

1-1-2005

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### Recommended Citation

KIM, KYUNG HO and LEE, JONG GEOL (2005) "On Intuitionistic Fuzzy Bi-Ideals of Semigroups," *Turkish Journal of Mathematics*: Vol. 29: No. 2, Article 9. Available at: <https://journals.tubitak.gov.tr/math/vol29/iss2/9>

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## On Intuitionistic Fuzzy Bi-Ideals of Semigroups

*K. H. Kim, J. G. Lee*

### Abstract

We consider the intuitionistic fuzzification of the concept of several ideals in a semigroup  $S$ , and investigate some properties of such ideals.

**Key Words:** Intuitionistic fuzzy  $(1, 2)$ -ideal, intuitionistic fuzzy bi-ideal, intuitionistic fuzzy ideal.

### 1. Introduction

After the introduction of fuzzy sets by L. A. Zadeh [8], several researchers explored on the generalization of the the notion of fuzzy set. The concept of intuitionistic fuzzy set was introduced by K. T. Atanassov [1, 2], as a generalization of the notion of fuzzy set. In [3], N. Kuroki gave some properties of fuzzy ideals and fuzzy bi-ideals in semigroups. The concept of  $(1, 2)$ -ideals in semigroups was introduced by S. Lajos [5]. In this paper, we consider the intuitionistic fuzzification of the concept of several ideals in a semigroup  $S$ , and investigate some properties of such ideals.

### 2. Preliminaries

Let  $S$  be a semigroup. By a *subsemigroup* of  $S$  we mean a non-empty subset  $A$  of  $S$  such that  $A^2 \subseteq A$ , and by a *left (right) ideal* of  $S$  we mean a non-empty subset  $A$  of  $S$  such that  $SA \subseteq A$  ( $AS \subseteq A$ ). By *two-sided ideal* or simply *ideal*, we mean a non-empty subset of  $S$  which is both a left and a right ideal of  $S$ . A subsemigroup  $A$  of a semigroup

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2000 *Mathematics Subject Classification*: 20M12, 04A72.

$S$  is called a *bi-ideal* of  $S$  if  $ASA \subseteq A$ . A subsemigroup  $A$  of  $S$  is called a  $(1, 2)$ -*ideal* of  $S$  if  $ASA^2 \subseteq A$ . A semigroup  $S$  is said to be  $(2, 2)$ -*regular* if  $x \in x^2Sx^2$  for any  $x \in S$ . A semigroup  $S$  is said to be *regular* if, for each  $x \in S$ , there exists  $y \in S$  such that  $x = xyx$ . A semigroup  $S$  is said to be *completely regular* if, for each  $x \in S$ , there exists  $y \in S$  such that  $x = xyx$  and  $xy = yx$ . For a semigroup  $S$ , note that  $S$  is completely regular if and only if  $S$  is a union of groups if and only if  $S$  is  $(2, 2)$ -regular. A semigroup  $S$  is said to be *left* (resp. *right*) *duo* if every left (resp. right) ideal of  $S$  is a two-sided ideal of  $S$ .

By a *fuzzy set*  $\mu$  in a non-empty set  $S$  we mean a function  $\mu : S \rightarrow [0, 1]$ , and the complement of  $\mu$ , denoted by  $\bar{\mu}$ , is the fuzzy set in  $S$  given by  $\bar{\mu}(x) = 1 - \mu(x)$  for all  $x \in S$ .

An intuitionistic fuzzy set (briefly, IFS)  $A$  in a non-empty set  $X$  is an object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\}$$

where the functions  $\mu_A : X \rightarrow [0, 1]$  and  $\gamma_A : X \rightarrow [0, 1]$  denote the degree of membership and the degree of nonmembership, respectively, and

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1$$

for all  $x \in X$ .

An intuitionistic fuzzy set  $A = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\}$  in  $X$  can be identified to an ordered pair  $(\mu_A, \gamma_A)$  in  $I^X \times I^X$ . For the sake of simplicity, we shall use the symbol  $A = (\mu_A, \gamma_A)$  for the IFS  $A = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\}$ .

### 3. Intuitionistic Fuzzy ideals

In what follows, let  $S$  denote a semigroup unless otherwise specified.

**Definition 3.1** [3] *An IFS  $A = (\mu_A, \gamma_A)$  in  $S$  is called an intuitionistic fuzzy subsemigroup of  $S$  if*

$$(i) \quad \mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\},$$

$$(ii) \quad \gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\},$$

for all  $x, y \in S$ .

**Definition 3.2** *An IFS  $A = (\mu_A, \gamma_A)$  in  $S$  is called an intuitionistic fuzzy left ideal of  $S$  if  $\mu_A(xy) \geq \mu_A(y)$  and  $\gamma_A(xy) \leq \gamma_A(y)$  for all  $x, y \in S$ . An intuitionistic fuzzy right ideal*

of  $S$  is defined in an analogous way. An IFS  $A = (\mu_A, \gamma_A)$  in  $S$  is called an intuitionistic fuzzy ideal of  $S$  if it is both an intuitionistic fuzzy right and an intuitionistic fuzzy left ideal of  $S$ .

It is clear that any intuitionistic fuzzy left (right) ideal of  $S$  is an intuitionistic fuzzy subsemigroup of  $S$ .

**Definition 3.3** An intuitionistic fuzzy subsemigroup  $A = (\mu_A, \gamma_A)$  of  $S$  is called an intuitionistic fuzzy bi-ideal of  $S$  if

$$(i) \quad \mu_A(xwy) \geq \min\{\mu_A(x), \mu_A(y)\},$$

$$(ii) \quad \gamma_A(xwy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$$

for all  $w, x, y \in S$ .

**Example 3.4** Let  $S := \{a, b, c, d, e\}$  be a semigroup with the following Cayley table:

$\cdot$	$a$	$b$	$c$	$d$	$e$
$a$	$a$	$a$	$a$	$a$	$a$
$b$	$a$	$a$	$a$	$a$	$a$
$c$	$a$	$a$	$c$	$c$	$e$
$d$	$a$	$a$	$c$	$d$	$e$
$e$	$a$	$a$	$c$	$c$	$e$

Define an IFS  $A = (\mu_A, \gamma_A)$  in  $S$  by  $\mu_A(a) = 0.6, \mu_A(b) = 0.5, \mu_A(c) = 0.4, \mu_A(d) = \mu_A(e) = 0.3, \gamma_A(a) = \gamma_A(b) = 0.3, \gamma_A(c) = 0.4$  and  $\gamma_A(d) = 0.5, \gamma_A(e) = 0.6$ . By routine calculation, we can check that  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy bi-ideal of  $S$ .

**Theorem 3.5** If  $\{A_i\}_{i \in \Lambda}$  is a family of intuitionistic fuzzy bi-ideals of  $S$ , then  $\cap A_i$  is an intuitionistic fuzzy bi-ideal of  $S$ , where  $\cap A_i = (\wedge \mu_{A_i}, \vee \gamma_{A_i})$  and

$$\wedge \mu_{A_i}(x) = \inf\{\mu_{A_i}(x) \mid i \in \Lambda, x \in S\},$$

$$\vee \gamma_{A_i}(x) = \sup\{\gamma_{A_i}(x) \mid i \in \Lambda, x \in S\}.$$

**Proof.** Let  $x, y \in S$ . Then we have

$$\begin{aligned}\wedge\mu_{A_i}(xy) &\geq \wedge\{\min\{\mu_{A_i}(x), \mu_{A_i}(y)\}\} \\ &= \min\{\min\{\mu_{A_i}(x), \mu_{A_i}(y)\}\}, \\ &= \min\{\min\{\mu_{A_i}(x)\}, \min\{\mu_{A_i}(y)\}\} \\ &= \min\{\wedge\mu_{A_i}(x), \wedge\mu_{A_i}(y)\},\end{aligned}$$

$$\begin{aligned}\vee\gamma_{A_i}(xy) &\leq \vee\{\max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}\} \\ &= \max\{\max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}\}, \\ &= \max\{\max\{\gamma_{A_i}(x)\}, \max\{\gamma_{A_i}(y)\}\} \\ &= \max\{\vee\gamma_{A_i}(x), \vee\gamma_{A_i}(y)\}.\end{aligned}$$

Hence  $\cap A_i$  is an intuitionistic fuzzy subsemigroup of  $S$ . Next for  $x, y, a \in S$ , we obtain

$$\begin{aligned}\wedge\mu_{A_i}(xay) &\geq \wedge\{\min\{\mu_{A_i}(x), \mu_{A_i}(y)\}\} \\ &= \min\{\min\{\mu_{A_i}(x), \mu_{A_i}(y)\}\}, \\ &= \min\{\min\{\mu_{A_i}(x)\}, \min\{\mu_{A_i}(y)\}\} \\ &= \min\{\wedge\mu_{A_i}(x), \wedge\mu_{A_i}(y)\},\end{aligned}$$

$$\begin{aligned}\vee\gamma_{A_i}(xay) &\leq \vee\{\max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}\} \\ &= \max\{\max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}\}, \\ &= \max\{\max\{\gamma_{A_i}(x)\}, \max\{\gamma_{A_i}(y)\}\} \\ &= \max\{\vee\gamma_{A_i}(x), \vee\gamma_{A_i}(y)\}.\end{aligned}$$

Hence  $\cap A_i$  is an intuitionistic fuzzy bi-ideal of  $S$ . This completes the proof.  $\square$

**Theorem 3.6** *If an IFS  $A = (\mu_A, \gamma_A)$  in  $S$  is an intuitionistic fuzzy bi-ideal of  $S$ , then so is  $\square A := (\mu_A, \overline{\mu_A})$ .*

**Proof.** It is sufficient to show that  $\overline{\mu}_A$  satisfies the condition (ii) in Definition 3.1, and (ii) in Definition 3.3. For any  $a, x, y \in S$ , we have

$$\begin{aligned}\overline{\mu}_A(xy) &= 1 - \mu_A(xy) \leq 1 - \min\{\mu_A(x), \mu_A(y)\} \\ &= \max\{1 - \mu_A(x), 1 - \mu_A(y)\} = \max\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}\end{aligned}$$

and  $\overline{\mu}_A(xay) = 1 - \mu_A(xay) \leq 1 - \min\{\mu_A(x), \mu_A(y)\} = \max\{1 - \mu_A(x), 1 - \mu_A(y)\} = \max\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$ . Therefore  $\square A$  is an intuitionistic fuzzy bi-ideal of  $S$ .  $\square$

**Definition 3.7** An intuitionistic fuzzy subsemigroup  $A = (\mu_A, \gamma_A)$  of  $S$  is called an intuitionistic fuzzy  $(1, 2)$ -ideal of  $S$  if

- (i)  $\mu_A(xw(yz)) \geq \min\{\mu_A(x), \mu_A(y), \mu_A(z)\}$ ,
- (ii)  $\gamma_A(xw(yz)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\}$

for all  $w, x, y, z \in S$ .

**Theorem 3.8** Every intuitionistic fuzzy bi-ideal is an intuitionistic fuzzy  $(1, 2)$ -ideal.

**Proof.** Let  $A = (\mu_A, \gamma_A)$  be an intuitionistic fuzzy bi-ideal of  $S$  and let  $w, x, y, z \in S$ . Then

$$\begin{aligned}\mu_A(xw(yz)) &= \mu_A((xwy)z) \\ &\geq \min\{\mu_A(xwy), \mu_A(z)\} \\ &\geq \min\{\min\{\mu_A(x), \mu_A(y)\}, \mu_A(z)\} \\ &= \min\{\mu_A(x), \mu_A(y), \mu_A(z)\},\end{aligned}$$

and

$$\begin{aligned}\gamma_A(xw(yz)) &= \gamma_A((xwy)z) \\ &\leq \max\{\gamma_A(xwy), \gamma_A(z)\} \\ &\leq \max\{\max\{\gamma_A(x), \gamma_A(y)\}, \gamma_A(z)\} \\ &= \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\}.\end{aligned}$$

Hence  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy  $(1, 2)$ -ideal of  $S$ .  $\square$

To consider the converse of Theorem 3.9, we need to strengthen the condition of a semigroup  $S$ .

**Theorem 3.9** *If  $S$  is a regular semigroup, then every intuitionistic fuzzy  $(1, 2)$ -ideal of  $S$  is an intuitionistic fuzzy bi-ideal of  $S$ .*

**Proof.** Assume that a semigroup  $S$  is regular and let  $A = (\mu_A, \gamma_A)$  be an intuitionistic fuzzy  $(1, 2)$ -ideal of  $S$ . Let  $w, x, y \in S$ . Since  $S$  is regular, we have  $xw \in (xSx)S \subseteq xSx$ , which implies that  $xw = xsx$  for some  $s \in S$ . Thus

$$\begin{aligned}\mu_A(xwy) &= \mu_A((xsx)y) = \mu_A(xs(xy)) \\ &\geq \min\{\mu_A(x), \mu_A(x), \mu_A(y)\} \\ &= \min\{\mu_A(x), \mu_A(y)\},\end{aligned}$$

and

$$\begin{aligned}\gamma_A(xwy) &= \gamma_A((xsx)y) = \gamma_A(xs(xy)) \\ &\leq \max\{\gamma_A(x), \gamma_A(x), \gamma_A(y)\} \\ &= \max\{\gamma_A(x), \gamma_A(y)\}.\end{aligned}$$

Therefore  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy bi-ideal of  $S$ . □

**Theorem 3.10** *Let  $A = (\mu_A, \gamma_A)$  be an intuitionistic fuzzy bi-ideal of  $S$ . If  $S$  is a completely regular, then  $A(a) = A(a^2)$  for all  $a \in S$ .*

**Proof.** Let  $a \in S$ . Then there exists  $x \in S$  such that  $a = a^2xa^2$ . Hence

$$\begin{aligned}\mu_A(a) &= \mu_A(a^2xa^2) \geq \min\{\mu_A(a^2), \mu_A(a^2)\} \\ &= \mu_A(a^2) \geq \min\{\mu_A(a), \mu_A(a)\} = \mu_A(a)\end{aligned}$$

and

$$\begin{aligned}\gamma_A(a) &= \gamma_A(a^2xa^2) \leq \max\{\gamma_A(a^2), \gamma_A(a^2)\} \\ &= \gamma_A(a^2) \leq \max\{\gamma_A(a), \gamma_A(a)\} = \gamma_A(a).\end{aligned}$$

It follows that  $\mu_A(a) = \mu_A(a^2)$  and  $\gamma_A(a) = \gamma_A(a^2)$  so that  $A(a) = A(a^2)$ . □

**Theorem 3.11** *Let  $A = (\mu_A, \gamma_A)$  be an intuitionistic fuzzy ideal of  $S$ . If  $S$  is an intra-regular, then  $A(a) = A(a^2)$  for all  $a \in S$ .*

**Proof.** Let  $a$  be any element of  $S$ . Then since  $S$  is intra-regular, there exist  $x$  and  $y$  in  $S$  such that  $a = xa^2y$ . Hence since  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy ideal,

$$\begin{aligned} \mu_A(a) &= \mu_A(xa^2y) \geq \mu_A(xa^2) \\ &\geq \mu_A(a^2) \geq \{\mu_A(a), \mu_A(a)\} = \mu_A(a) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(a) &= \gamma_A((xa^2y)) = \gamma_A(xa^2) \leq \gamma_A(a^2) \\ &\leq \max\{\gamma_A(a), \gamma_A(a)\} = \gamma_A(a). \end{aligned}$$

Hence we have  $\mu_A(a) = \mu_A(a^2)$  and  $\gamma_A(a) = \gamma_A(a^2)$ . Therefore  $A(a) = A(a^2)$  for all  $x, y \in S$ .  $\square$

**Theorem 3.12** *Let  $A = (\mu_A, \gamma_A)$  be an intuitionistic fuzzy ideal of  $S$ . If  $S$  is an intra-regular, then  $A(ab) = A(ba)$  for all  $a, b \in S$ .*

**Proof.** Let  $a, b \in S$ . Then by Theorem 3.14, we have

$$\begin{aligned} \mu_A(ab) &= \mu_A((ab)^2) \geq \mu_A(a(ba)b) \\ &\geq \mu_A(ba) = \mu_A((ba)^2) \geq \mu_A(b(ab)a) \geq \mu_A(ab) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(ab) &= \gamma_A((ab)^2) = \gamma_A(a(ba)b) \leq \gamma_A(ba) \\ &= \gamma_A((ba)^2) = \gamma_A(b(ab)a) \leq \gamma_A(ab). \end{aligned}$$

So we have  $\mu_A(ab) = \mu_A(ba)$  and  $\gamma_A(ab) = \gamma_A(ba)$ . Therefore  $A(ab) = A(ba)$ .  $\square$

**Theorem 3.13** *An IFS  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy bi-ideal of  $S$  if and only if the fuzzy sets  $\mu_A$  and  $\overline{\gamma_A}$  are fuzzy bi-ideals of  $S$ .*



**Proof.** Let  $A = (\mu_A, \gamma_A)$  be an intuitionistic fuzzy bi-ideal of  $S$ . Then clearly  $\mu_A$  is a fuzzy bi-ideal of  $S$ . Let  $x, a, y \in S$ . Then

$$\begin{aligned}\overline{\gamma_A}(xy) &= 1 - \gamma_A(xy) \\ &\geq 1 - \max\{\gamma_A(x), \gamma_A(y)\} \\ &= \min\{(1 - \gamma_A(x)), (1 - \gamma_A(y))\} \\ &= \min\{\overline{\gamma_A}(x), \overline{\gamma_A}(y)\}, \text{ and}\end{aligned}$$

$$\begin{aligned}\overline{\gamma_A}(xay) &= 1 - \gamma_A(xay) \\ &\geq 1 - \max\{\gamma_A(x), \gamma_A(y)\} \\ &= \min\{(1 - \gamma_A(x)), (1 - \gamma_A(y))\} \\ &= \min\{\overline{\gamma_A}(x), \overline{\gamma_A}(y)\}.\end{aligned}$$

Hence  $\overline{\gamma_A}$  is a fuzzy bi-ideal of  $S$ .

Conversely, suppose that  $\mu_A$  and  $\overline{\gamma_A}$  are fuzzy bi-ideals of  $S$ . Let  $a, x, y \in S$ . Then

$$\begin{aligned}1 - \gamma_A(xy) &= \overline{\gamma_A}(xy) \geq \min\{\overline{\gamma_A}(x), \overline{\gamma_A}(y)\} \\ &= \min\{(1 - \gamma_A(x)), (1 - \gamma_A(y))\} \\ &= \max\{\gamma_A(x), \gamma_A(y)\}, \text{ and}\end{aligned}$$

$$\begin{aligned}1 - \gamma_A(xay) &= \overline{\gamma_A}(xay) \\ &\geq \min\{\overline{\gamma_A}(x), \overline{\gamma_A}(y)\} \\ &= 1 - \max\{\gamma_A(x), \gamma_A(y)\}\end{aligned}$$

which imply that  $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$  and  $\gamma_A(xay) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ . This completes the proof.  $\square$

**Corollary 3.14** *An IFS  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy bi-ideal of  $S$  if and only if  $\square A = (\mu_A, \overline{\mu_A})$  and  $\diamond A = (\overline{\gamma_A}, \gamma_A)$  are intuitionistic fuzzy bi-ideals of  $S$ .*

**Proof.** It is straightforward by Theorem 3.14.  $\square$

Let  $f$  be a map from a set  $X$  to a set  $Y$ . If  $A = (\mu_A, \gamma_A)$  and  $B = (\mu_B, \gamma_B)$  are IFSs in  $X$  and  $Y$  respectively, then the *preimage* of  $B$  under  $f$ , denoted by  $f^{-1}(B)$ , is an IFS in  $X$  defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

**Theorem 3.15** *Let  $f : S \rightarrow T$  be a homomorphism of semigroups. If  $B = (\mu_B, \gamma_B)$  is an intuitionistic fuzzy bi-ideal of  $T$ , then the preimage  $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$  of  $B$  under  $f$  is an intuitionistic fuzzy bi-ideal of  $S$ .*

**Proof.** Assume that  $B = (\mu_B, \gamma_B)$  is an intuitionistic fuzzy bi-ideal of  $T$  and let  $x, y, \in S$ . Then

$$\begin{aligned} f^{-1}(\mu_B)(xy) &= \mu_B(f(xy)) \\ &= \mu_B(f(x)f(y)) \\ &\geq \min\{\mu_B(f(x)), \mu_B(f(y))\} \\ &= \min\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(y))\}, \text{ and} \end{aligned}$$

$$\begin{aligned} f^{-1}(\gamma_B)(xy) &= \gamma_B(f(xy)) \\ &= \gamma_B(f(x)f(y)) \\ &\leq \max\{\gamma_B(f(x)), \gamma_B(f(y))\} \\ &= \max\{f^{-1}(\gamma_B(x)), f^{-1}(\gamma_B(y))\}. \end{aligned}$$

Hence  $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$  is an intuitionistic fuzzy subsemigroup of  $S$ . For any  $a, x, y \in S$  we have

$$\begin{aligned} f^{-1}(\mu_B)(xay) &= \mu_B(f(xay)) \\ &= \mu_B(f(x)f(a)f(y)) \\ &\geq \min\{\mu_B(f(x)), \mu_B(f(y))\} \\ &= \min\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(y))\} \text{ and} \end{aligned}$$

$$\begin{aligned}
f^{-1}(\gamma_B)(xay) &= \gamma_B(f(xay)) \\
&= \gamma_B(f(x)f(a)f(y)) \\
&\leq \max\{\gamma_B(f(x)), \gamma_B(f(y))\} \\
&= \max\{f^{-1}(\gamma_B(x)), f^{-1}(\gamma_B(y))\}.
\end{aligned}$$

Therefore  $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$  is an intuitionistic fuzzy bi-ideal of  $S$ .  $\square$

### Acknowledgements

This thesis has been funded by Research Fund of Chungju National University in 2003.

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Received 16.03.2004