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Affine Manifolds, Log Structures, and Mirror Symmetry

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Mark Gross, Bernd Siebert

Abstract

We outline work in progress suggesting an algebro-geometric version of the Strominger-Yau-Zaslow conjecture. We define the notion of a toric degeneration, a special case of a maximally unipotent degeneration of Calabi-Yau manifolds. We then show how in this case the dual intersection complex has a natural structure of an affine manifold with singularities. If the degeneration is polarized, we also obtain an intersection complex, also an affine manifold with singularities, related by a discrete Legendre transform to the dual intersection complex. Finally, we introduce log structures as a way of reversing this construction: given an affine manifold with singularities with a suitable polyhedral decomposition, we can produce a degenerate Calabi-Yau variety along with a log structure. Hopefully, in interesting cases, this object will have a well-behaved deformation theory, allowing us to use the discrete Legendre transform to construct mirror pairs of Calabi-Yau manifolds. We also connect this approach to the topological form of the Strominger-Yau-Zaslow conjecture.

Introduction.

Mirror symmetry between Calabi-Yau manifolds is inherently about degenerations: a family $f : \mathcal{X} \to S$ of Calabi-Yau varieties where S is a disk, $\mathcal{X}_t$ is a non-singular Calabi-Yau manifold for $t \neq 0$, and $\mathcal{X}_0$ a singular variety. Much information about the singular fibre is carried in the geometry and topology of the family over the punctured disk: $f^* : \mathcal{X}^* = \mathcal{X} \setminus f^{-1}(0) \to S^* = S \setminus \{0\}$. For example, in some sense, the degree to which $\mathcal{X}_0$ is singular can be measured in terms of the monodromy operator $T : H^*(\mathcal{X}_t, \mathbb{Z}) \to H^*(\mathcal{X}_t, \mathbb{Z})$, where $t \in S^*$ is a basepoint of a simple loop around the origin of S.

An appropriate form of the mirror symmetry conjecture suggests that associated to any sufficiently "bad" degeneration of Calabi-Yau manifolds, i.e. a maximally unipotent degeneration or large complex structure limit point, there should be a mirror manifold $\check{X}$, defined as a symplectic manifold. Furthermore, if the family $f^* : \mathcal{X}^* \to S^*$ is polarized, i.e. given a choice of a relatively ample line bundle $\mathcal{L}$ on $\mathcal{X}^*$, one should expect to be able to construct a degenerating family of complex manifolds $\check{\mathcal{X}}^* \to S^*$ along with a polarization $\check{\mathcal{L}}$. This correspondence between degenerating polarized families is not precise, though it can be made more precise if one allows multi-parameter families of Calabi-Yau manifolds and subcones of the relatively ample cone of $f^* : \mathcal{X}^* \to S^*$: this is essentially the form in which a general mirror symmetry conjecture was stated in [24] and we do not wish to elaborate on that point of view here.

This article was presented at the 9^{th} Gökova Geometry-Topology Conference

n

n

0 g l v

0 0

$\prime$ $\prime$

-1 $\sim$ $\prime$ -1 $\sim$ $\prime$

0

0

$\subseteq \mathbb{P}^3 \times$ 4 $z_0 z_1 z_2 z_3$

z_0 z_3 $\mathbb{P}^3$ 4

0 $\mathbb{P}^3$

4 $\cap$ ing 0 0

0

0

0

$\subseteq$ n fl

$\mathbb{P}_\Xi$ $\mathcal{O}_{\mathbb{P}_\Xi}$ s Γ $\mathbb{P}_\Xi$ $\mathcal{O}_{\mathbb{P}_\Xi}$

s_0 s_0

$\mathbb{P}_\Xi$

n $\mathbb{P}_\Xi$ $\subseteq \mathbb{P}_\Xi \times$

s s_0 0 $\mathbb{P}_\Xi$

0

0 l c g g l

0

c *g* §

Z ⊆

0

n

B 0 *n* 0

B *g l ffi m f l w h g l*

B

B *Z* ⊆

B *n* ⊆ *n*

B

§

0

0 0

l g hm c c 0

0 *l g ch m*

† 0

0 ⊆

† 0

$^{\dagger}_{0}$ B $\mathcal{P}$

$^{\dagger}_{0}$

$_{0}$

$_{0}$

B $\mathcal{P}$

B $\mathcal{P}$ B $\mathcal{P}$ $^{\dagger}_{0}$

φ B

c L g f m B $\mathcal{P}$ φ B $\mathcal{P}$ φ B $\mathcal{P}$

φ B

^{n}

B $\mathcal{P}$ φ $_{0}$

$^{\dagger}_{0}$ $^{\dagger}_{0}$

B

B

B

Ac wl g m :
..

1. Toric degenerations

Definition 1.1. fl n

c g

$\omega_{\mathcal{X}}$

0 ν 0 0

n 0 $U \subseteq$ 0

0 ν U ν U ν^{-1} ν U U

ν 0 $\subseteq$ 0

ν ν ν

$Z \subseteq$ $\geq$ Z

Z ν

0 Z U_x

n Y_x x

Y_x

$$\begin{array}{ccc} U_x & \xrightarrow{\psi_x} & Y_x \\ \downarrow{\scriptstyle f|_{U_x}} & & \downarrow{\scriptstyle f_x} \\ & \xrightarrow{\varphi_x} & \end{array}$$

ψ_x φ_x x

Y_x

0

Z

$\mathbb{P}^3$ $z_0 z_1 z_2 z_3$

s_0 s s_0 s

$\mathbb{P}_\Xi$ Z

$$Z \quad \cap \quad ing \quad {}_0 \cap \; s \quad \times \quad \subseteq \mathbb{P}_\Xi \times$$

s

$$s_0 \quad \sum \quad {}^{h(s)+1} s$$
$$s$$

h

$\Gamma \ \mathbb{P}_\Xi \ \mathcal{O}_{\mathbb{P}_\Xi}$

$\mathbb{P}_\Xi \times$

2. From toric degenerations to affine manifolds: the dual intersection complex

M ^{n} N $_\mathbb{Z}$ M $M_\mathbb{R}$ $M \otimes_\mathbb{Z}$ $N_\mathbb{R}$ $N \otimes_\mathbb{Z}$

$$M_\mathbb{R} \quad M_\mathbb{R} \rtimes GL_n$$

$$M_\mathbb{R}$$

$$_\mathbb{R} \ M \quad M_\mathbb{R} \rtimes GL_n$$
$$M \quad M \rtimes GL_n$$

Definition 2.1. B n ffi c B
U ψ U $M_\mathbb{R}$
$\psi \circ \psi_j^{-1}$ $M_\mathbb{R}$ g l
M B B' n n'
B B'
n n' n n'

Definition 2.2. C^0 B
$\subseteq B$
B_0 B
g l B_0
i B_0 $\hookrightarrow$ B B B'
$^{-1}$ $B_0' \cap B_0$ B
$f^{-1}(B_0') \cap B_0$ $^{-1}$ $B_0' \cap B_0$ B_0'

Definition 2.3. p l h l c mp $R \subseteq M_\mathbb{R}$
$\mathcal{P}$ R c ll

σ $\mathcal{P}$ $\tau \subseteq \sigma$ σ τ $\mathcal{P}$
σ σ' $\mathcal{P}$ $\sigma \cap \sigma'$ σ σ'
g l $\mathcal{P}$
M

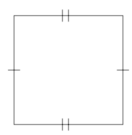
$\mathcal{P}$ σ $\mathcal{P}$

$$Int\,\sigma \quad \sigma \quad \bigcup_{\tau \in \mathcal{P}, \tau \subsetneq \sigma} \tau$$

$M_{\mathbb{R}}$ B
B

B
n

B ^{2} ^{2}

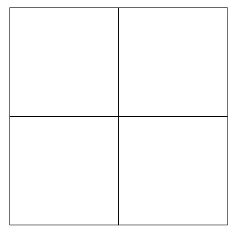


Definition 2.4. B p l h l
c mp B $\mathcal{P}$ B B c ll
v $\mathcal{P}$ v B v
$\mathcal{P}_v$ $R_v \subseteq M_{\mathbb{R}}$
$_v$ R_v B
$_v$ v
$_v$

n σ $\mathcal{P}_v$ $_v$ Int σ $\cap$ ϕ $_v$
Int σ
σ $\mathcal{P}$ v $\sigma \Leftrightarrow \sigma$ $_v$ σ σ $\mathcal{P}_v$ σ
σ $\mathcal{P}$ v σ v $\mathcal{P}$
c

σ $\mathcal{P}$ $U_\sigma \subseteq B$ Int σ
s_σ U_σ $M'_{\mathbb{R}}$ M' $B -$ σ
s_σ $\sigma \cap U_\sigma$

Example 2.5. B $M_{\mathbb{R}}$ ϕ B
$M_{\mathbb{R}}$ B $M_{\mathbb{R}}$
$\subseteq M$ B
$M_{\mathbb{R}}$
B ^{2} ^{2} v
R_v ^{2}

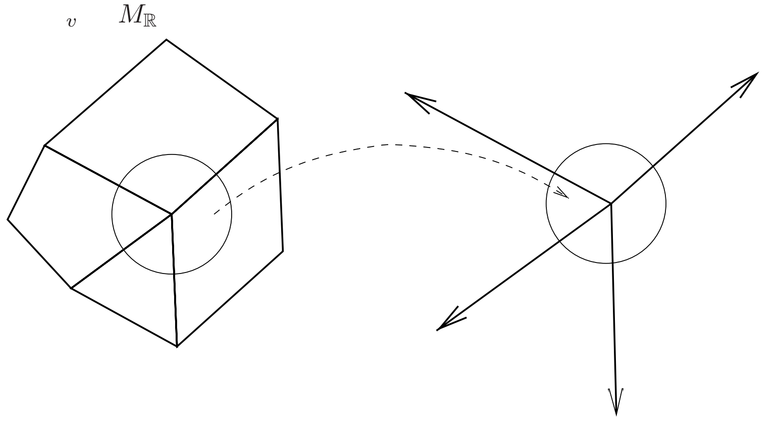


$_v$ R_v ^{2} ^{2} ^{2}
R_v $M_{\mathbb{R}}$ $_v$
R_v
B

$\mathcal{P}$

R m . $\mathcal{P}$ B v $\mathcal{P}$
R_v $M_{\mathbb{R}}$

$_v$ $M_{\mathbb{R}}$



$\mathcal{P}$
v B

Definition 2.7. $\sigma \subset M_{\mathbb{R}}$ B r σ σ
σ
σ σ

σ

$\mathcal{P}$ B

B r $\mathcal{P}$ $\mathcal{P}$

σ $\mathcal{P}_v$ v

B

$\mathcal{P}'$ n

$M_{\mathbb{R}}$

$\mathcal{P}'$

B $\mathcal{P}$

$\mathcal{P}'$

$$\pi \quad \coprod_{\sigma'\in\mathcal{P}'} \sigma' \quad B$$

B

$\mathcal{P}'$

B

B r $\mathcal{P}$ $\mathcal{P}$ $' \subseteq B$

B r $\mathcal{P}$ $\mathcal{P}$

n $\mathcal{P}$

$\mathcal{P}$

v $\mathcal{P}$ W_v B r $\mathcal{P}$

v W_v v

W_v v $\mathcal{P}$ $\cup$ In σ σ $\mathcal{P}$

B $'$ B $'$

W_v

v $\mathcal{P}$

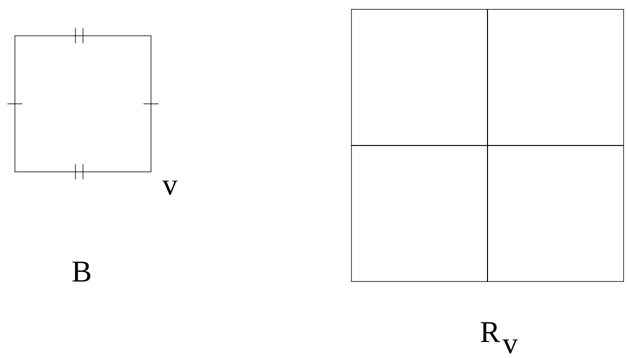
$\mathcal{P}'_v$ v' σ' v' σ' $\mathcal{P}'$ π v' v

R_v $\coprod_{(v',\sigma')\in\mathcal{P}'_v} \sigma'$

$\omega' \subsetneq \sigma'$ i v' σ' $\mathcal{P}'_v$ π ω'_1 π ω'_2 v' ω'

$\mathcal{P}'$ 2 B

v $\mathcal{P}$



π_v R_v B

b $\sigma' \subseteq R_v$ π b U_v
π_v^{-1} W_v U_v W_v
R_v $\mathcal{P}_v$ v' U_v
v i_v R_v $M_{\mathbb{R}}$
ψ_v W_v $M_{\mathbb{R}}$ $i_v \circ \pi_v^{-1}$ W_v
i_v R_v B $\pi_v \circ i_v^{-1}$ B $'$ v
$\mathcal{P}$ B
f c v $\mathcal{P}$
v v $M_{\mathbb{R}}$
$\mathcal{P}_v$ v' v
σ $\sigma_{v'}$
i_σ σ v'
$\sigma_{v'}$

i_σ

i_v R_v $M_{\mathbb{R}}$

ψ_v
B_0 $\mathcal{P}$ v

B $'$ $\mathcal{P}$ $'$
B
$'$ B $'$ B

m m l c m l c

B

B §

0 0

$\coprod$ ν 0 0 0

r 0 ν i

n

r 0 M' $n+1$ $M'_{\mathbb{R}}$ $M' \otimes_{\mathbb{Z}}$

N' $\mathbb{Z}$ M' $N'_{\mathbb{R}}$ $N' \otimes_{\mathbb{Z}}$

Y_x $\sigma_x \subseteq$

$M'_{\mathbb{R}}$ x Y_x

x x N'

σ_x m σ_x $\langle$ x $m\rangle$

Y_x σ_x Y_x

x

τ σ_x τ

τ x τ $\langle$ x $\tau\rangle$

σ_x

σ_x M m M' $\langle$ x $m\rangle$

$M_{\mathbb{R}}$ m $M'_{\mathbb{R}}$ $\langle$ x $m\rangle$ σ_x $M_{\mathbb{R}}$

0 σ_x n

Example 2.8. 0 $n+1$ z_0 z_n $\prod^n_{=0} z$

σ_x

$n+1$

$\mathbb{Z}$ $n+1$ σ

$$0 \quad n \quad \sum_{=0}^{n}$$

$n+1$

B

σ_x r 0

0

ν ν

σ_x r 0 σ_x $\sigma_{x'}$

0

B

Lemma 2.9. *If* 0 *n c mpl x m l h* B *n l m l m f l .*

B

B v_1 v_m

B 1 m

0 $M_{\mathbb{R}}$

v

σ B

v σ

σ v

v

$\mathcal{P}$

Examples 2.10. Z B

B v_1 v_m

1 m 0 p

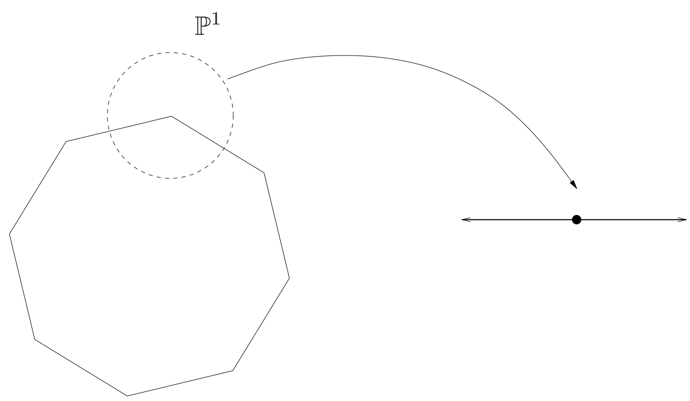
v_0 v_p $_0\cap\cdots\cap$ $_p$ ϕ

0

I_m m

0 $m \geq$

B m



B m

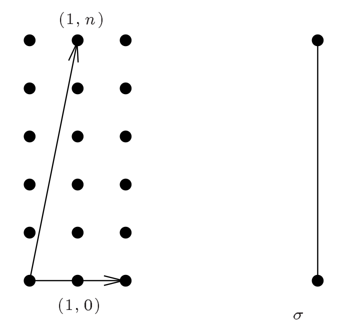
0

y z^n 3 n $n-$

y z z

σ n 2 $M'_{\mathbb{R}}$

N' σ n



$\mathbb{P}^1$ B

4 0 1 2 3 $\times \mathbb{P}^3$

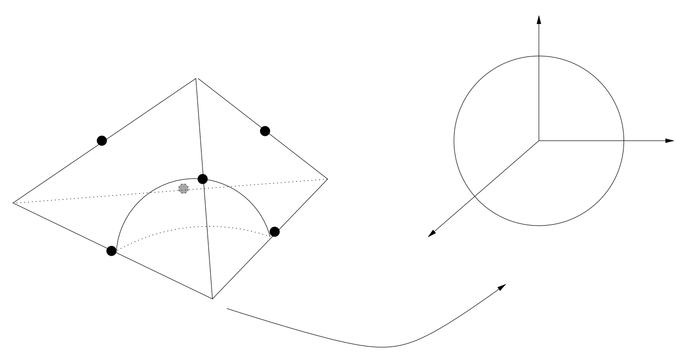
0 B

v

$\mathbb{P}^2$ B

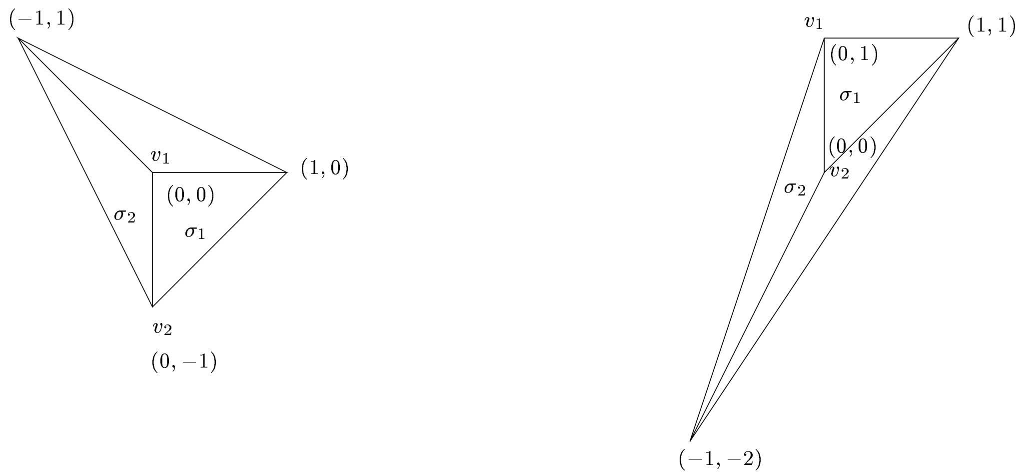
v

v



v_1

v_2 R_{v_1} R_{v_2}



v_1 v_2

σ_1 v_i

$\left(\quad \right)$

σ_2 R_{v_1} σ_2 R_{v_2}

v_2 σ_1 v_1 σ_2 v_2

$\left(\quad \right)$

v_1 v_2

3. From affine manifolds to toric degenerations

B $\mathcal{P}$

0 B $\mathcal{P}$

σ $\mathcal{P}$

0

$\mathcal{P}$ v_1 v_m $\mathcal{P}$

$\mathcal{T}_{B,v_i}$

∂ ∂y_1 ∂ ∂y_n y_1 y_n

M $\mathcal{T}_{B,v_i}$ $M_{\mathbb{R}}$

v $M_{\mathbb{R}}$ $\mathcal{P}$ v $M_{\mathbb{R}}$

1 m 0

$\mathcal{P}$

v v_j $\mathcal{P}$

j D D_j j

D D_j

j

0

D D_j

D

i j D

B

0

0 $\mathcal{P}$

$\mathcal{P}$

0

0

Y_x

l g c

0 fl

0 $\subseteq$

m

Definition 3.1.

α_X $\mathcal{M}_X$ $\mathcal{O}_X$

α_X^{-1} $\mathcal{O}_X^\times$ $\mathcal{O}_X^\times$

$\mathcal{M}_X$ α_X l g p c †

F $\dagger$ $Y^\dagger$ $\underline{F}$ Y
$F^\#$ $\underline{F}^{-1}$ $\mathcal{M}_Y$ $\mathcal{M}_X$

$$\alpha_X \circ F^\# \quad \underline{F} \quad \circ \alpha_Y$$

Examples 3.2. $D \subseteq$
j D

$$\alpha_X \quad \mathcal{M}_X \quad j \quad \mathcal{O}^\times_{X\setminus D} \cap \mathcal{O}_X \quad \mathcal{O}_X$$

D
p l g c φ $\mathcal{P}$
$\mathcal{O}_X$ $\mathcal{M}_X$

$$\mathcal{M}_X \quad \mathcal{P} \oplus \mathcal{O}_X^\times \quad p \ \varphi \ p \ ^{-1} \ p \quad \varphi^{-1} \ \mathcal{O}_X^\times$$

α_X p h $h \cdot \varphi$ p
Y α_Y $\mathcal{M}_Y$ $\mathcal{O}_Y$
Y $^{-1}$ $\mathcal{M}_Y$ $\mathcal{O}_X$
p ll- c l g c
D
$_0 \subseteq$

$$^0 \quad \mathcal{M}_0 \quad ^\times \oplus \mathbb{N} \quad \mathbb{N}$$

$$\alpha_0 \ h \ n \quad \begin{cases} h & n \\ & n \end{cases}$$

$\dagger$ $\dagger$ $\dagger_0$ $\dagger$
$\sigma \subseteq M_\mathbb{R}$ $\sigma^\vee \subseteq N_\mathbb{R}$ P $\sigma^\vee \cap N$
σ P P
P z^p p P
$z^p \cdot z^{p'}$ $z^{p+p'}$

P P

p z^p p P
z^p $\mathbb{N}$
$_0$ P z^p
$_0$ $\dagger_0$ $\dagger$
Z $\dagger$ $\dagger_0$

R m .

$\mathcal{O}_X^\times$ $\mathcal{M}_X$ $\overline{\mathcal{M}}_X$

$\overline{\mathcal{M}}_X$ D

$_1$ $_2$ $\subseteq$ k $_1$ $_2$ D $D_1 \cup D_2$ D i_j D_j

$\overline{\mathcal{M}}_X$ i_1 $\mathbb{N} \oplus i_2$ $\mathbb{N}$ $\mathcal{M}_X$ n_1 n_2

n_j D_j D

$\overline{\mathcal{M}}_D$ $\overline{\mathcal{M}}_X$

$^\dagger_0$ †

l g m h †

B $\mathcal{P}$ $_0$

$_0$ $Z \subseteq$ ing $_0$

† † ^{0} $^\dagger_0$ Z $^\dagger_0$

B $\mathcal{P}$ $_0$

σ $\mathcal{P}$ σ $M_\mathbb{R}$ σ

σ $M_\mathbb{R} \oplus$

σ rm r r $_{\geq 0}$ m σ

σ Y_σ $_\mathbb{Z}$ $M \oplus$ $N \oplus$

$N \oplus$ z Y_σ $_\sigma$

z Y_σ § B

$_\sigma$ σ $\mathcal{P}$

$_0$

$_\sigma$

$_\sigma \subseteq Y_\sigma$ $_0$

† $^\dagger_\sigma$ † σ $\mathcal{P}$

Z ϕ

$_\sigma$ †

$\overline{\mathcal{M}}_X$ $\mathcal{O}_X^\times$

$\overline{\mathcal{M}}_{X_\sigma}$

$\overline{\mathcal{M}}_{\mathcal{X}_0}$ $_0$

L $_0$ $\mathcal{E}x$ ^{1} $\overline{\mathcal{M}}^{gp}_{\mathcal{X}_0}$ $\mathcal{O}^\times_{\mathcal{X}_0}$

gp

Examples 3.4.

$\subseteq$ ^{4} y z w

y zw

$\times$ $\subseteq Y$ Y $\dagger$ $y -$ zw $\dagger$

$\dagger$, ,

$^\dagger_0$ $\mathcal{P}$

D ing $_0$ $\mathcal{N}_D$ D

$\mathcal{E}x$ ^{1} $\Omega^1_{\mathcal{X}_0}$ $\mathcal{O}_{\mathcal{X}_0}$ L $_0$

$\mathcal{O}_D^\times$ $\mathcal{N}_D$ $\mathcal{N}_D$

$_0$ $\mathcal{N}_D \sim \mathcal{O}_D$

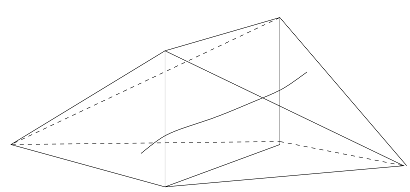
$\subseteq B$

$\mathcal{N}_D$

B

Z

$\mathcal{P}$



$U \subseteq$ $_0$

$_0$ U Z

L $_0$

Z $_0$

$\mathcal{N}_D$ fl

0

0

$\dagger_0$

$\leq$ $\dagger_0$

$\geq$

B

4. The intersection complex, the discrete Legendre transform, and mirror symmetry

0 n

B $\mathcal{P}$

B $\mathcal{P}$ § B $\mathcal{P}$

B

0 0 σ

X_i $N_{\mathbb{R}}$

σ

σ

σ_j j 0

B B

B B r 0

$\sigma_x \subseteq M_{\mathbb{R}}$ x σ_x

τ

σ_x τ

$N_{\mathbb{R}}$ τ $\langle\ z\rangle \geq \langle\ y\rangle$ z σ_x y τ

x B

Proposition 4.1. *B g l ffi m f l w h g l $\mathcal{P}$ c p l h l c mp B. If $\subseteq B$ h m m l c m l c f h ffi c h h h m m ph m α B B w h α . h m h ffi c B B l h h h h l m p f h fl c c $\mathcal{T}_B$ $\mathcal{T}_{\check{B}}$ c h p c v ffi c ll l.*

B B $\mathcal{P}$

Definition 4.2. B B B B_0 $\mathcal{P}$ B $U \subseteq B$ p c w l U U $U \cap In$ σ σ $\mathcal{P}$ y U y In σ σ $\mathcal{P}$ V y g Γ V B $-g$ $V \cap In$ σ

Definition 4.3. m l -v l p c w l f c B $\mathcal{P}$ U φ U B φ $-\varphi_j$ Γ $U \cap U_j$ B i j φ c l c v x v $\mathcal{P}$ φ v

φ B B B B $\mathcal{P}$ φ B B $\mathcal{P}$ φ c L g f m B $\mathcal{P}$ φ $\mathcal{P}$ σ $\mathcal{P}$ σ B r $\mathcal{P}$ σ σ $\mathcal{P}$ σ σ $\mathcal{P}$ σ σ $n-$ σ σ B B § v $\mathcal{P}$ v $\mathcal{T}_{B,v}$ φ φ_v v

v' $\mathcal{T}_{B,v}$ $\langle$ $y\rangle \geq -\varphi_v$ y $\forall y$ $\mathcal{T}_{B,v}$

φ_v v' v τ v $\tau \subseteq v'$

τ v' $\langle$ $y\rangle$ $-\varphi_v$ y $\forall y$ τ

v' φ_v v' v $\mathcal{P}$ v'

v $N_{\mathbb{R}}$
B
B B
σ $\mathcal{P}$ σ $\mathcal{P}$
σ $_{\sigma}$ σ
φ φ
φ σ $\mathcal{P}$ σ $\mathcal{P}$
$\varphi_{\check{\sigma}}$ $_{\sigma}$ σ σ
$M_{\mathbb{R}}$
σ $\varphi_{\check{\sigma}}$
$_{\sigma}$ $\varphi_{\check{\sigma}}$

$$\varphi_{\check{\sigma}}\ y \quad - \quad \langle y \quad \rangle \quad \sigma$$

y $N_{\mathbb{R}}$ $_{\sigma}$
$\varphi_{\check{\sigma}}$ σ σ
$M_{\mathbb{R}}$ $\varphi_{\check{\sigma}}$

$$B\ \mathcal{P}\ \varphi \qquad B\ \mathcal{P}\ \varphi$$

B
v $\mathcal{P}$ $_0$
$_{X_i}$
φ v B
B
$_{X_i}$ i $B\ \mathcal{P}\ \varphi$
g
$B\ \mathcal{P}$
$B\ \mathcal{P}\ \varphi$ $B\ \mathcal{P}\ \varphi$
m mm c
l w c g w h l g

$B\ \mathcal{P}\ \varphi$ $B\ \mathcal{P}\ \varphi$

$$\overset{\dagger}{_0} \qquad \overset{\dagger}{_0}$$
$$B\ \mathcal{P}\ \varphi \qquad B\ \mathcal{P}\ \varphi \qquad \overset{\dagger}{_0} \qquad \overset{\dagger}{_0}$$

..

$h^{1,1}$ $h^{1,n-1}$ $h^{1,n-1}$ $h^{1,1}$

$^{\dagger}_{0}$

B

Example 4.4.

B m m_1 m_p $\sum m$ m

$_0$ p n

n_1 n_p p $_0$ $n \geq$ $\sum n$ n B

n_1 n_p B n

m_1 m_p

$_0$

$\mathcal{P}$ φ $^{-1}$

$_1$ $\big(\quad m \big)$ n

m n

R m .

5. Connections with the Strominger-Yau-Zaslow conjecture

fl

B $_{\mathbb{R}}$ M

M $M_{\mathbb{R}}$ $\mathcal{T}_B$

∂ ∂y_1 ∂ ∂y_n y_1 y_n

GL_n $\subseteq \mathcal{T}_B$

B $\mathcal{T}_B$

z B B

y_1 y_n

$$z \left(\sum_j \quad {}_j \frac{\partial}{\partial y_j} \right) \quad 2 \quad \overline{-1}(x_j + \quad \overline{-1} y_j)$$

φ B $\partial^2 \varphi$ ∂y ∂y_j

y_1 y_n π B B

$\varphi \circ \pi$ ¨ ¨ B fl φ

$\partial^2 \varphi$ ∂y ∂y_j $cons$ n

B B

y $\partial \varphi$ ∂y

$$\varphi \; y_1 \quad y_n \quad \sum_{=1}^{n} y \, y - \varphi \; y_1 \quad y_n$$

φ

B φ ¨ B B

B B B

g l

π B B B
B d B $M_{\mathbb{R}}$
v l p g m p

B h l m p
π_1 B B γ π_1 B
γ B B $\gamma_1 \circ$ γ_2 $\gamma_2\gamma_1$
γ $M_{\mathbb{R}}$ $\gamma \circ d \circ$ γ d
π_1 B $M_{\mathbb{R}}$ h l m p
$\subseteq$ M
B
d B $M_{\mathbb{R}}$ π_1 B M d' B $M_{\mathbb{R}}$
$'$
B $M_{\mathbb{R}}$ $'$ γ $\circ d' \circ$ γ d' $'$
$M_{\mathbb{R}}$ GL $M_{\mathbb{R}}$
$r > R$ rd d' B $M_{\mathbb{R}}$

$$B \quad B \times \quad R$$

$$d \quad B \times \quad R \qquad M_{\mathbb{R}} \times$$

$$d \; b \; r \qquad rd \; b \qquad d' \; b \quad r$$

B

$$M_{\mathbb{R}} \oplus \qquad \rtimes GL \; M \oplus$$

d
B
B
B n B R
B R
-2 R n

U B α α j
ﬂ $\mathcal{T}_B$ U U_j

α j B

B α

B

B α

U B ψ U $M_{\mathbb{R}}$

U

$M_{\mathbb{R}}$ $\sim M_{\mathbb{R}} \otimes$ $\times$ B α

U U_j i j $U \cap U_j$

φ j $U \cap U_j$ $U \cap U_j$ α α

B α

y $M_{\mathbb{R}}$ $M_{\mathbb{R}}$ $iM_{\mathbb{R}}$ M

d y $\| \; - y\|$ y y $M_{\mathbb{R}}$ $iM_{\mathbb{R}}$

$\| \cdot \|$ $M_{\mathbb{R}} \otimes$

f m f B α f z C

φ U

- φ^{-1} $\cap$ j y $U \cap U_j$ d y C y $U \cap U_j$ φ^{-1} $\cap$ j d y C C
- φ^{-1} $\cap$ j j U_j φ_j j φ y $U \cap U_j$ d y C d j φ j y C

Theorem 5.1. *L c g B h c p g g l ffi m f l w h g l . Th h x*

- *p* $U \subseteq B$ *ch h* B U *c h g l .*
- *A m p d′* U $M_{\mathbb{R}}$ *v fi g ffi c* U $U \times R$ *.*
- *A Č ch 1-c c cl* α *f fl c f* $\mathcal{T}_{\bar{U}}$ *h c g v g m p*

g U α R $^{-2}$ R

- *A fic f* R *w h p c p gh h f .*
- *A p* $\mathcal{U} \subseteq$ *.*
- *C* C_1 C_2

ch h f ffic l m ll $^{-1}$ $\cap \mathcal{U}$ *f m f* g^{-1} *f z* C_1 ^{C_2}. *H* g^{-1} *lf f h f m* B α' *f m ffi c* B *Č ch 1-c c cl* α' *m l m ll f m f* g^{-1} *.*

B

$\mathcal{P}$ B $_0$

U

U

B $\mathcal{P}$ φ

B $\mathcal{P}$ φ

B_0

B_0 B_0

r $-\frac{\log|\ |}{2}$ R g^{-1} $\sim$ B α' B

rd d' α'

ϵ r d $\epsilon d'$

d B α'

$\mathcal{T}_B$ ϵ $\subseteq \mathcal{T}_B$

d $\epsilon d'$ ϵ

ϵ O $^{-C/\epsilon}$

φ $U \subseteq B_0$

fl $^{-1}$ $\cap\,\mathcal{U}$

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