theta & \frac{mn}{2y_3} \quad \theta & h_{44} \end{pmatrix},$$

$$h_{44} \quad \frac{x_1x_2}{y_3} \quad {}^2\,\theta \quad \frac{x_3}{y_1y_2} \; x_1y_1 \quad {}^2\,\psi \quad x_2y_2 \quad {}^2\,\psi \quad {}^2\,\theta.$$

$$A_i \quad \sum_{k=1}^{4} h_{ik}^{-1}\widetilde{A}_k,$$

$$\widetilde{A}_1 \quad \frac{mn}{y_1} \quad \psi d\theta,$$
$$\widetilde{A}_2 \quad -\frac{mn}{y_2} \quad \psi d\theta,$$
$$\widetilde{A}_3 \quad \frac{mn}{y_3} d\psi,$$
$$\widetilde{A}_4 \quad \frac{x_1x_2}{y_3} \quad \theta d\psi \quad \left(\frac{x_2x_3}{y_1} - \frac{x_1x_3}{y_2}\right) \quad \psi \quad \psi \quad \theta d\theta.$$

$$ds^2_{1,3} \quad -d\tau^2 \quad dr^2 \quad \frac{x_1x_2}{y_3} d\psi^2 \quad \left(\frac{x_2x_3}{y_1} \quad {}^2\,\psi \quad \frac{x_1x_3}{y_2} \quad {}^2\,\psi\right) d\theta^2 - \sum_{i,j=1}^{4} A_i h_{ij}^{-1} A_j.$$

5.1. Spherically Symmetric Monopoles

SU

$$x_1 \quad x_2 \quad x_3, \quad y_1 \quad y_2 \quad y_3.$$

$$A_1 \quad -\left(\frac{n}{x}\right) \quad \psi d\theta - \quad \psi \quad \psi \quad \theta d\psi \;,$$

$$A_2 \quad \left(\frac{n}{x}\right) \quad \psi d\theta \quad \psi \quad \theta \quad \theta d\psi \ ,$$
$$A_3 \quad -\left(\frac{n}{x}\right) \quad {}^2\,\theta d\psi,$$
$$A_4 \quad \theta d\psi.$$

A_1 A_2 A_3 n A_4
SU

$$ds_E^2 \quad \left(-\frac{mx}{y^{1/2}}\right) -d\tau^2 \quad dr^2 \quad y^{3/2}d\Omega_2^2.$$

n , G_2 X fl
$\mathbb{R}^4 \times T^3$ x r y r

$$x \quad -\frac{m^{1/3}}{\quad}r^2, \quad y \quad \frac{m^{2/3}}{\quad}r^2.$$

$$h \quad m^{2/3}, m^{2/3}, m^{2/3}, \varphi \ , \quad \varphi \quad -r^2,$$

$$ds_E^2 \quad -mr \ -d\tau^2 \quad dr^2 \quad \frac{m}{\quad}r^3 d\Omega_2^2.$$

A_1 A_2
A_3
U A_4

$$A_4 \quad \theta d\psi.$$

A_4 φ

$$S \quad -\frac{}{\pi\kappa_5}\int d^5x\sqrt{-g_5}R_5.$$

φ $\mathbb{R}^3$
ϕ f.

$x_i x_j / y_k$ r

OV Z LO

$\mathbb{R}^4$

${}^4 \times T^3$

SU

fl $\mathbb{R}^4 \times T^3$ T^3

5.2. Axially Symmetric Monopoles

U

SU

x_1 x_2 y_1 y_2

U

$$A_4 \quad \frac{z \quad \theta}{z \quad \beta \quad {}^2\,\theta} d\psi,$$

z $x_1 y_1$ A_4

ψ

$$ds_E^2 \quad \sqrt{\quad h}\Big[-d\tau^2 \quad dr^2 - \frac{y_1^2 y_3}{mz}\Big(d\theta^2 \quad \frac{z \quad {}^2\,\theta}{z \quad \beta \quad {}^2\,\theta} d\psi^2\Big)\Big]$$
$$\frac{m\sqrt{z^2 \quad \beta z \quad {}^2\,\theta}}{y_1\sqrt{y_3}}\Big(-d\tau^2 \quad dr^2 - \frac{y_1^2 y_3}{mz} d\theta^2\Big) - \frac{y_1\sqrt{y_3}z \quad {}^2\,\theta}{\sqrt{z^2 \quad \beta z \quad {}^2\,\theta}} d\psi^2.$$

$|z| \gg |\beta|$

θ

$SO \quad \sim SU$ U

β $\beta \to$

OV Z LO

$z \to$

$$ds_E^2 \approx \frac{m}{\sqrt{\ }} \quad \theta\Big(- d\tau^2 \quad dr^2 \quad \frac{|\beta|}{m} d\theta^2 \quad \sqrt{\frac{|\beta|}{m}} r^2 d\psi^2 \Big).$$

θ θ π

Acknowledgments

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