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Galois symmetry on Floer cohomology

KENJI FUKAYA

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Kenji Fukaya

Abstract

In this article, we define an action of $\hat{\mathbb{Z}}$ on the A_∞ category whose objects are rational Lagrangian submanifolds together with some other data. The group $\hat{\mathbb{Z}}$ is a Galois group of the (universal) Novikov ring (field) over Laurent polynomial field. By mirror symmetry it will corresponds to the algebraic fundamental group of a punctured disk which parametrize the maximal degenerate family of the mirror complex manifold.

1. Introduction

The Floer cohomology of a Lagrangian submanifold defined in [Fl, Oh, FOOO] is in general a module over a kind of formal power series ring, which is called the Novikov ring [No]. In [FOOO, Fu2] we used the universal Novikov ring $\Lambda_{\mathbb{C}}$, which consists of a formal sum

$$\sum_i a_i T^{\lambda_i} \tag{1}$$

where $a_i \in \mathbb{C}$ and $\lambda_i \in \mathbb{R}$ such that $\lambda_i < \lambda_{i+1}$, $\lim_{i\to\infty} \lambda_i = +\infty$ and T is a formal parameter.

For a rational Lagrangian submanifold (we define it later in Definition 2.2), we take a smaller ring $\Lambda_{\mathbb{C}}^{\mathbb{Q}}$ which consists of elements (1) satisfying in addition $\lambda_i \in \mathbb{Q}$.

Both of them are complete non-Archimedean valued fields whose norm is defined by

$$\left\| \sum_i a_i T^{\lambda_i} \right\| = \exp\left(-\min\{\lambda_i | a_i \neq 0\}\right). \tag{2}$$

The continuous Galois group of $\Lambda_{\mathbb{C}}^{\mathbb{Q}}$ over $\mathbb{C}[[T]][T^{-1}]$, the Laurent power series ring, is $\hat{\mathbb{Z}}$, the profinite completion of $\mathbb{Z}$. In fact, the (topological) generator $\rho \in \hat{\mathbb{Z}}$ acts by

$$\sum_i a_i T^{\lambda_i} \mapsto \sum_i a_i e^{2\pi\sqrt{-1}\lambda_i} T^{\lambda_i} \tag{3}$$

We denote the map (3) by $x \mapsto x^\rho$. The rationality of λ_i implies that the $\mathbb{Z}$ action defined by (3) can be extended to a $\hat{\mathbb{Z}}$ action.

In the same way, $\mathbb{Z}$ acts as automorphisms on $\Lambda_{\mathbb{C}}/\mathbb{C}[[T]][T^{-1}]$ by the formula (3).

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A
A fl

fl
§

2. Statement of main result

M, ω n c_1 M
c_1 M M
M M
M
ω H^2 M H^2 M

Definition 2.1. ξ, ∇^ξ M
pr q t m b dl F_{∇^ξ} $\pi\sqrt{-}\ \omega$ F_{∇^ξ}
∇^ξ

ω H^2 M ξ, ∇^ξ
L M L
n M ω L
ξ, ∇^ξ L fl hol_ξ $\pi_1 L$ U

Definition 2.2. L *r t o l* hol_ξ
$\pi_1 L$ U

L hol_ξ $\pi_1 L$ U
A $L, \mathcal{L}, s, b$
L fl U $\mathcal{L}$
s, b L
fl
L H^2 M 2
L M
L
L g M M M
M ${}_x M$
L g M M
c_1 M $\widetilde{L\ g}$ M
L g M L g M M
L s ${}_x L$ L g M L *gr d d*
s L $\widetilde{L\ g}$ M s s *gr d g* L

F Y

A $L, \mathcal{L}, s, b$

L

$\mathcal{L}$ fl L

L s

b

b

L b

b b

b $n-$ b

H^k L H^k L

Q H^1 L H^2 L

Q Q $\otimes$

$=1$

Q H^1 L H^2 L

Q Q L $\mathcal{L}$ s L s $Q_{L,\mathcal{L}}$ $Q_{,L,\mathcal{L}}$

Q Q L $\mathcal{L}$

Q b

Q

Q b $b \equiv$ $^+$ b $\sum b$

$>$

$\mathcal{M}$ $L, \mathcal{L}$ b H^1 L $b \equiv$ $^+$, Q b

$\mathcal{M}$ $L, \mathcal{L}$ $\mathcal{M}$ $L, \mathcal{L}$ $\cap H^1$ L

$\mathcal{M}$ $L, \mathcal{L}$ *bo* *d* *g* *coch*

b

$L, \mathcal{L}, s, b$ A

Remark 2.1.

Remark 2.2. b
Q b

CF $L_1, \mathcal{L}_1, s_1, b_1$, $L_2, \mathcal{L}_2, s_2, b_2$
b

A

HF $L_1, \mathcal{L}_1, s_1, b_1$, $L_2, \mathcal{L}_2, s_2, b_2$
b $\mathcal{M}$ $L, \mathcal{L}$

Proposition 2.1. *If* L *s r t o l d f* b $\mathcal{M}$ $L, \mathcal{L}$ *th lo r cohomolog s d- fi d ov r . N m l w h v coch compl x* CF $L_1, \mathcal{L}_1, s_1, b_1$, $L_2, \mathcal{L}_2, s_2, b_2$ *ov r wh ch d fi s* HF $L_1, \mathcal{L}_1, s_1, b_1$, $L_2, \mathcal{L}_2, s_2, b_2$.

§
$\mathfrak{m}_k$
A
$\mathcal{LA}$

$\mathcal{LA}$

Proposition 2.2. *Th r x sts filt r d A c t gor* $\mathcal{LAG}$ *whos ob cts r* $L, \mathcal{L}, s, b$ *wh r* L $\mathcal{LA}$, $\mathcal{L}$ *s fl t b dl o t, s s gr d g of* L *d* b $\mathcal{M}$ $L, \mathcal{L}$. *Th s t of morph sms b tw two ob cts s* CF $L_1, \mathcal{L}_1, s_1, b_1$, $L_2, \mathcal{L}_2, s_2, b_2$ *Propos t o 2.1.*

A $\mathfrak{m}_k$
§ $\mathfrak{m}_k$

$\mathcal{LAG}$

F Y

$\mathcal{L}$ $\mathcal{L} \otimes \xi$ L

L m

$\mathcal{L}$ ^{m} $\mathcal{L}$

L s

b

$\mathcal{M}$ $L, \mathcal{L}$ -1

$\mathcal{M}$ $L, \mathcal{L}$ -1

Proposition 2.3. *Th follow g d gr m comm t s.*

$$\begin{array}{ccc} H^1 \; L & \xrightarrow{Q_{L,\mathcal{L}}} & H^2 \; L \\ {\scriptstyle 1\otimes}\downarrow & & {\scriptstyle 1\otimes}\downarrow \\ H^1 \; L & \xrightarrow{Q_{L,\mathcal{L}^\rho}} & H^2 \; L \end{array}$$

$\otimes$ H^k L H^k L H^k L

H^k L $\otimes$

$\mathcal{M}$ $L, \mathcal{L}$ $\mathcal{M}$ $L, \mathcal{L}$

b b

Theorem 2.4. *Th r x sts ct o o* $\mathcal{LAG}$*, wh ch s comp t bl w th th ct o o d whos ct o o ob cts s g v b*

$$L, \mathcal{L}, s, b \qquad L, \mathcal{L} \; , s, b$$

$\mathcal{LAG}$

$\mathcal{LAG}$

$$HF \quad L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2$$
$$\sim HF \quad L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2$$

F Y

$^{-1}$

v v

v HF $L_1, \mathcal{L}_1, s_1, b_1$, $L_2, \mathcal{L}_2, s_2, b_2$
L_1 L_2
§

$$CF \quad L_1, \mathcal{L}_1, s_1, b_1 \ , \ L_2, \mathcal{L}_2, s_2, b_2 \qquad \bigoplus_{p \in L_1 \cap L_2}$$

$$CF \quad L_1, \mathcal{L}_1, s_1, b_1 \ , \ L_2, \mathcal{L}_2, s_2, b_2 \qquad \sim CF \quad L_1, \mathcal{L}_1, s_1, b_1 \ , \ L_2, \mathcal{L}_2, s_2, b_2$$

$\mathfrak{m}_1$ §
$\mathfrak{m}_1$
$\mathfrak{m}_k$
$\mathfrak{m}_k$

$$\mathfrak{m}_k \quad \prod_{\ell=0}^{k-1} CF \quad L_\ell, \mathcal{L}_\ell, s_\ell, b_\ell \ , \ L_{\ell+1}, \mathcal{L}_{\ell+1}, s_{\ell+1}, b_{\ell+1}$$
$$CF \quad L_0, \mathcal{L}_0, s_0, b_0 \ , \ L_k, \mathcal{L}_k, s_k, b_k$$

$$\mathfrak{m}_k \circ \quad \otimes \cdots \otimes \qquad \circ \, \mathfrak{m}_k$$

3. Proofs

fl

$Q_{L,\mathcal{L}}$ β π_2 M, L

$$E \ \beta \qquad \int_{D^2} \varphi^* \omega$$

φ D^2 M β

H β $\mathcal{L}$ $hol_{\mathcal{L}}$ φ S^1 U

fl $\mathcal{L}$ φ S^1

Lemma 3.1. *If* L *s r t o l th* E β .

Proof.

$$\pi\sqrt{-}\, E\ \beta \qquad \int_{D^2} F_{\nabla_\xi}$$

$$\pi\sqrt{-}\, E\ \beta \qquad hol_\xi\ \partial\beta\ ,$$

φ S^1 L m hol_ξ $\partial\beta$ ^{m} E β

□

Lemma 3.2. $\pi\sqrt{-}\, E\ \beta\ \ H\ \beta\ \mathcal{L}\ \ H\ \beta\ \mathcal{L}^\sigma$

Proof. □

$\mathcal{M}_{k+1}\ L\ \beta$

J M $\mathcal{M}_k\ L\ \beta$ φ D^2, S^1 M, L

β k ∂D^2 S^1

Definition 3.1. $\widetilde{\mathcal{M}}_{k+1}\ L\ \beta$ $\varphi\ \vec{z}$ φ D^2, S^1 M, L J β $\vec{z}$ $z_1, \cdots, z_{k+1}$ $\partial D^{2\ k+1}$ z ∂D^2

$Aut\ D^2, J_{D^2}\ \sim PSL$ $\widetilde{\mathcal{M}}_{k+1}\ L\ \beta$

$$u\cdot\ \varphi\ z_1, \cdots, z_{k+1} \qquad \varphi\circ u^{-1}\ u\ z_1\ , \cdots, u\ z_{k+1}$$

$\mathcal{M}_{k+1}\ L\ \beta$

$\mathcal{M}_{k+1}\ L\ \beta$ §

$\mathcal{C}\mathcal{M}_{k+1}\ L\ \beta$

n $k-$ β

$c_1\ M$ L

L $\mathcal{C}\mathcal{M}_{k+1}\ L\ \beta$

$$v\ \ \mathcal{C}\mathcal{M}_{k+1}\ L\ \beta \qquad L^{k+1}$$

$$v\ \varphi, \vec{z} \qquad \varphi\ z_0\ , \cdots, \varphi\ z_k$$

v $v_0, \cdots,\ v_k$

$\mathcal{C}\mathcal{M}_{k+1}\ L\ \beta$

L^{k+1} $\mathcal{C}\mathcal{M}_{k+1}\ L\ \beta$ Q

$\beta_1, \beta_2, \cdots$ π_2 M, L
E β $\leq E$ β $_{+1}$ C i
E β $\leq C$ β $\beta_1, \beta_2, \cdots$ $\mathcal{CM}_{k+1}$ L β

$\overline{S}^k$ L k L
$n-k$ §

C^k L C^k L C^k L $\otimes$
Q

$$\mathfrak{m}_{k,\beta} \quad C^1 \ L \quad {}^{\otimes k} \quad C^2 \ L$$

$$\mathfrak{m}_{k,\beta} \ \ P_1, \cdots, P_k \quad \pm \ v_{0,*} \ \ \mathcal{CM}_{k+1} \ L \ \beta \ {}_{ev_1,\cdots,ev_k} \times_{L^k} \ P_1 \times \cdots P_k$$

$\mathfrak{m}_{k,\beta}$

$$Q'_{L,\mathcal{L}} \quad C^1 \ L \quad {}^{\otimes k} \quad C^2 \ L$$

$$Q'_{L,\mathcal{L}} \qquad Q'_{L;\beta} \otimes H \ \beta \ \ \mathcal{L} \quad {}^{E(\beta \ )}$$

$$Q'_{L,\mathcal{L};\beta} \quad C^1 \ L \qquad C^2 \ L$$

$$Q'_{L,\mathcal{L};\beta} \ \ b \qquad \sum_k \mathfrak{m}_{k,\beta} \ \ b, \cdots, b$$

$b \equiv$ $Q'_{L,\mathcal{L}}$ b
E β
C^2 L

Lemma 3.3. *Th follow g d gr m comm t s.*

$$\begin{array}{ccc} C^1 \ L & \xrightarrow{Q'_{L,\mathcal{L}}} & C^2 \ L \\ {\scriptstyle 1\otimes}\downarrow & & {\scriptstyle 1\otimes}\downarrow \\ C^1 \ L & \xrightarrow{Q'_{L,\mathcal{L}^\rho}} & C^2 \ L \end{array}$$

Proof. $Q'_{L;\beta}$ $\mathcal{L}$

$$Q'_{L,\mathcal{L}}\ b \quad Q'_{L;\beta}\ b \ \otimes\ H\ \beta\ \mathcal{L}\ \ ^{E(\beta\ )}$$

$$Q'_{L;\beta}\ b \ \otimes\ \ \pi\sqrt{-}\ E\ \beta\ \ H\ \beta\ \mathcal{L}\ \ ^{E(\beta\ )}$$

$$Q'_{L;\beta}\ b \ \otimes H\ \beta\ \mathcal{L}\ \ ^{E(\beta\ )}\ \ Q'_{L,\mathcal{L}^{\rho}}\ b$$

∎

$Q_{L,\mathcal{L}}$ $Q'_{L,\mathcal{L}}$ §

L_1 L_2 L_1 L_2
L_1 L_2

Lemma 3.4. *W ss m th t* L_1 *d* L_2 *r r t o l. If* φ $S^1\times$, M *s smooth m p s ch th t* $S^1\times$ $\subset L_1$, $S^1\times$ $\subset L_2$, *th*

$$\int_{S^1\times[0,1]}\varphi^*\omega$$

$$CF\ \ L_1,\mathcal{L}_1,s_1,b_1\ ,\ L_2,\mathcal{L}_2,s_2,b_2 \qquad \bigoplus_{p\in L_1\cap L_2} Hom\ \mathcal{L}_{1,p},\mathcal{L}_{2,p}\ \otimes$$

$\mathcal{L}$ $\mathcal{L}$

$\mathfrak{m}_1$
p,q $L_1\cap L_2$ q
p

Remark 3.1. s

$\mathfrak{m}'_1$
$\mathfrak{m}'_1$

$$\mathfrak{m}_1'^{p,q}\ \ Hom\ \mathcal{L}_{1,p},\mathcal{L}_{2,p}\ \otimes \qquad Hom\ \mathcal{L}_{1,q},\mathcal{L}_{2,q}\ \otimes$$

$\partial_1 D^2$ z ∂D^2 z , $\partial_2 D^2$ z ∂D^2 $z >$

Definition 3.2. π_2 M L_1, L_2 p, q
φ D^2 M

φ q φ $-$ p
φ $\partial_1 D^2$ $\subset L_1$ φ $\partial_2 D^2$ $\subset L_2$

β π_2 M L_1, L_2 p, q

$$E\ \beta \quad \int_{D^2} \varphi^* \omega,$$

φ β
φ L

$$E\ \beta\ - E\ \beta'$$

β, β' π_2 M L_1, L_2 p, q

$$H\ \beta\ \mathcal{L}_1, \mathcal{L}_2 \quad Hom\ Hom\ \mathcal{L}_{1,p}, \mathcal{L}_{2,p}\ , Hom\ \mathcal{L}_{1,q}, \mathcal{L}_{2,q}\ ,$$

$$H\ \beta\ \mathcal{L}_1, \mathcal{L}_2\ \ v \quad {}_{\mathcal{L}_2}\ \varphi\ \partial_2 D^2 \ \circ v \circ \quad {}_{\mathcal{L}_1}\ \varphi\ \partial_1 D^2 \ \ ^{-1}$$

${}_{\mathcal{L}_1}$ φ $\partial_1 D^2$ $\mathcal{L}_{1,p}$ $\mathcal{L}_{1,q}$ fl $\mathcal{L}_1$
φ $\partial_1 D^2$

β π_2 M L_1, L_2 p, q
φ D^2 M J β
φ $\widetilde{\mathcal{M}}$ L_1, L_2 p, q β
D^2 $\pm$ $\widetilde{\mathcal{M}}$ L_1, L_2 p, q β
$\mathcal{M}$ L_1, L_2 p, q β
$\mathcal{CM}$ L_1, L_2 p, q β § p
q $\mathcal{CM}$ L_1, L_2 p, q β
$\sharp\mathcal{CM}$ L_1, L_2 p, q β
$\mathfrak{m}_1'^{p,q}$
$\mathfrak{m}_{1;0}'^{p,q}$

$$\mathfrak{m}_{1;0}'^{p,q} \quad \sharp\mathcal{CM}\ L_1, L_2\ p, q\ \beta\ \ H\ \beta\ \ \mathcal{L}_1, \mathcal{L}_2\ \otimes \quad ^{E(\beta\ )}$$

β π_2 M L_1, L_2 p, q $\mathcal{CM}$ L_1, L_2 p, q β

$\mathfrak{m}_1'^{p,q}$ $\mathfrak{m}_1' \circ \mathfrak{m}_1'$
b_1 b_2 $\mathfrak{m}_1'^{p,q}$ $\mathfrak{m}_{1;0}'^{p,q}$

b_1 b_2 Q

$E\ \beta$ L_1, L_2

$\mathfrak{m}'^{p,q}_{1;0}$

$\mathfrak{m}'^{p,q}_{1;0}$

$$\pi_2\ M\ L_1, L_2\ p, q\ \times \pi_2\ M\ L_1, L_2\ q, r\qquad \pi_2\ M\ L_1, L_2\ p, r\ ,$$

$$\beta, \beta'\qquad \beta\sharp\beta'$$

$$E\ \beta\sharp\beta'\qquad E\ \beta\qquad E\ \beta'\ ,\quad H\ \beta\sharp\beta'\ \mathcal{L}_1, \mathcal{L}_2\qquad H\ \beta'\ \mathcal{L}_1, \mathcal{L}_2\ \circ H\ \beta\ \mathcal{L}_1, \mathcal{L}_2$$

p_0 $L_1 \cap L_2$ p $L_1 \cap L_2$ β^p $\pi_2\ M\ L_1, L_2\ p_0, p$

Definition 3.3. p, q $L_1 \cap L_2$ β $\pi_2\ M\ L_1, L_2\ p, q$

$$E'\ \beta\qquad E\ \beta\ - E\ \beta^p\qquad E\ \beta^q\ ,$$

$$H'\ \beta\ \mathcal{L}_1, \mathcal{L}_2\qquad H\ \beta^q\ \mathcal{L}_1, \mathcal{L}_2\ ^{-1} \circ H\ \beta\ \mathcal{L}_1, \mathcal{L}_2\ \circ H\ \beta^p\ \mathcal{L}_1, \mathcal{L}_2$$

$$H'\ \beta\ \mathcal{L}_1, \mathcal{L}_2\qquad Hom\ \mathcal{L}_{1,p_0}, \mathcal{L}_{2,p_0}\ , Hom\ \mathcal{L}_{1,p_0}, \mathcal{L}_{2,p_0}\quad \sim$$

$H'\ \beta\ \mathcal{L}_1, \mathcal{L}_2$ U

$E'\ \beta$

$$CF\quad L_1, \mathcal{L}_1, s_1, b_1\ ,\ L_2, \mathcal{L}_2, s_2, b_2\qquad \bigoplus_{p \in L_1 \oplus L_2} \qquad p$$

Remark 3.2.

$$CF\quad L_1, \mathcal{L}_1, s_1, b_1\ ,\ L_2, \mathcal{L}_2, s_2, b_2\qquad \bigoplus_{p \in L_1 \cap L_2} \qquad p\ \otimes\ Hom\quad \mathcal{L}_1\ _{p_0},\ \mathcal{L}_2\ _{p_0}$$

p_0 $L_1 \cap L_2$ $Hom\quad \mathcal{L}_1\ _{p_0},\ \mathcal{L}_2\ _{p_0}$

p

$$\mathfrak{m}^{p,q}_{1;0} \qquad \sum_{\beta\ \in\ _2(M; L_1, L_2; p, q)} \sharp\mathcal{CM}\ L_1, L_2\ p, q\ \beta\ \ H'\ \beta\ \ \mathcal{L}_1, \mathcal{L}_2\quad ^{E'(\beta\ )}\qquad ,$$

$$\mathfrak{m}_{1,0}\ \ p\qquad \sum_q \mathfrak{m}^{p,q}_{1;0}\ q$$

b_1 b_2 $\mathfrak{m}_{1,0} \circ \mathfrak{m}_{1,0}$

$\mathfrak{m}_1$ $\mathfrak{m}_{1,0}$ $\cdots$

$\ldots$

$CF\quad L_1, \mathcal{L}_1, s_1, b_1\ ,\ L_2, \mathcal{L}_2, s_2, b_2\qquad , \mathfrak{m}_1$

Lemma 3.5. *Th ch compl x* $CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2 \quad , \mathfrak{m}_1 \otimes_{\Lambda_{\mathbb{C}}^{\mathbb{Q}}}$ *s somorph c to th ch compl x* $CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2 \quad , \mathfrak{m}'_1$.

$\otimes$

Proof.

$$CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2 \qquad \bigoplus_{p \in L_1 \cap L_2} Hom \; \mathcal{L}_{1,p}, \mathcal{L}_{2,p} \; \otimes$$

$$I \; CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , L_2, \mathcal{L}_2, s_2, b_2 \qquad \otimes_{\Lambda_{\mathbb{C}}^{\mathbb{Q}}}$$
$$\sim CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2$$

$p \quad L_1 \cap L_2 \qquad \varphi \; D^2 \quad M \qquad \beta^p$

$$I_0 \; p \qquad {}_{\mathcal{L}_2} \; \varphi \; \partial_2 D^2 \quad \circ \quad {}_{\mathcal{L}_1} \; \varphi \; \partial_1 D^2 \quad ^{-1} \quad Hom \; \mathcal{L}_{1,p}, \mathcal{L}_{2,p}$$

$$I \; p \qquad I_0 \; p \; \otimes \quad ^{E(\beta^p)}$$

$I \qquad I$

∎

$\mathfrak{m}_k$

$CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2 \qquad CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2$

$CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2 \qquad p \quad p$ $L_1 \cap L_2 \qquad \mathcal{L} \quad b$

$p \qquad p$

$\mathfrak{m}_k \; k \quad , \; , \cdots$

k

Proposition 3.6. *Th follow g d gr m comm t s.*

$$\begin{array}{ccc}
CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2 & \xrightarrow{\mathfrak{m}_1} & CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2 \\
\downarrow & & \downarrow \\
CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2 & \xrightarrow{\mathfrak{m}_1} & CF \; L_1, \mathcal{L}_1, s_1, b_1 \; , \; L_2, \mathcal{L}_2, s_2, b_2
\end{array}$$

Proof.

Lemma 3.7. *L t* φ $S^1 \times$, M *b smooth m p s ch th t* $S^1 \times \quad \subset L_1$, $S^1 \times \quad \subset L_2$*; th w h v*

$$\left(\pi\sqrt{-} \int_{S^1\times[0,1]} \varphi^*\omega \right) hol_{\mathcal{L}_1} \ \varphi \ S^1 \times \quad {}^{-1} hol_{\mathcal{L}_2} \ \varphi \ S^1 \times$$

$$hol_{\mathcal{L}_1^\rho} \ \varphi \ S^1 \times \quad {}^{-1} hol_{\mathcal{L}_2^\rho} \ \varphi \ S^1 \times$$

Proof.

$$\left(\pi\sqrt{-} \int_{S^1\times[0,1]} \varphi^*\omega \right) \quad hol_\xi \ \varphi \ S^1 \times \quad {}^{-1} hol_\xi \ \varphi \ S^1 \times$$

□

β π_2 M L_1, L_2 p, q

$$E' \quad \pi\sqrt{-}\,\beta \quad H' \ \beta \ \mathcal{L}_1, \mathcal{L}_2 \qquad H' \ \beta \ \mathcal{L}_1, \mathcal{L}_2$$

b

□

4. Relation to Mirror symmetry

§

M, ω c_1 M ω H^2 M
$M^\vee$ M $M^\vee$
ω $M^\vee, J_\omega$

Ω ω $\sqrt{-}\,B$ B ω M
$M^\vee$ J J_Ω Ω

$\Omega - \Omega'$ $\sqrt{-}\,H^2$ M
$M^\vee, J_\Omega$ $M^\vee, J_{\Omega'}$

$M, -\sqrt{-}\,\tau\omega$
M, ω τ $\tau >$ $M^\vee, J_\tau$

F Y

J_τ $J_{-\overline{-1}\omega}$ $-\sqrt{-}\,\tau\omega - \sqrt{-}\;\tau$ ω $\sqrt{-}\,H^2$ M

$$\psi_\tau \quad M^\vee, J_\tau \sim M^\vee, J_{\tau+1}$$

q $\pi\sqrt{-}\,\tau$ D^2_*

D^2_* $D^2 -$ J_τ q J_q J_τ

ψ

$$\pi \quad \mathfrak{M}^\circ \quad D^2_*$$

q $M^\vee, J_q$

D^2_*

$$\pi \quad \mathfrak{M} \quad D^2$$

π^{-1} π^{-1}

π^{-1} q

q π^{-1}

$\mathcal{P}_m$ D^2 D^2 $\mathcal{P}_m$ z z^m

$$\mathfrak{M}_m \quad \mathfrak{M} \quad \times_{\mathcal{P}_m} D^2$$

π_m $\mathfrak{M}_m$ D^2 $\mathcal{P}_{m,m'}$ $\mathfrak{M}_{mm'}$ $\mathfrak{M}_m$

U D^2 $\mathfrak{F}$ $\pi^{-1}\mathcal{P}_m^{-1}$ U $\subseteq$

$\mathfrak{M}_m$ $\mathfrak{F}, m, U$

$\mathfrak{F}, m, U$ $\mathfrak{G}, \ell, V$ $\mathfrak{F}, m, U \sim \mathfrak{G}, \ell, V$

W $W \subset U \cap V$ d

$$\Psi \quad \mathcal{P}^*_{m,d\ell}\mathfrak{F} \quad {}_{-1}\mathcal{P}^{-1}_{dm\ell}(W-\{0\}) \sim \mathcal{P}^*_{\ell,dm}\mathfrak{G} \quad {}_{-1}\mathcal{P}^{-1}_{dm\ell}(W-\{0\})$$

$q^k\Psi$ $\mathcal{P}^*_{m,d\ell}\mathfrak{F}$ ${}_{-1}\mathcal{P}^{-1}_{dm\ell}(W)$ $\mathcal{P}^*_{\ell,dm}\mathfrak{G}$ ${}_{-1}\mathcal{P}^{-1}_{dm\ell}(W)$

q^k Ψ^{-1} $\mathcal{P}^*_{\ell,dm}\mathfrak{G}$ ${}_{-1}\mathcal{P}^{-1}_{dm\ell}(W)$ $\mathcal{P}^*_{m,d\ell}\mathfrak{F}$ ${}_{-1}\mathcal{P}^{-1}_{dm\ell}(W)$

k $\sim$

$\mathcal{SH}$ $\mathfrak{M}$ $\sim$ $\mathfrak{F}, m, U$

$\mathfrak{OB}$ $\mathcal{SH}$ $\mathfrak{M}$

$\mathfrak{F}, m, U$, $\mathfrak{G}, \ell, V$ $\mathfrak{OB}$ $\mathcal{SH}$ $\mathfrak{M}$

$\mathcal{SH}$ $\mathfrak{M}$ $\mathfrak{F}, m, U$
$\mathfrak{F}$ $\mathbf{R}\mathfrak{F}^*$
$\mathfrak{F}, m, U$, $\mathfrak{G}, \ell, V$ $\mathcal{SH}$ $\mathfrak{M}$ $W \subseteq U \cap V$ W

$$Hom\left(\mathcal{P}^*_{\ell,m}\mathbf{R}\mathfrak{F}^* \quad {}^{-1}\mathcal{P}^{-1}_{m\ell}(W), \mathcal{P}^*_{m,\ell}\mathbf{R}\mathfrak{G} \quad {}^{-1}\mathcal{P}^{-1}_{m\ell}(W)\right)$$

$\pi^{-1}\mathcal{P}^{-1}_{m\ell}$ W $\mathfrak{O}_{\mathcal{P}^{-1}_{m\ell}(W)}$
$\mathcal{P}^{-1}_{m\ell}$ W $\subset D^2$ $\mathfrak{O}_{\mathcal{P}^{-1}_{m\ell}(W)}$
q $\mathcal{P}^{-1}_{m\ell}$ W

$$\Gamma\left(\pi^{-1}_{\ell m} \; q \quad Hom\left(\mathcal{P}^*_{\ell,m}\mathbf{R}\mathfrak{F}^*, \mathcal{P}^*_{m,\ell}\mathbf{R}\mathfrak{G}\right)\right)$$

C^*_W $\mathbf{R}\mathfrak{F}, \mathbf{R}\mathfrak{G}$
W' W s
C^*_W $\mathbf{R}\mathfrak{F}, \mathbf{R}\mathfrak{G}$ $\mathcal{P}^{-1}_{\ell m}$ W' $-$ $q^k s$ $\mathcal{P}^{-1}_{\ell m}$ W' k
Γ $\mathcal{P}^{-1}_{\ell m}$ W'_* C^*_W $\mathbf{R}\mathfrak{F}, \mathbf{R}\mathfrak{G}$

$$C^*_0 \; \mathbf{R}\mathfrak{F}, \mathbf{R}\mathfrak{G} \quad \longleftarrow \quad \Gamma \; \mathcal{P}^{-1}_{\ell m} \; W'_* \quad C^*_W \; \mathbf{R}\mathfrak{F}, \mathbf{R}\mathfrak{G}$$

W'
$\langle\langle q^{1/\ell m}\rangle\rangle$ q^{-1} $\pi^{-1}_{\ell m}$ W $\subseteq D^2$
C^*_0 $\mathbf{R}\mathfrak{F}, \mathbf{R}\mathfrak{G}$ $\langle\langle q^{1/\ell m}\rangle\rangle$ q^{-1}
$\langle\langle q^{1/\ell m}\rangle\rangle$ q^{-1} $q^{1/mm'}$ $1/mm'$

$$C^* \quad \mathfrak{F}, m, U \; , \; \mathfrak{G}, \ell, V \qquad C^*_0 \; \mathbf{R}\mathfrak{F}, \mathbf{R}\mathfrak{G} \; \otimes \; {}_{\langle\langle q^{1/\ell m}\rangle\rangle[q^{-1}]}$$

$\otimes$

$\mathfrak{F}, m, U$, $\mathfrak{G}, \ell, V$
A
A
A

Proposition 4.1. *Th r x sts filt r d A c t gor* $\mathcal{SH}$ $\mathfrak{M}$ *s ch th t th s t of ts ob cts s* $\mathfrak{OB}$ $\mathcal{SH}$ $\mathfrak{M}$ *, d th mod l s of morph sms r* C^* $\mathfrak{F}, m, U$, $\mathfrak{G}, \ell, V$ *.*

Mor ov r th r x sts ct o o $\mathcal{SH}$ $\mathfrak{M}$ *comp t bl w th ts ct o o .*

Proof. $\mathfrak{m}_1$ C $\mathfrak{F}, \mathfrak{G}$ $\mathfrak{m}_2$
$\mathfrak{m}_k$ $k \geq$
A
$\mathcal{SH}$ $\mathfrak{M}$
D^2_* $\mathcal{SH}$ $\mathfrak{M}$

$\mathcal{P}_{1,m}$

$\mathfrak{M}_m$ $\mathfrak{M}$ $\mathfrak{F}, m, U$ $\mathcal{SH}$ $\mathfrak{M}$ ${}^*\mathfrak{F}, m, U$ $\mathcal{SH}$ $\mathfrak{M}$

$\mathfrak{F}, m, U$ ${}^*\mathfrak{F}, m, U$

$\mathfrak{F}$ $\mathbf{R}\mathfrak{F}^*$ $\mathfrak{F}$ ${}^*\mathfrak{F}$
${}^*\mathbf{R}\mathfrak{F}^*$ ${}^*\mathfrak{F}$

$$C^* \ \mathfrak{F}, m, U \ , \ \mathfrak{G}, \ell, V \qquad C^* \ {}^*\mathfrak{F}, m, U \ , \ {}^*\mathfrak{G}, \ell, V$$

□

A
§

$\mathfrak{m}_k$

Conjecture 4.2. *L t M b C l b -Y m fold M w th ¨hl r form* ω. *W ss m* ω H^2 M . *L t s ss m th t th r x sts m rror f m l* $\mathfrak{M}$ D^2_* *s bov .* *Th th r x sts filt r d A f ctor* $\mathcal{LAG}$ $\mathcal{SH}$ $\mathfrak{M}$ *wh ch pr s rv s ct o s. Mor ov r th d c d homomorph sms* HF $L_1, \mathcal{L}_1, b_1$, $L_2, \mathcal{L}_2, b_2$ H C^* $L_1, \mathcal{L}_1, b_1$, $L_2, \mathcal{L}_2, b_2$, $\mathfrak{m}_1$ *o cohomolog s r somorph sms.*

2

$\mathfrak{m}_1, \mathfrak{m}_2$ $\mathfrak{m}_k$
$k \geq$

ξ $^2\times$ $,\pi\sqrt{-}$ $dy - yd$ 2 2 $, y , v$
$, y ,$ $\overline{-1}y$ v $, y , v$ $, y$ $, -$ $\overline{-1}x$ v

L_{pt} $, y \ y$ S^1 L_{st} b $, b$ S^1
L , S^1
L_{pt} L_{st} b $, b$ L
$\xi^{\otimes 2}_{L}$ ξ_L $, y \ v$ $, y$ $-v$
$L,$ 2
L_{st} , L_{st} ,
$L,$ E
$, y^*$ L_{x,y^*} L_{pt} $, \mathcal{L} \ y^*$
$\mathcal{L} \ y^*$ $S^1 \sim L_{\text{pt}}$ 2 $\overline{-1}y^*$
L_{x,y^*} $, y^*$ $\frac{2}{\tau}$

$^{2}_{\tau}$ ^{2} ^{2} $, y^*$ τ $- y^*$

z z $, y^*$ τ $- y^*$ $^{2}_{\tau}$ $\oplus \tau$

Remark 4.1. τ $\sqrt{-}\ b$

$b - \sqrt{-}$ ω $b - \sqrt{-}$ $\sqrt{-}\ y^*$

$b - \sqrt{-}$ $\sqrt{-}\ y^*$ $\sqrt{-}\ z$

fl ξ $_{L_{\mathrm{pt}}(x)}$

$$\left(\int_0^1 \pi\sqrt{-}\ dy\right) \quad (\pi\sqrt{-}\ ),$$

$$, y\ , v \quad , y \quad , - \overline{-1}x v$$

$$L_{x,y^*} \quad L_{x,y^*+x}$$

$^{2}_{\tau}$ $\mathfrak{F}_z$

z $^{2}_{\tau}$

$$\mathfrak{m}_2 \quad ^0\ ^2_\tau\ , E \otimes\ ^1\ ^2_\tau\ E, \mathfrak{F}_z \quad ^1\ ^2_\tau\ , \mathfrak{F}_z$$

$^1\ ^2_\tau\ E, \mathfrak{F}_z \sim E_z^*$ E z

$^1\ ^2_\tau\ , \mathfrak{F}_z$ $^0\ ^2_\tau\ , E$

s $^0\ ^2_\tau\ , E$

E v $E_z^* \sim\ ^1\ ^2_\tau\ E, \mathfrak{F}_z$

$$\mathfrak{m}_2\ s, v \quad v\ s\ z \quad \sim\ ^1\ ^2_\tau\ , \mathfrak{F}_z$$

$$\mathfrak{m}_2 \quad HF^0 \quad L_{\mathrm{st}} \quad , \quad , \ L, \quad \otimes HF^1 \quad L, \quad , \quad L_{\mathrm{pt}} \quad , \mathcal{L}\ y^*$$

$$HF^1 \quad L_{\mathrm{st}} \quad , \quad , \quad L_{\mathrm{pt}} \quad , \mathcal{L}\ y^*$$

Remark 4.2. HF

$\mathfrak{m}_k$ $^{-1}$

HF^0 L_{st} , , $L,$

$$HF^1 \quad L, \quad , \quad L_{\mathrm{pt}} \quad , \mathcal{L}\ y^* \quad \sim \mathcal{L}\ y^* \ _{(x,0)}$$

$$HF^1 \quad L_{\mathrm{st}} \quad , \quad , \quad L_{\mathrm{pt}} \quad , \mathcal{L}\ y^* \quad \sim \mathcal{L}\ y^* \ _{(x,x)}$$

$\mathcal{L}\ y^*$

$\sqrt{-}\ y^* dy$ y L_{pt} $, y$ y

F Y

$\tau \quad , y^* \quad \pi\sqrt{-}\,\tau \quad n\ ^2 - \ \pi\sqrt{-} \quad n\ y^*$

n

$^2\ L, \quad L, \quad L_{\mathrm{st}}, \quad L_{\mathrm{st}}, \quad L_{x,y^*} \quad L_{x,y^*+x}$

$\tau \quad , y^* \quad - \ \pi\sqrt{-}\ ^2 \quad \tau \quad , y^*$

$\xi\ _{L_{\mathrm{pt}}(x)} \otimes \ L_{\mathrm{pt}} \quad \times \quad , \ \pi\sqrt{-}\ y^* dy \ \sim \ L_{\mathrm{pt}} \quad \times \quad , \ \pi\sqrt{-}\ y^* \quad dy$

$, y\ , v \quad , \ - \quad \overline{-1} xy_{\mathcal{V}} \quad , \quad - \quad \overline{-1} x^2$

$HF^1\ \ L, \quad , \quad L_{\mathrm{pt}} \quad , \mathcal{L}\ y^* \quad \text{———}$

$^2\downarrow \qquad \exp(-2 \quad \overline{-1} x^2)\downarrow$

$HF^1\ \ L, \quad , \quad L_{\mathrm{pt}} \quad , \mathcal{L}\ y^* \quad \text{———}$

$^2 \qquad \mathfrak{m}_2$

$HF^0\ \ L_{\mathrm{st}} \quad , \quad , \ L, \qquad E$

$s \qquad HF^1\ \ L, \quad , \quad L_{\mathrm{pt}} \quad , \mathcal{L}\ y^* \quad \sim$

$E \qquad \pi\sqrt{-}\,\tau\ ^2 - \ \pi\sqrt{-}\ \ y^*$

$_z \quad E_z$

$\mathfrak{m}_2\ s, \ _z \quad _z\ s\ z \quad \tau \quad , y \quad -\pi\sqrt{-}\,\tau\ ^2 \quad \pi\sqrt{-}\ \ y^*$

$\pi\sqrt{-}\ n^2\tau \quad \pi\sqrt{-}\ nz$

n

$z \quad \tau \quad - y^* \qquad \vartheta\ \tau, z$

$\vartheta\ \tau, z \quad \vartheta\ \tau \quad , z\ ,$

$\vartheta\ - \quad \tau, z\ \tau \quad - \quad \overline{-1}^{1/4} \tau^{1/2} \vartheta\ \tau, z$

PSL

$\tau \qquad -1/\tau$

P^1

$PSL \qquad \mathfrak{m}_k$

F Y

$\mathfrak{m}_k$ $k \geq$ $\mathfrak{m}_3$
PSL

$\mathfrak{h}$
$\times\,\mathfrak{h}$ ${}^2\times PSL$,
v, σ w, z σw $v, \sigma z$ σ $\mathfrak{h}$
$\sim$ ^{2} P^1

$\mathfrak{h}$ PSL $\cup$
$\mathfrak{M}$ D^2
F_0 ^{2} F_0
P^1 D_1 D_2 $D_1 \cap D_2$ D_1 D_2

^{2} $F_{0,\mathrm{reg}}$ F_0 $F_{0,\mathrm{reg}} \sim$ ${}_* \cup$ ${}_*$
p $F_{0,\mathrm{reg}}$ s_p D^2 $\mathfrak{M}$ s_p p

[1]

^{2} L_{pt}
s_p D^2 $\mathfrak{M}$ ^{2}
${}_2$

L_{pt} $\xi^{2\otimes}$
L_{pt} $, L_{\mathrm{pt}}$
s_p p ${}_* \cup$ ${}_*$
$\mathcal{LAG}$ $L, \mathcal{L}, s, b$
b L L_{pt} $, L_{\mathrm{pt}}$
Q H^1 L H^2 L b
L b H^1 S^1 $\otimes$ ${}_{,+}$

${}_{,+}$ ${}_{,+} \cap$ ${}_{,+}$
$\sum$ $>$ H^1 S^1 $>$
Q

Q
b $\sum$ ${}_0$ $>$ $i >$ b
H^1 L H^1 L
fl $\mathcal{L}$
H^1 L

---

[1]

L_{pt} , L_{pt} $*$ $S^1 \times$ $+$ S^1
$\mathcal{L}$ $+$ $* \cup$ $*$
H^1 S^1 $,+$ b
s_p s s p

Remark 4.3.

L' ,
L' L
$\mathfrak{h}$ PSL

Remark 4.4.

$\mathfrak{m}_1$
P^1 $\mathfrak{m}_k$ P^1

$\mathfrak{m}_2$

M H^2 M
M $M^\vee$

τ τ τ $\frac{\tau\cos 2 \quad /5-\sin 2 \quad /5}{\tau\sin 2 \quad /5+\cos 2 \quad /5}$
PSL , $\sqrt{-}$

O ,

Remark 4.5.

F Y

$\mathcal{LAG}$

$M^\vee$ $\mathcal{LAG}$ M

ψ_τ $M^\vee, J_\tau$ $\sim$ $M^\vee, J_{\tau+1}$

Conjecture 4.3. *I th s t t o of Co ct r 4.2, th mo odrom* ψ_τ $M^\vee, J_\tau$ $\sim$ $M^\vee, J_{\tau+1}$ *c b t s "compl t l t gr bl s st m".*

N m l th follow g holds. Th r x sts B r s t $M_0^\vee$ *of* $M^\vee$ *s ch th t f* $M_0^\vee$ *th th clos r of th orb t* ψ_τ^k *k s n d m s o l L gr g tor s. (n* $M^\vee$*). Mor ov r, for , th clos r of* ψ_τ^k *k s fi t o of sotrop c tor .*

$, y^*$, y^*

S^1

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Department of Mathematics, Faculty of Sciences, Kyoto University, Kitashirakawa, Sakyo-ku, Kyoto, Japan

E-mail address `fukaya@kusm.kyoto-u.ac.jp`