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Abstract

We consider hypersurfaces $S \subset \mathbb{R}^3$ with zero Gaussian curvature at every ordinary point with surface measure dS and define the surface measure $d\mu = \psi(x)dS(x)$ for smooth function ψ with compact support. We obtain uniform estimates for the Fourier transform of measures concentrated on such hypersurfaces. We show that due to the damping effect of the surface measure the Fourier transform decays faster than $O(|\xi|^{-1/h})$, where h is the height of the phase function. In particular, Fourier transform of measures supported on the exceptional surfaces decays in the order $O(|\xi|^{-1/2})$ (as $|\xi| \to +\infty$).

Key Words: Oscillatory Integrals, oscillation index, singular hypersurfaces, curvature.

1. Introduction

It is well-known that the L^p−estimates of the maximal operators associated to hypersurfaces in Euclidean spaces are strongly related to the decay of the Fourier transform of measures carried on S, i.e. to oscillatory integrals of the form

$$d\hat{\mu}(\xi) = \int_S e^{i(\xi,x)}\psi(x)dS(x), \tag{1.1}$$

where $d\mu = \psi(x)dS(x)$ is a compactly supported density on S, (x,ξ) is the inner product of the vectors x and ξ.

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The decay of the oscillatory integral (1.1) as $|\xi| \to \infty$, in return, is connected to geometric properties of the surface S and may be very complicated depending on the direction of ξ. The problem on the decay of such oscillatory integrals has been considered by various authors, including van Der Corput [25], E. Hlawka [8], C.S. Hertz [7], W. Littman [14], B. Randol [17], [18], I. Svenson [24], A. Varchenko [26], C.D. Sogge, E.M. Stein [23], J.J. Duistermaat [5], Colin de Verdier [3]. We refer the reader to [23] for references, also to results on maximal operators associated to surfaces.

On the other hand some problems of mathematical physics are connected to uniform estimates of the oscillatory integrals (1.1) [21].

An optimal uniform estimates for oscillatory integrals (1.1) in the case $dim(S) = 1$ were obtained by B. Randol [17], and for analytic hypersurfaces in the case $dim(S) = 2$ were obtained by A.N. Varchenko and V.N. Karpushkin [13], [26]. The optimal estimates based on decomposition of the phase function were obtained by H. Schulz [19] (see also [18], [12]) in the case of convex smooth finite type hypersurfaces.

In this paper we consider the problem on a behavior of $d\hat{\mu}(\xi)$ in a special case, when $S \subset \mathbb{R}^3$ has zero Gaussian curvature at every ordinary point. Such hypersurfaces may have singularities.

It is well-known that the hypersurfaces in $\mathbb{R}^3$ with zero Gaussian curvature in general may be cylindric, cone or ruled surface. We define ruled surface as a tangent space to a space curve [22].

Following [9], [10] we define ruled surfaces. Let $\gamma : (\mathbb{R}, 0) \mapsto \mathbb{R}^3$ be a germ of C^∞ parametrized space curve at the origin of $\mathbb{R}$. Representing γ as
$x(t) = (x_1(t), x_2(t), x_3(t))$, we say that γ is of finite type at 0 if the infinite number of vectors $x'(0)$, $x''(0), \ldots$ generate the three dimensional space. Then for some affine coordinates and for some positive integers m, n, k the curve γ is written in the form

$$x_1(t) = t^m g_1(t), \quad x_2(t) = t^{m+n} g_2(t), \quad x_3(t) = t^{m+n+k} g_3(t),$$

where g_1, g_2, g_3 are smooth functions and $g_1(0) = g_2(0) = g_3(0) = 1$. The triplet $(m, m+n, m+n+k)$ is independent of the choice of affine local coordinates, and is called the type of the curve-germ γ : $typ(\gamma) = (m, m+n, m+n+k)$. Notice that if a curve germ does not have an infinite tangency with any affine plane, then it is of finite type.

A type of space curve-germ is called smoothly determinative (respectively topologically determinative) if it determines the tangent developable up to local diffeomorphism (respectively local homeomorphism).

Recently, the list of developable surfaces has been given by G. Ishikawa [9],[10], continuing the results of O.P. Shcherbak [19]. We consider estimates for Fourier transform of Borel's measures associated to hypersurfaces with zero Gaussian curvature. In this case there is a finite list of smoothly determinative singularities, namely $(1, 2, 2+k)(k \geq 1)$ type surfaces and exceptional surfaces of types $(2, 3, 4)$, $(1, 3, 4)$, $(3, 4, 5)$ and $(1, 3, 5)$. It is interesting that, although the phase function associated to exceptional singularities has degenerate singularities depending on their types, oscillatory integrals with this phase function decays faster due to the damping effect of the surface measure.

More precisely, it should be noted that in the case $(1, 2, 2+k)$, the optimal form of decay is defined by "height" h of the phase function and it has the form $O(|\xi|^{-\frac{1}{h}})$. But in the exceptional cases the decay of oscillatory integrals more faster than $O(|\xi|^{-\frac{1}{h}})$.

The main results of this paper are the following statements.

Theorem 3.1 *Let γ be a smooth curve-germ at zero of type $(m, m+n, m+n+k)$. Then there exists a neighborhood U of the origin such that for any $\psi \in C_0^\infty(U)$ the estimate*

$$|J(\xi)| \leq \frac{c\|\psi\|_{C^1}}{(1+|\xi|)^\beta}$$

holds, where $\beta = \min\{\frac{1}{2}, \frac{n}{n+k}\}$ and $\|\cdot\|_{C^1}$ is the norm of the space of continuous differentiable functions.

Note that the exponent β in the Theorem 3.1 is optimal.

Corollary 3.2 *1) If S is a developable hypersurface of type $(1, 2, 2+k)(k \geq 1)$. then for the oscillatory integral J the estimate*

$$|J(\xi)| \leq \frac{C\|\psi\|_{C^1}}{(1+|\xi|)^{\frac{1}{k+1}}}$$

holds;
2) If the hypersurface has one of the types $(2, 3, 4)$, $(1, 3, 4)$, $(3, 4, 5)$, or $(1, 3, 5)$ then for any $\psi \in C_0^\infty(\mathbb{R}^3)$ the following estimate

$$|J(\xi)| \leq \frac{C\|\psi\|_{C^1}}{(1+|\xi|)^{1/2}}$$

holds.

Note that in the case of hypersurfaces of types $(2,3,4)$ and $(3,4,5)$, the "height" h of phase functions is 2 by Varchenko terminology . But in the case $(1,3,4)$ $\quad h=3$, and in the other case $(1,3,5)$ $\quad h=4$. Nevertheless, the associated oscillatory integrals decay in the order $O(|\xi|^{-\frac{1}{2}})$(as $|\xi|\to\infty$).

The paper is organized as follows. In Section 2 we consider estimates for oscillatory integrals and also we consider some auxiliary statements about Lebesgue measure of sublevel sets. In Section 3 we consider some applications of results of the Section 2. In particular, we prove our main theorem. The Section 4 is devoted to estimates for Fourier transform of measures supported on the cone surfaces.

2. Estimates for Oscillatory Integrals with Smooth Phases

In this section we consider oscillatory integrals having the form

$$J(\lambda,s)=\int_{\mathbb{R}^2} e^{i\lambda(yp(x,s)+g(x,s))}|y|a(x,y,s)dxdy,$$

where $p(x,s)$ is the polynomial function in the form

$$p(x,s)=x^{n+1}+s_1x^{n-1}+\cdots+s_{n-1}x+s_n,$$

where $g(x,s)$ is a smooth function and a is a smooth function with compact support.

Theorem 2.1 *Let $U\times V$ be a bounded neighborhood of the origin in $\mathbb{R}^2\times\mathbb{R}^n$. There exists a constant C such that for any $a\in C_0^\infty(U\times V)$ the estimate*

$$|J(\lambda,s)|\le\frac{C\|a(.,s)\|_{C^1(U)}}{|\lambda|^{\frac{1}{n+1}}}$$

holds.

If g is an analytic function and the amplitude function is smooth then the result follows from Karpushkin's theorem on uniform estimates for two-dimensional oscillatory integrals [13]. We give more elementary proof of Theorem 2.1 for a wider class of amplitude functions.

Lemma 2.2 There exists a constant C such that for any $\lambda\neq 0$ and $s\in V\subset\mathbb{R}^n$ the following inequality

$$J_1=\int_{\{x:\,|\lambda p(x,s)|>1\}}\frac{dx}{|p(x,s)|}\le C|\lambda|^{\frac{n}{n+1}}$$

holds.

Proof of Lemma 2.2 We prove Lemma 2.2 by the induction method over n. If $n = 1$ then we deal with the integral

$$\int_{\{x:\,|\lambda(x^2+s_1)|>1\}} \frac{dx}{|x^2+s_1|}.$$

Let's note that if $s_1 \geq 0$ then we have

$$\int_{\{x:\,|\lambda(x^2+s_1)|>1\}} \frac{dx}{|x^2+s_1|} \leq \int_{\{x:\,|\lambda x^2|>1\}} \frac{dx}{x^2} = 2|\lambda|^{1/2}.$$

Further, assume that $s_1 < 0$. In this case we use the change of variables $x = |s_1|^{1/2}y$ and obtain

$$\int_{\{x:\,|\lambda(x^2+s_1)|>1\}} \frac{dx}{|x^2+s_1|} = \frac{1}{|s_1|^{1/2}} \int_{\{y:\,|\lambda s_1(y^2-1)|>1\}} \frac{dy}{|y^2-1|}.$$

First, we consider the case $|\lambda s_1| < \frac{1}{3}$. If $|\lambda s_1| < \frac{1}{3}$ then, straightforward computations show that

$$\frac{1}{|s_1|^{1/2}} \int_{\{y:\,|\lambda s_1(y^2-1)|>1\}} \frac{dy}{|y^2-1|} \leq 4|\lambda|^{1/2}.$$

Suppose $\frac{1}{3} < |\lambda s_1| < 4$. In this case the last integral has an upper bound $c|s_1|^{-1/2}$. This bound gives a required estimate for our integral.

Finally, we consider the case $|\lambda s_1| > 4$. Then it is easy to show that the following estimate

$$\int_{\{y:\,|\lambda s_1(y^2-1)|>1\}} \frac{dy}{|y^2-1|} \leq C \log(|\lambda s_1|)$$

holds. From the last inequality the required estimate follows immediately. Thus, we obtain the inequality

$$\int_{\{x:\,|\lambda(x^2+s_1)|>1\}} \frac{dx}{|x^2+s_1|} \leq C|\lambda|^{1/2}.$$

In other words, we get the conclusion of Lemma 2.2 in the case $n = 1$.

From now, we suppose that $n \geq 2$ and the conclusion of Lemma 2.2 is fulfilled for any $k \leq n-1$. We shall prove it for the case $k = n$.

First, we introduce a number ρ defined by

$$\rho := \rho(s) = |s_1|^{\frac{n+1}{2}} + |s_2|^{\frac{n+1}{3}} + \cdots + |s_n|.$$

Let us use a change of variables $x = \rho^{\frac{1}{n+1}} y$ in the integral J_1 and obtain

$$J_1 = \frac{1}{\rho^{\frac{n}{n+1}}} \int_{\{y:\, |\lambda\rho p(y,\xi)|>1\}} \frac{dy}{|p(y,\xi)|},$$

where $\xi_k = \frac{s_k}{\rho^{\frac{k+1}{n+1}}}$, $k = \overline{1,n}$.

We introduce the so-called quasisphere Σ by $\Sigma := \{s \in \mathbb{R}^n : \rho(s) = 1\}$. So, $\xi \in \Sigma$.

Since Σ is a compact set there exists a number N and a function $\varphi(y,\xi)$ defined on the set $\{|y| > N\} \times \Sigma$ such that $p(y,\xi) = y^{n+1}\varphi(y,\xi)$. Moreover, there exist positive numbers C_1, C_2 such that for any $(y,\xi) \in \{|y| > N\} \times \Sigma$ the inequalities $C_1 \leq \varphi(y,\xi) \leq C_2$ hold. Note that if ε is a sufficiently small positive number then we have the inclusion $\{y : \varepsilon|p(y,\xi)| > 1\} \subset \{|y| > N\} \times \Sigma$.

Consequently, if ε is a sufficiently small positive real number, then for $|\lambda\rho| < \varepsilon$ we get the estimate

$$\int_{\{|y:\, \lambda\rho p(y,\xi)|>1\}} \frac{dy}{|p(y,\xi)|} \leq \int_{\{y:\, C_1|\lambda\rho y^{n+1}|>1\}} \frac{dy}{C_1|y^{n+1}|} \leq C|\lambda\rho|^{\frac{n}{n+1}}.$$

Thus, in this case we have the required upper bound for the integral J_1.

Now, we consider the case $0 < \varepsilon < |\lambda\rho| < M$, where M is a fixed positive number. Then there exists a positive number $C(M,n)$ such that the inequality

$$\int_{\{y:\, |\lambda\rho p(y,\xi)|>1\}} \frac{dy}{|p(y,\xi)|} \leq C(M,n)$$

holds. Consequently, the estimate

$$\frac{1}{\rho^{\frac{n}{n+1}}} \int_{\{y:\, |\lambda\rho p(y,\xi)|>1\}} \frac{dy}{|p(y,\xi)|} \leq C|\lambda|^{\frac{n}{n+1}}$$

is fulfilled. Finally, we consider the case $|\lambda\rho| > M$, where M is a sufficiently large fixed positive real number. We fix $\xi = \xi^0 \in \Sigma$. Assume that the polynomial $p(y,\xi)$ has some real roots $y_1, \ldots, y_l$ with multiplicities $k_1, \ldots, k_l$ satisfying the conditions $k_j \leq n$ for

$j=\overline{1,l}$, otherwise the conclusion of Lemma 2.2 is obvious for some neighborhood of the point ξ^0.

Due to the Weierstrass-Malgrange Theorem [15], there exist a neighborhood V of ξ^0 and neighborhoods $U_1,\dots,U_l$ of points $y_1,\dots y_l$ such that in $U_j\times V$ we have the factorization

$$p(y,\xi)=p_j(y,\xi)Q_j(y,\xi),$$

where $p_j(y,\xi)=(y-y_j)^{k_j}+\eta_1(\xi)(y-y_j)^{k_j-1}+\cdots+\eta_{k_j}(\xi)$ is a pseudopolynomial the coefficients of which are real analytic vanishing at ξ_0 functions. Q_j is a real analytic function satisfying the condition $|Q_j(y,\xi)|\geq\delta>0$ for any $(y,\xi)\in U_j\times V$.

We can choose a neighborhood V of ξ^0 such that $p(y,\xi)\neq 0$ for any $(y,\xi)\in(\mathbb{R}\setminus\{\cup_{j=1}^{l}U_j\})\times V$.

Therefore, there exists a number C such that for any $\xi\in V$ the following estimate

$$\int_{\mathbb{R}\setminus(\cup_{j=1}^{l}U_j)}\frac{dy}{|p(y,\xi)|}\leq C$$

holds.

On the other hand, due to the induction hypothesis, we have

$$\int_{U_j\cap\{y:\,|\lambda\rho p(y,\xi)|>1\}}\frac{dy}{|p(y,\xi)|}\leq\int_{U_j\cap\{y:\,|\lambda\delta\rho p_j(y,\xi)|>1\}}\frac{dy}{|\delta p_j(y,\xi)|}\leq C(\delta)|\lambda\rho|^{\frac{k_j}{k_j+1}}\leq C|\lambda\rho|^{\frac{n}{n+1}}.$$

Thus for any $\xi\in V(\xi^0)$ we obtain the estimate

$$\frac{1}{\rho^{\frac{n}{n+1}}}\int_{\{y:\,|\lambda\rho p(y,\xi)|>1\}}\frac{dy}{|p(y,\xi)|}\leq C|\lambda|^{\frac{n}{n+1}}.$$

Since Σ is a compact set by standard arguments we get the last estimate on the quasisphere.

This proves the Lemma 2.2.

Now we prove an analog of Lemma 2.2 for special type of functions. Let $p(x,s_1,s_2)$ be a function defined by

$$p(x,s_1,s_2)=x^{r_1}+s_1xg_1(x^r)+s_2g_2(x^r),$$

with domain $\mathbb{R}_+$, where $g_1(y)$, $g_2(y)$ are smooth bounded functions on $\mathbb{R}$, satisfying the condition $g_1(0)=g_2(0)=1$, and r_1, r are fixed positive real numbers, $r_1>1$.

Lemma 2.3 There exist a positive number δ and C such that the estimate

$$I = \int_{\{x \in \mathbb{R}_+ :\, \lambda|p(x,s_1,s_2)|>1\}} \frac{dx}{|p(x,s_1,s_2)|} \le C\lambda^{\beta}$$

holds, for any $\lambda > 2$ and $s \in \{s : |s| < \delta\}$, where $\beta = \max\{\frac{1}{2}, \frac{r_1-1}{r_1}\}$.

Proof of Lemma 2.3

First, we introduce a quasidistance $\rho(s) = |s_1|^{\frac{r_1}{r_1-1}} + |s_2|$ and the quasisphere $\Sigma_1 = \{s \in \mathbb{R}^2 :\, \rho(s) = 1\}$. Let's use a change of variables: $x = \rho^{\frac{1}{r_1}} y$. Then we obtain

$$I = \rho^{\frac{1-r_1}{r_1}} \int_{\{y \in \mathbb{R}_+ :\, |\lambda\rho\tilde{p}(y,\sigma_1,\sigma_2,\rho)|>1\}} \frac{dy}{|\tilde{p}(y,\sigma_1,\sigma_2,\rho)|},$$

where

$$\tilde{p}(y,\sigma_1,\sigma_2,\rho) = y^{r_1} + \sigma_1 y g_1(\rho^{\frac{r}{r_1}} y^r) + \sigma_2 g_2(\rho^{\frac{r}{r_1}} y^r), \quad \sigma_1 = s_1\rho^{\frac{1-r_1}{r_1}},\ \sigma_2 = \frac{s_2}{\rho},\ \sigma := (\sigma_1, \sigma_2).$$

Note that $\sigma \in \Sigma_1$. Since g_1, g_2 are bounded functions, there exist a positive number $N > 0$ and a function $\varphi(y,\sigma,\rho)$ such that for any $y > N$ and $\sigma \in \Sigma_1$, $\rho \in \mathbb{R}_+$ the identity

$$\tilde{p}(y,\sigma_1,\sigma_2,\rho) = y^{r_1}\varphi(y,\sigma_1,\sigma_2,\rho)$$

holds. Moreover, φ is essentially constant function on that set, i.e. there exist positive real numbers c_1, c_2 such that the inequalities $c_1 \le \varphi(y,\sigma_1,\sigma_2,\rho) \le c_2$ hold for any $(y,\sigma,\rho) \in (N,+\infty) \times \Sigma_1 \times \mathbb{R}_+$.

Let N be a fixed real number with above-mentioned property. There exists a positive number $\varepsilon > 0$ such that the inclusion

$$\{(y,\sigma,\rho) : \varepsilon|\tilde{p}(y,\sigma_1,\sigma_2,\rho)| > 1\} \subset (N,+\infty) \times \Sigma_1 \times \mathbb{R}_+$$

is fulfilled.

Now, we consider the case $|\lambda|\rho < \varepsilon$. Then

$$I = \rho^{\frac{1-r_1}{r_1}} \int_{\{y \in \mathbb{R}_+ :\, |\lambda\rho\tilde{p}(y,\sigma_1,\sigma_2,\rho)|>1\}} \frac{dy}{|\tilde{p}(y,\sigma_1,\sigma_2,\rho)|} \le \rho^{\frac{1-r_1}{r_1}} \int_{\{y :\, c_1|\lambda|\rho y^{r_1}>1\}} \frac{dy}{c_1 y^{r_1}} \le c|\lambda|^{\frac{r_1-1}{r_1}}.$$

If M is any fixed real number and $\varepsilon < \lambda\rho < M$ then the integral I has an upper bound $c\rho^{\frac{1-r_1}{r_1}}$. This upper bound implies the estimate $I \le c|\lambda|^{\frac{r_1-1}{r_1}}$.

Finally, we consider the case $|\lambda|\rho > M$, where M is a sufficiently large fixed positive number. We divide the set of integration into two parts and consider two integral over that sets:

$$I_1 = \rho^{\frac{1-r_1}{r_1}} \int_{(N,\delta\rho^{\frac{-1}{r_1}})\cap\{y:\, |\lambda\rho\tilde{p}(y,\sigma_1,\sigma_2,\rho)|>1\}} \frac{dy}{|\tilde{p}(y,\sigma_1,\sigma_2,\rho)|}$$

and

$$I_2 = \rho^{\frac{1-r_1}{r_1}} \int_{(0,N)\cap\{y:\, |\lambda\rho\tilde{p}(y,\sigma_1,\sigma_2,\rho)|>1\}} \frac{dy}{|\tilde{p}(y,\sigma_1,\sigma_2,\rho)|}.$$

Then we write the integral I as a sum of the two integrals: $I = I_1 + I_2$.

The integral I_1 can be estimate by $c\rho^{\frac{1-r_1}{r_1}}$; therefore it satisfies a required inequality. Take a positive real number Δ and introduce the new integral

$$I_{21} = \rho^{\frac{1-r_1}{r_1}} \int_{(0,\Delta)\cap\{y:\, \lambda\rho|\tilde{p}(y,\sigma_1,\sigma_2,\rho)|>1\}} \frac{dw}{|\tilde{p}(y,\sigma_1,\sigma_2,\rho)|}.$$

Now we fix $\sigma = \sigma^0 \in \Sigma_1$. If $\sigma_1^0 \neq 0$ then we can choose Δ and a neighborhood $V(\sigma^0)$ such that the condition $\tilde{p}(y,\sigma_1,\sigma_2,\rho) \neq 0$ is fulfilled for any $(y,\sigma) \in (0,\Delta)\times V(\sigma^0)$. Therefore, the integral I_{21} is bounded by $c\rho^{\frac{1-r_1}{r_1}}$. Hence, it satisfies the required inequality. Let $\sigma = \sigma^0 \in \Sigma_1$ be a fixed point and $\sigma_1^0 = 0$. Then $\sigma_2^0 = \pm 1$ because $\rho(\sigma^0) = 1$. In this case we can use a change of variables:

$$z = \frac{\tilde{p}(y,\sigma_1,\sigma_2,\rho)}{g_2(\rho^{\frac{r}{r_1}} y^r)}.$$

The last fraction is a continuous differentiable function in a small neighborhood of the origin and it is invertible.

Then it is easy to show that the following estimate

$$I_{21} \leq c\rho^{\frac{1-r_1}{r_1}} log(\rho\lambda)$$

holds for the integral I_{21}. The last inequality gives a right estimate for the integral I_{21}.

Finally, we consider the function $\tilde{p}(y,\sigma_1,\sigma_2,\rho)$ on the set $[\Delta, N]$. Let $\sigma^0 \in \Sigma$ be a fixed point. Note that the function $\tilde{p}(y,\sigma_1,\sigma_2,\rho)$ can be considered as a smooth deformation of the function $\tilde{p}(y,\sigma_1^0,\sigma_2^0,0) = y^{r_1} + \sigma_1^0 y + \sigma_2^0$ in a small neighborhood of its roots belonging to the set $[\Delta, N]$. The function $y^{r_1} + \sigma_1^0 y + \sigma_2^0$ has no roots of multiplicity greater than 2 in that interval. Therefore, we can use Malgrange preparation theorem [15] to the function $\tilde{p}(y,\sigma_1,\sigma_2,\rho)$ and have the factorization $\tilde{p}(y,\sigma_1,\sigma_2,\rho) = g(y,\sigma,\rho)p_2(y,\sigma,\rho)$, where $g(y,\sigma,\rho)$ is a smooth nonzero function and $p_2(y,\sigma,\rho)$ is the polynomial function of the second order with respect to variable y. Its coefficients are smooth functions of $(\sigma, \rho^{\frac{r}{r_1}})$.

Now, we can use Lemma 2.2 and have

$$|I_2 - I_{21}| \leq \frac{C|\lambda\rho|^{\frac{1}{2}}}{\rho^{\frac{1-r_1}{r_1}}} \leq C|\lambda|^{\beta}.$$

Lemma 2.3 is proved. □

The following lemma is needed for the sequel.

Lemma 2.4 There exist a positive number δ and a constant c such that the estimate

$$\mu(\{x > 0 : |p(x,s_1,s_2)| < h\}) \leq ch^{\beta}$$

holds for any $h > 0$ and $|s| < \delta$, where $\beta = \min\{\frac{1}{2}, \frac{1}{r_1}\}$ and $\mu(\{x > 0 : |p(x,s_1,s_2)| < h\})$ is the Lebesgue measure of the set $\{x > 0 : |p(x,s_1,s_2)| < h\}$.

Lemma 2.4 can be proved as Lemma 2.3.

Lemma 2.5 Let g_1, g_2, g_3 be smooth functions and $g_1(0) = g_2(0) = g_3(0) = 1$. There exist a constant δ and C such that for any $\eta \in S^2$ and $\lambda > 2$ the following estimate

$$\mu((0,\delta) \cap \{t : |\eta_3 t^{\frac{n+k}{n}} g_3(t^{\frac{1}{n}}) + \eta_2 t g_2(t^{\frac{1}{n}}) + \eta_1 g_1(t^{\frac{1}{n}})| < \lambda^{-1}\}) \leq C\lambda^{-\beta_1}$$

holds, where $\beta_1 = \min\{\frac{1}{2}, \frac{k}{n+k}\}$ and S^2 is the unit sphere centered at the origin of $\mathbb{R}^3$.

Lemma 2.5 follows from Lemma 2.4.

The following lemma is needed to prove the Theorem 2.1 ([2], [4]).

Lemma 2.6 Let F be $n-$times differentiable function on $I = [a,b]$ and $|f^{(n)}(x)| \geq 1$ for any $x \in I$. Then there exists a constant $C(n)$ (depending only on n) such that the estimate

$$\mu(\{x \in I : |f(x)| < h\}) \leq C(n)h^{1/n}$$

holds.

Proof of Theorem 2.1 Note that, due to Lemma 2.6, measure of the set $\{x : |\lambda p(x,s)| \leq 1\}$ is estimated by $C(n)\lambda^{-\frac{1}{n+1}}$ for any $s \in \mathbb{R}^n$.

Thus, by using Lemma 2.2 and 2.6 for the oscillatory integral $J(\lambda, s)$ we have

$$|J(\lambda,s)| \leq \int_{\{x:\, |\lambda p(x,s)|\leq 1\}} |ya(x,y,s)|dxdy + |\int_{\{x:\, |\lambda p(x,s)|>1\}} e^{i\lambda g(x,s)}dx \times$$

$$\int_{\mathbb{R}} e^{i\lambda p(x,s)y}|y|a(x,y,s)dy| \leq C|\lambda|^{-\frac{1}{n+1}}\|a\|_{C(U)} + \int_{\{x:\, |\lambda p(x,s)|>1\}} \frac{\|a(x,.,s)\|_{C^1}dx}{|p(x,s)|}.$$

Now, we use the inequality

$$\sup_x \|a(x,.,s)\|_{C^1} \leq \|a(.,.,s)\|_{C^1(U)}.$$

Then due to Lemma 2.2 we have

$$\int_{\{x:\, |\lambda p(x,s)|>1\}} \frac{\|a(x,.,s)\|_{C^1}dx}{|p(x,s)|} \leq \|a(.,.,s)\|_{C^1(U)} \int_{\{x:\, |\lambda p(x,s)|>1\}} \frac{dx}{|p(x,s)|}$$

$$\leq C|\lambda|^{\frac{-1}{n+1}}\|a(.,s)\|_{C^1(U)}.$$

The last estimate establishes Theorem 2.1. □

3. Estimates for Fourier Transform of Measures Supported on Ruled Surfaces

Let γ be a space curve-germ at the origin of $\mathbb{R}$and S be a surface which is the tangent developable of the curve. Consider the measure $d\mu = \psi(x)dS(x)$ and its Fourier transform $J(\xi) := d\hat{\mu}(\xi)$.

Theorem 3.1 *Let γ be a smooth curve-germ at zero of type $(m, m+n, m+n+k)$. Then there exists a neighborhood U of the origin such that for any $\psi \in C_0^\infty(U)$ the estimate*

$$|J(\xi)| \leq \frac{c\|\psi\|_{C^2}}{(1+|\xi|)^\beta},$$

holds, where $\beta = \min\{\frac{1}{2}, \frac{n}{n+k}\}$.

Proof. If γ is a curve germ of type $(m, m+n, m+n+k)$, then the surface defined by the curve has the form

$$x_1(t,v) = t^m g_1(t) + (mg_1(t) + tg_1'(t))v,\ x_2(t,v) = t^{m+n} g_2(t) + t^n((m+n)g_2(t) + g_2'(t))v,$$

$$x_3(t,v) = t^{m+n+k} g_3(t) + ((m+n+k)t^{n+k} g_3(t) + t^{n+k+1} g_3'(t))v.$$

Straightforward computations show that $|x_v \wedge x_t| = |vt^{n-1}|g(t)$, where $x_v \wedge x_t$ is an exterior product of the vectors $x_v,\ x_t$ and $g(t)$ is a smooth function, $g(0) = mn(m+n) \neq 0$.

We define the function $p(\eta, t)$ by the relation

$$p(\eta,t) = \eta_1(mg_1(t) + tg_1'(t)) + \eta_2((m+n)t^n g_2(t) + t^{n+1} g_2'(t)) +$$

$$\eta_3((m+n+k)t^{n+k} g_3(t) + t^{n+k+1} g_3'(t)),$$

where $\eta \in S^2$ is a unit vector.

Note that due to Lemma 2.5 we get

$$\mu((0,\delta) \cap \{t : |\eta_3 t^{\frac{n+k}{n}} g_3(t^{\frac{1}{n}}) + \eta_2 t g_2(t^{\frac{1}{n}}) + \eta_1 g_1(t^{\frac{1}{n}}))| < \lambda^{-1}\}) \leq C\lambda^{-\beta}.$$

We write the integral $J(\xi)$ as a sum of two integrals: $J(\xi) = J_-(\xi) + J_+(\xi)$, where

$$J_+(\xi) = \int_D e^{i(\xi, x(t,v))} \psi(x(t,v)) |x_t(t,v) \wedge x_v(t,v)| \theta(t) dt dv, \quad J_-(\xi) := J(\xi) - J_+(\xi),$$

θ is the Heaviside's function. We consider estimates for the integral $J_+(\xi)$. The integral $J_-(\xi)$ can be estimated analogously:

$$|J_+(\xi)| \leq \int_{\{|\xi||\eta_3 t^{\frac{n+k}{n}} g_3(t^{\frac{1}{n}}) + \eta_2 t g_2(t^{\frac{1}{n}}) + \eta_1 g_1(t^{\frac{1}{n}})| < 1\}} |\psi(t^{\frac{1}{n}}, v)| dt dv +$$

$$|\xi|^{-1} \int_{\{|\xi||\eta_3 t^{\frac{n+k}{n}} g_3(t^{\frac{1}{n}}) + \eta_2 t g_2(t^{\frac{1}{n}}) + \eta_1 g_1(t^{\frac{1}{n}})| > 1\}} \frac{\|\psi(u_1^{\frac{1}{n}}, .)\|_{C^1} du}{|\eta_3 t^{\frac{n+k}{n}} g_3(t^{\frac{1}{n}}) + \eta_2 t g_2(t^{\frac{1}{n}}) + \eta_1 g_1(t^{\frac{1}{n}})|}.$$

Due to the Lemma 2.5 we obtain

$$|\xi|^{-1} \int_{\{|\xi||4\eta_3 u_1^{3/2} + 3\eta_2 w + \eta_1| > 1\}} \frac{\|\psi(u_1^{1/n}, .)\|_{C^1} du_1}{|4\eta_3 u_1^{3/2} + 3\eta_2 w + \eta_1|} \leq \frac{C\|\psi\|_{C^1(D)}}{|\xi|^{\beta}}.$$

We remind that $1-\beta_1=\beta$. Thus we get the estimate

$$|J_+(\xi)| \le \frac{C\|\psi\|_{C^1(D)}}{|\xi|^\beta}.$$

From this inequality we obtain a proof of Theorem 3.1. □

Corollary 3.2 *1) If S is a developable hypersurface of type $(1,2,2+k)(k\ge 1)$ then for the oscillatory integral J the estimate*

$$|J(\xi)| \le \frac{C\|\psi\|_{C^1}}{(1+|\xi|)^{\frac{1}{k+1}}}$$

holds;
2) Let the hypersurface has one of the types $(2,3,4)$, $(1,3,4)$, $(3,4,5)$, or $(1,3,5)$ then for any $\psi\in C_0^\infty(\mathbb{R}^3)$ the estimate

$$|J(\xi)| \le \frac{C\|\psi\|_{C^1}}{(1+|\xi|)^{1/2}}$$

holds.

Let $S\subset\mathbb{R}^3$ be a hypersurface with zero Gaussian curvature at any ordinary point. Let $H:=\frac{k_1+k_2}{2}$ be a mean curvature of the surface, where k_1, k_2 are principal curvatures of the surface. It is a smooth function defined on the set of ordinary (non-singular) points of the hypersurface. We assume that H is a finite type function and its type no greater than n at the non-singular point $x^0\in S$. A type of the function H at the point x^0 is defined as a minimal non-negative integer number n such that $d^nH(x^0)\neq 0$.

Theorem 3.3 *Let $S\subset\mathbb{R}^3$ be a hypersurface with zero Gaussian curvature at any ordinary point and the function H has a type n at the ordinary point $x^0\in S$. There exists a neighborhood U of the point x^0 such that for any $\psi\in C_0^\infty(U)$ the associated integral $J(\xi)$ has the estimate*

$$|J(\xi)| \le \frac{c\|\psi\|_{L_2^1(U)}}{(1+|\xi|)^{\frac{1}{n+2}}}.$$

The following Lemma is needed for the sequel [6]

Lemma 3.4 If x^0 is an ordinary point of the hypersurface then the curvature lines passing through that point are smooth curves.

Proof of Theorem 3.3 Let $x^0 \in S$ be an ordinary point and $\rho = \rho(u)$ be the orthogonal curve passing from the point x^0 and $\mathbf{e} = \mathbf{e}(u)$ be the direction of generators [22]. Without loss of generality we may assume that u is the natural parameter. Then for the surface we have

$$\mathbf{r} = \mathbf{r}(u, v) = \rho(u) + v\mathbf{e}(u).$$

It is well-known that if $\rho = \rho(u)$ is orthogonal to $\mathbf{e} = \mathbf{e}(u)$ for any u then $\mathbf{e}(u)$ can be written as

$$\mathbf{e}(u) = \cos(c - \int_0^u \kappa(\tau)d\tau)\mathbf{n}(u) + \sin(c - \int_0^u \kappa(\tau)d\tau)\mathbf{b}(u),$$

where $\mathbf{n}$ is the principal normal, $\mathbf{b}$ is the binormal and κ is a torsion of the curve $\rho = \rho(u)$, and c is a constant depending only on the surface.

It is easy to show that $\rho = \rho(u)$ is a line of curvature passing from the point $x^0 = \rho(0)$. Indeed, the unit normal vector to the surface alone the curve $\rho = \rho(u)$ has the form

$$\mathbf{m}(u) = -\sin(c - \int_0^u \kappa(\tau)d\tau)\mathbf{n}(u) + \cos(c - \int_0^u \kappa(\tau)d\tau)\mathbf{b}(u).$$

The direct computations based on the Frenet formulas show that [22]

$$\mathbf{m}'(u) = k(u)\sin(c - \int_0^u \kappa(\tau)d\tau)\mathbf{t}(u),$$

where $k(u)$ is the curvature of the curve $\rho = \rho(u)$. Therefore, due to Rodrique's theorem, the principal curvature along the curve $\rho = \rho(u)$ is defined by the formula

$$k_1(u) = k(u)\sin(c - \int_0^u \kappa(\tau)d\tau).$$

Now we consider the Fourier transform of the surface:

$$J(\xi) = \int e^{i|\xi|(\eta_1 x_1(u,v) + \eta_2 x_2(u,v) + \eta_3 x_3(u,v))}\varphi(u, v)dudv,$$

where $\eta = \frac{\xi}{|\xi|}$ is a unit vector. And also we define associated one-dimensional oscillatory integral

$$J_1(\xi) = \int e^{i|\xi|(\eta_1 x_1(u,v)+\eta_2 x_2(u,v)+\eta_3 x_3(u,v))}\varphi(u,v)du.$$

We consider behavior of the integral near critical direction. Let $u = 0$, $v = 0$ be a fixed point and $\eta^{\mathbf{0}} = \mathbf{m}(0,0)$ be the unite normal vector to the hypersurface at the point x^0. We assume that the support of the amplitude function is concentrated in a small neighborhood of the point $(0,0)$. In this case the phase function $F(\eta,u,v) := (\eta,\mathbf{r}(u,v))$ can be considered as a smooth deformation of the function $f(u) := (\eta^0,\mathbf{r}(u,0))$.

If the curvature $k_1 = k_1(u)$ has a root of order n at $\mathbf{r}(0,0)$, then we have

$$f(u) = u^{n+2}\psi(u) + c,$$

where ψ is a smooth function satisfying the condition $\psi(0) \neq 0$. Therefore, due to the Mather theorem [1], [16], there exists a smooth function $z = z(u,\eta,v)$ such that $\frac{\partial z(0,\eta^0,0)}{\partial u} \neq 0$ and the function F has the form

$$F(\eta,u(z,\eta,v),v) = z^{n+2} + \lambda_1(\eta,v)z^n + \cdots + \lambda_n(\eta,v)z + \lambda_{n+1}(\eta,v),$$

where $\lambda_k(\eta^0,0) = 0$ for $(k = 1, \dot{;} n)$ and λ_k, $(k = 0,\ldots,n+1)$ are real analytic functions.

Now, we consider the interior integral

$$J_1(\xi) = \int e^{i|\xi|(\eta_1 x_1(u(z,\eta,v),v)+\eta_2 x_2(u(z,\eta,v),v)+\eta_3 x_3(u(z,\eta,v),v))}\varphi(u(z,\eta,v),v)\frac{\partial u(z,\eta,v)}{\partial z}dz.$$

The Generalized Van der Corpute Lemma yields [4]:

$$|J_1(\xi)| \leq \frac{c\|\psi\|_{C^1(U)}}{|\xi|^{\frac{1}{n+2}}}.$$

Moreover, the last inequality is fulfilled uniformly with respect to the other variables. Therefore, by integrating in v variable we obtain

$$|J(\xi)| \leq \frac{c\|\psi\|_{C^1(U)}}{|\xi|^{\frac{1}{n+2}}}.$$

Theorem 3.3 is proved. □

Let $S \subset \mathbb{R}^3$ be the cylindric hypersurface and $\psi \in C_0^\infty(S)$ be a fixed cut-off function. We consider the measure defined by $d\mu = \psi(x)dS(x)$, where dS is the induced Lebesgue measure on S.

From Theorem 3.3 it follows the following corollary

Corollary 3.4. If $S \subset \mathbb{R}^3$ is a cylindric hupersurface and its mean curvature H has no roots of order more than n and $\psi \in C_0^\infty(S)$ is any fixed function, then for the integral $J(\xi)$ the uniform estimate

$$|J(\xi)| \leq \frac{c\|\psi\|_{C^1(S)}}{(1+|\xi|)^{\frac{1}{n+2}}}$$

holds.

4. Estimates for Fourier Transforms of Surface-Carried Measures Supported on the Cone Surfaces

Let $S \subset \mathbb{R}^3$ be a cone surface and $x^0 \in S$ be a fixed point of the surface. Without loss of generality we assume that the origin of $\mathbb{R}^3$ is the vertex of the cone and $x^0 \neq 0$. Thus, straight line passing from the origin and the point x^0 lies on the surface S. We assume that the straight line is transversal to the hyperplane $x_1 0 x_2$.

Lemma 4.1 There exist a cone neighborhood U of the point x^0 and a smooth (out of the origin) homogeneous function f of the order 1 such that the set $U \cap S$ can be written as a graph of the function f.

Proof. Of Lemma 4.1 is straightforward. □

Lemma 4.2 Let (x_1^0, x_2^0) be a fixed point and $x_2^0 \neq 0$. Then order of mean curvature H at that point and order of $f_{x_1 x_1}$ coincide. The same conclusion holds if $x_1^0 \neq 0$.

Proof. Note that f is homogeneous function. Therefore, by Euler's homogeneity relation we have $f(x_1, x_2) = x_1 f_{x_1}(x_1, x_2) + x_2 f_{x_2}(x_1, x_2)$ for any $x \neq 0$. Moreover, both f_{x_1} and f_{x_2} are also homogeneous functions of order zero. Hence

$$x_1 f_{x_1 x_1}(x_1, x_2) + x_2 f_{x_1 x_2}(x_1, x_2) = 0, \quad x_1 f_{x_2 x_1}(x_1, x_2) + x_2 f_{x_2 x_2}(x_1, x_2) = 0.$$

Consequently, we obtain $x_1^2 f_{x_1 x_2}(x_1, x_2) - x_2^2 f_{x_2 x_2}(x_1, x_2) = 0$. Moreover, if $x_2 \neq 0$ then we have

$$f_{x_2 x_2}(x_1, x_2) = \frac{x_1^2}{x_2^2} f_{x_1 x_1}(x_1, x_2).$$

Note that for the mean curvature one holds the relation [6]

$$H = div\left(\frac{\nabla f}{(1+|\nabla f|^2)^{1/2}}\right).$$

Therefore, straightforward calculations show that

$$H = \left((1+x_1^2x_2^{-2})(1+|\nabla f|^2) - (f_{x_1} + x_1x_2^{-1}f_{x_2})^2\right)\frac{f_{x_1x_1}}{(1+|\nabla f|^2)^{3/2}}.$$

Due to the Cauchy-Schwartz inequality we have

$$(1+x_1^2x_2^{-2})|\nabla f(x_1,x_2)|^2 - (f_{x_1}(x_1,x_2) + x_1x_2^{-1}f_{x_2}(x_1,x_2))^2 \geq 0.$$

Therefore, if x^0 is a fixed point satisfying the condition $x_2^0 \neq 0$, then we obtain

$$H = f_{x_1x_2}(x_1,x_2)b(x_1,x_2),$$

where

$$b(x_1,x_2) = \frac{(1+x_1^2x_2^{-2})(1+|\nabla f(x_1,x_2)|^2) - (f_{x_1}(x_1,x_2) + x_1x_2^{-1}f_{x_2}(x_1,x_2))^2}{(1+|\nabla f|^2)^{3/2}}$$

is a smooth function in some cone neighborhood of the point x^0. Moreover, it is a homogeneous function of order zero and $b(x^0) \neq 0$.

Consequently if $f_{x_1x_1}(x_1^0, x_2^0) \neq 0$, then one principal curvature is non-zero at the point x^0. If $f_{x_1x_1}(x_1^0, x_2^0) = 0$ and it is a finite type function then due to the results by [11] (Lemma 3.2) in some neighborhood of the point (x_1^0, x_2^0) we have

$$f_{x_1x_1}(x_1,x_2) = (x_1 - cx_2)^n g(x),$$

and $g(g(x_1^0, x_2^0) \neq 0)$ is a smooth function in some cone neighborhood of the point x^0, and also $c = x_1^0(x_2^0)^{-1}$. Thus, the orders of roots of $f_{x_1x_1}$ and H at that point coincide. Lemma 4.2 is proved. □

Theorem 4.3 *Let $S \subset \mathbb{R}^3$ be the conic hypersurface and H be mean curvature of the surface. If H has no roots of order greater than n, then for the Fourier transform of the measure $\mu(X) = \psi(X)dS(X)$ supported on that hypersurface, the estimate*

$$|J(\xi)| \leq \frac{c\|\psi\|_{L_2^1(S)}}{(1+|\xi|)^{\frac{1}{n+2}}}$$

holds.

Note that the last estimate holds not only in a small neighborhood of ordinary points, but, in any compact neighborhood of the vertex of the cone.

Proof. Without loss of generality, we may assume that the cone is given as a graph of the homogeneous function f. In this case we consider the oscillatory integral

$$J(\xi) = \int_D e^{i(x_1\xi_1 + x_2\xi_2 + f(x_1,x_2)\xi_3)} \tilde{\psi}(x_1, x_2) dx_1 dx_2,$$

where $\tilde{\psi}(x_1, x_2) = \psi(x_1, x_2, f(x_1, x_2))(1 + |\nabla f(x_1, x_2)|^2)^{1/2}$.

Without loss of generality we may assume that the support of $\tilde{\psi}$ is contained in a ball of radius 1 centered at the origin. We choose a non-negative function $\psi_0 \in C_0^\infty(\{\frac{1}{2} < |x| < 2\})$ such that the relation

$$\sum_{k=0}^{\infty} \psi_0(2^k x) = 1$$

holds for any $x \in \mathrm{supp}\tilde{\psi} \setminus \{(0,0)\}$.

It is obvious that for any fixed ξ we have the relation

$$J(\xi) = \sum_{k=0}^{\infty} J_k(\xi),$$

where

$$J_k(\xi) = \int_D e^{i(x_1\xi_1 + x_2\xi_2 + f(x_1,x_2)\xi_3)} \psi_0(2^k x) \tilde{\psi}(x) dx.$$

Further, we consider estimates for the integral $J_k(\xi)$. Let's use scaling $2^k x \mapsto x$ then we obtain

$$J_k(\xi) = 2^{-2k} \int_D e^{i2^{-k}(x_1\xi_1 + x_2\xi_2 + f(x_1,x_2)\xi_3)} \psi_0(x) \tilde{\psi}_k(x) dx,$$

where the function $\tilde{\psi}_k$ has the form

$$\tilde{\psi}_k(x) = \tilde{\psi}(2^{-k}x_1, 2^{-k}x_2, 2^{-k} f(x_1, x_2))(1 + |\nabla f(x_1, x_2)|^2)^{1/2},$$

because $f(|\nabla f|)$ is a homogeneous function of degree $1(0)$, respectively. Note that both functions are smooth on the support of ψ_0. If $|\xi_1| \geq M \max\{|\xi_2|, |\xi_3|\}$ or $|\xi_2| \geq$

$M\max\{|\xi_1|, |\xi_3|\}$, where M is a sufficiently large fixed positive real number, then the integration by parts formula yields

$$|J_k(\xi)| \leq \frac{c\|\tilde{\psi}\|_{L_2^1(D)}2^{-2k}}{(1+2^{-k}|\xi|)}.$$

Now, we consider the case $|\xi_3| \geq M^{-1}\max\{|\xi_1|, |\xi_2|\}$. In this case we deal with the phase function

$$F(x,s) = f(x_1,x_2) + s_1x_1 + s_2x_2, \quad \text{where,} \quad s_1 = \frac{\xi_1}{\xi_3},\ s_2 = \frac{\xi_2}{\xi_3}.$$

Note that both numbers s_1, s_2 are bounded. We fix $s = s^0 \in \{|s_1| \leq M, |s_2| \leq M\}$ and consider the set of critical points of the phase function $F(x, s^0)$. The set of critical points of the phase function coincides with the projection into $x_1 0 x_2$ plane of the set of points on S such that the unit normal to the surface S is co linear to the vector $(s_1^0, s_2^0, 1)$.

Due to Lemma 4.2, multiplicity of roots of the mean curvature and $f_{x_1x_1}(x_1,x_2)$ $(f_{x_2x_2}(x_1,x_2))$ if $x_2^0 \neq 0 (x_2^0 \neq 0)$ at critical point coincide. Therefore, we can use the Van der Corpute method [2], [4], [11] and obtain:

$$|J_k(\xi)| \leq \frac{c2^{-2k}\|\tilde{\psi}\|_{L_2^1(D)}}{(1+2^{-k}|\xi|)^{\frac{1}{n+2}}}.$$

Note that, if $2^{-k}|\xi| \leq 1$, then the trivial estimate for the integrals gives

$$\sum_{\{2^{-k}|\xi|\leq 1\}} |J_k(\xi)| \leq C2^{-2k_0}\|\tilde{\psi}\|_{L^1(D)} \leq \frac{C}{|\xi|^2}\|\tilde{\psi}\|_{L^1(D)},$$

where k_0 is the minimal natural number satisfying the inequality $2^{-k}|\xi| \leq 1$. Otherwise, we obtain

$$\sum_{\{2^{-k}|\xi|\geq 1\}} |J_k(\xi)| \leq C\sum_{k=0}^{\infty} 2^{-k(2-\frac{1}{n+2})}\frac{\|\tilde{\psi}\|_{L_2^1(D)}}{(1+|\xi|)^{\frac{1}{n+2}}} \leq \frac{c\|\tilde{\psi}\|_{L_2^1(D)}}{(1+|\xi|)^{\frac{1}{n+2}}}.$$

Theorem 4.3. is proved. □

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References

[1] Arnold V.I., Gusein-zade S.M. and Varchenko A.N.: Singularities of Differentiable Maps, Vol. II, Birhäuser, Boston, 1988.

[2] Arkhipov G.I., Karatsuba A.A. and Chubarikov B.N.: Trigonometric integrals. Izv. Akad. Nauk. SS SR, ser. Mat., 43(5)(1979) 971-1003. English transl. in Math. USSR, Izv. 15(1980).

[3] Colin de Verdier Y.: Nombre de points entiers dans une famille homothetique de domains de $\mathbb{R}^n$. Ann. Sci. Ecole Norm. Super ser. 4, 10:4, 559-575, 1977.

[4] Carbery A., Christ M. and Wright J.: Multidimensional van Der Corput and sublevel set estimates. Journal of AMS, 12(4), 981-1015, 1999.

[5] Duistermaat J.: Oscillatory integrals, Lagrange immersions and unfolding of singularities, Comm. pure and Appl. Math., 27(2), 207-281, 1974.

[6] Dubrovin B.A., Fomenko A.T. and Novikov S.P.: Modern Geometry, Method and applications. P. 1, The Geometry of surfaces, transformation Groups, and Fields. Springer-Verlag, Berlin, Heidelberg, New York, and Tokyo. 1984.

[7] Hertz, C.S.: Fourier transforms related to convex sets. Ann. of Math. 75, 81-92, 1962.

[8] Hlawka, E.: Über Integrale auf konvexen Körper I, Monatsh Math., 50, 1-36, 1950.

[9] Ishikawa, G.: Determinacy of envelope of the osculating hyperplanes to a curve, Bull. London Math. Soc. 25, 603-610, 1993.

[10] Ishikawa, G.: Developable of a curve and its determinacy relatively to the osculation-type, Quart. J. Math. 46, 437-451, 1995.

[11] Ikromov, I.A., Müller, D. and Kempe, M.: Damped oscillatory integrals and boundedness of maximal operators associated to mixed homogeneous hypersurfaces, Duke Math. Journal, Vol. 126, No.3 471-490, 2005.

[12] Iosevich, A. and Sawyer E.: Maximal operators over convex hypersurfaces. Adv. in Math. 132, No. 1, 46-119, 1997.

[13] Karpushkin, V.N.: Theorem on uniform estimates of oscillating integrals Tr. Sem. I.G. Petrovskogo, 10, 3-38, 1983.

[14] Littman, W.: Fourier transform of surface-carried measures and differentiability of surface averages, Bull. Amer. Math. Soc., 69, 766-770, 1963.

[15] Malgrange, B.: Ideals of differentiable functions, Oxford Univ. press, 1966.

[16] Mather, J.: Stability of C^∞ mappings. I-VI. Ann. Math. 87, 89-104, 1968.

[17] Randol, B.: On the Fourier transform of the indicator function of a planar set, Trans. Amer. Math. Soc. 139, 271-278, 1969.

[18] Randol, B.: On the asymptotic behavior of the Fourier transform of the indicator function of a convex set, Trans. Amer. Math. Soc. 139, 279-285, 1969.

[19] Schulz, H.: Convex hypersurfaces of finite type and the asymptotics of their Fourier transforms, Indiana Univ. Math. J. 40, No. 4, 1267-1275, 1991.

[20] Scherbak, O.P.: Wavefront and reflection groups, Russian Math. Surveys 43-3, 149-194, 1988.

[21] Sjölin P.: Estimates of averages of Fourier transforms of measures with finite energy. Ann. Acad. Sci. Fenn. A 22, 227-236, 1997.

[22] Stoker, J.: Differential Geometry, New York, Sydney, TorontoLondon, 1988.

[23] Stein, E.M.: Harmonic analysis: Real-valued methods, Orthogonality, and oscillatory integrals. princeton Univ. press, 43, 1993.

[24] Swenson, I.: Estimates for the Fourier transform of the characteristic function of a convex set, Ark. f. Mat. 9, 11-22, 1971.

[25] Van Der Corput, J.G.: "Zahlentheoretische Abschatzungen". Math. Ann. 84, 53-79, 1921.

[26] Varchenko, A.N.: Newton polyhedra and estimates of oscillating integrals. Funct. anal. and Appl., 10:2, 79-83, 1976.

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