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ROVSHAN Z. GUMBATALIEV

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## On Generalized Solution of a Class of Higher Order Operator-Differential Equations

*Rovshan Z. Humbataliyev*

### Abstract

In this paper the sufficient conditions on the existence and uniqueness of a generalized solution on the axis are obtained for higher order operator-differential equations, the main part of which is multi characteristic.

**Key Words:** Operator-differential equations, Hilbert spaces, existence of generalized solution.

### 1. Introduction

Let  $H$  be a separable Hilbert space, and  $A$  be a positive-definite self-adjoint operator in  $H$  with domain of definition  $D(A)$ . Denote by  $H_\gamma$  a scale of Hilbert spaces generated by the operator  $A$ , i.e.  $H_\gamma = D(A^\gamma)$ ,  $(\gamma \geq 0)$ ,  $(x, y)_\gamma = (A^\gamma x, A^\gamma y)$ ,  $x, y \in D(A^\gamma)$ .

We denote by  $L_2((a, b); H_\gamma)$   $(-\infty \leq a < b \leq +\infty)$  a Hilbert space of vector-functions  $f(t)$  determined in  $(a, b)$  almost everywhere with values from  $H$  measurable, square integrable in the Bochner's sense

$$\|f\|_{L_2((a,b);H)} = \left( \int_a^b \|f\|_\gamma^2 dt \right)^{1/2}.$$

Assume

$$L_2((-\infty, +\infty); H) \equiv L_2(R; H).$$

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Further, we define a Hilbert space for natural  $m \geq 1$  [1].

$$W_2^m((a; b); H) = \left\{ u \mid u^{(m)} \in L_2((a; b); H), A^m u \in L_2((a, b); H_m) \right\}$$

with norm

$$\|u\|_{W_2^m((a,b);H)} = \left( \|u^{(m)}\|_{L_2((a,b);H)}^2 + \|A^m u\|_{L_2((a,b);H)}^2 \right)^{1/2}.$$

Here and in sequel the derivatives are understood in the sense of distributions theory [1]. Here we assume

$$W_2^m((-\infty, +\infty); H) \equiv W_2^m(R; H).$$

We denote by  $D(R; H)$  a set of infinitely-differentiable functions with values in  $H$ .

In the space  $H$  we consider the operator – differential equation

$$P \left( \frac{d}{dt} \right) u(t) \equiv \left( -\frac{d^2}{dt^2} + A^2 \right)^m u(t) + \sum_{j=0}^{2m} A_j u^{(2m-j)}(t) = f(t), \tag{1}$$

$$t \in R = (-\infty, +\infty),$$

where  $f(t)$  and  $u(t)$  are vector-valued functions from  $H$ , and coefficients  $A$  and  $A_j$  ( $j = \overline{0, 2m}$ ) satisfy the following conditions:

- 1)  $A$  is a positive-definite self-adjoint operator in  $H$  ;
- 2) the operators  $A_j$  ( $j = \overline{0, 2m}$ ) are linear in  $H$ .

In this paper we'll give definition of a generalized solution of equation (1) and prove a theorem on the existence and uniqueness of generalized solution (1). Notice that another definition of generalized solution of operator-differential equations and their existence is given in the book [2]. In the paper [3] by S. S. Mirzoyev, and in [4] by M. B. Orazov it was investigated a boundary-value problem when the principal part of equation has the form  $(-1)^m \frac{d^{2m}}{dt^{2m}} + A^{2m}$  where  $A$  is a self-adjoint operator  $A$  boundary-value problem when  $m = 2$  on the semi-axis  $R_+ = (0, +\infty)$  was studied by the author in [5].

**2. Some Auxiliary Facts**

First of all we consider some facts that we'll need in future. Denote

$$P_0 \left( \frac{d}{dt} \right) u(t) = \left( -\frac{d^2}{dt^2} + A^2 \right)^m u(t), \quad u(t) \in D(R; H)$$

and

$$P_1 \left( \frac{d}{dt} \right) u(t) = \sum_{j=0}^{2m} A_j u^{(2m-j)}(t), \quad u(t) \in D(R; H).$$

Now let's formulate a lemma that shows the conditions on operator coefficients (1) under which the solution of the equation from the class  $W_2^m(R; H)$  has sense.

**Lemma 2.1** *Let conditions 1) and 2) be fulfilled, moreover,  $B_j = A_j \times A^{-j}$  ( $j = \overline{0, m}$ ) and  $D_j = A^{-m} A_j A^{m-j}$  ( $j = \overline{m+1, 2m}$ ) be bounded in  $H$ . Then a bilinear functional  $L(u, \psi) \equiv (P_1(d/dt)u, \psi)_{L_2(R; H)}$  determined for all vector-functions  $u \in D(R; H)$  and  $\psi \in D(R; H)$  continues on the space  $W_2^m(R; H) \oplus W_2^m(R; H)$  that acts in the following way:*

$$\begin{aligned} L(u, \psi) &= (P_1(d/dt)u, \psi)_{L_2(R; H)} = \sum_{j=0}^{2m} (A_j u^{(2m-j)}, \psi)_{L_2(R; H)} \\ &= \sum_{j=0}^{2m} (-1)^m (A_j u^{(m-j)}, \psi^{(m)})_{L_2(R; H)} + \sum_{j=m+1}^{2m} (A_j u^{(2m-j)}, \psi)_{L_2(R; H)}. \end{aligned}$$

**Proof.** Let  $u \in D(R; H)$ ,  $\psi \in D(R; H)$ . Then integrating by parts we get

$$\begin{aligned} L(u, \psi) &= (P_1(d/dt)u, \psi)_{L_2(R; H)} = \sum_{j=0}^{2m} (A_j u^{(2m-j)}, \psi)_{L_2(R; H)} \\ &= \sum_{j=0}^m (-1)^m (A_j u^{(m-j)}, \psi^{(m)})_{L_2(R; H)} + \sum_{j=m+1}^{2m} (A_j u^{(2m-j)}, \psi^{(m)})_{L_2(R; H)}. \end{aligned} \tag{2}$$

On the other hand, for  $j = \overline{0, m}$  we apply the intermediate derivatives theorem [1] and get

$$\begin{aligned} & \left| \left( A_j u^{(m-j)}, \psi^{(m)} \right)_{L_2(R;H)} \right| = \left| \left( B_j A_j u^{(m-j)}, \psi^m \right)_{L_2(R;H)} \right| \leq \\ & \leq \|B_j\| \cdot \left\| A_j u^{(m-j)} \right\|_{L_2(R;H)} \cdot \left\| \psi^{(m)} \right\|_{L_2(R;H)} \leq C_{m-j} \|D_j\| \cdot \|u\| \cdot \|\psi\|_{W_2^m(R;H)}. \end{aligned} \quad (3)$$

And for  $j = \overline{m+1, 2m}$  we again use the theorem on intermediate derivatives [1] and get

$$\begin{aligned} & \left| \left( A_j u^{(2m-j)}, \psi^m \right)_{L_2(R;H)} \right| = \left| D_j \left( A^{m-j} u^{(2m-j)}, A^m \psi \right)_{L_2(R;H)} \right| \leq \\ & \leq \|D_j\| \left\| A^{m-j} u^{(2m-j)} \right\|_{L_2(R;H)} \cdot \|A^m \psi\|_{L_2(R;H)} \leq \\ & \leq C_{2m-j} \|D_j\| \cdot \|u\|_{W_2^m(R;H)} \|\psi\|_{W_2^m(R;H)}. \end{aligned} \quad (4)$$

Since the set  $D(R; H)$  in dense is the space  $W_2^m(R; H)$ , allowing for inequality (3) and (4) in (2) we get that the inequality

$$\|L(u, \psi)\| \leq const \|u\|_{W_2^m(R;H)} \cdot \|\psi\|_{W_2^m(R;H)}$$

is true for all  $u, \varphi \in W_2^m(R; H)$ , i.e.  $L(u, \psi)$  continues by continuity up to a bilinear functional acting on the spaces  $W_2^m(R; H) \oplus W_2^m(R; H)$ . We denote this functional by  $L(u, \psi)$  as well. The lemma is proved.  $\square$

**Definition** The vector function  $u(t) \in W_2^m(R; H)$  is said to be a generalized solution of (1) if for any vector-function  $\psi(t) \in W_2^m(R; H)$  it holds the identity

$$(u, \psi)_{W_2^m(R;H)} + \sum_{k=1}^{2m-1} C_{2m}^k \left( A^{m-k} u^{(k)}, A^{m-k} \psi^{(k)} \right)_{L_2(R;H)} = (f, \psi)_{L_2(R;H)}, \quad (5)$$

where  $C_{2m}^k = \frac{2m(2m-1)\dots(2m-k+1)}{k!}$ .

To find the solvability conditions of equation (1) we prove the following Lemma by using the method of paper [3].

**Lemma 2.2** For any  $u(t) \in W_2^m(R; H)$ , hold the following estimates:

$$\left\| A^{m-j} u^{(j)} \right\|_{L_2(R;H)} \leq d_{m,j}^{m/2} \|u\|_{W_2^m(R;H)}, \quad (j = \overline{0, m}), \quad (6)$$

where

$$\|u\|_{W_2^m(R;H)} = \left( \|u\|_{W_2^m(R;H_m)}^2 + \sum_{k=1}^{2m-1} c_{2m}^k \left\| \|A^{m-k} u^{(k)}\|_{L_2(R;H)} \right\|^2 \right)^{1/2};$$

and the numbers from inequalities (6) are determined as follows

$$d_{m,j} = \begin{cases} \left(\frac{j}{m}\right)^{\frac{j}{m}} \cdot \left(\frac{m-j}{m}\right)^{\frac{m-j}{m}}, & j = \overline{1, m-1} \\ 1, & j = 0, m \end{cases}$$

**Proof.** Obviously, the norm  $\|u\|_{W_2^m(R;H)}$  is equivalent to the norm  $\|u\|_{W_2^m(R;H)}$ . Then it follows from the intermediate derivatives theorem that the final numbers

$$b_j = \sup_{0 \neq u \in W_2^m(R;H)} \left\| A^{m-j} u^{(j)} \right\|_{L_2(R;H)} \cdot \|u\|_{W_2^m(R;H)}^{-1}, \quad j = \overline{0, m}.$$

Show that  $b_j = d_{m,j}^{m/2}$ ,  $j = \overline{0, m}$ . Then  $u(t) \in D(R;H)$ .

For all  $\beta \in [0, b_j^{-2}]$ , where

$$b_j = \sup_{\xi \in R} \left| \xi^j (\xi^2 + 1)^{-m/2} \right| = d_{m,j}^{m/2},$$

we use the Plancherel theorem and get

$$\begin{aligned} \|u\|_{W_2^m(R;H)} - \beta \left\| A^{m-j} u^{(j)} \right\|_{L_2(R;H)}^2 &= \sum_{k=0}^{2m} C_{2m}^k \left\| A^{m-k} \xi^k \hat{u}(\xi) \right\|_{L_2(R;H)}^2 - \\ - \beta \left\| A^{m-j} \xi^j \hat{u}(\xi) \right\|_{L_2(R;H)}^2 &= \sum_{k=0}^{2m} C_{2m}^k (A^{m-k} \xi^k \hat{u}(\xi), A^{m-k} \xi^k \hat{u}(\xi))_{L_2(R;H)} - \\ - \beta (A^{m-j} \xi^j \hat{u}(\xi), A^{m-j} \xi^j \hat{u}(\xi))_{L_2(R;H)} &= \\ = \int_{-\infty}^{+\infty} \left( \left[ (\xi^2 E + A^2)^m - \beta \xi^{2j} A^{(2m-j)} \right] \hat{u}(\xi), \hat{u}(\xi) \right)_{L_2(R;H)} d\xi, & \quad (7) \end{aligned}$$

where  $\hat{u}(\xi)$  is a Fourier transformation of the vector-function  $u(t)$ . Since for  $\beta \in [0, b_j^{-2})$ , it follows from the spectral expansion of the operator  $A$  that

$$\begin{aligned} & \left( \left( (\xi^2 E + A^2)^m - \beta \xi^{2j} A^{2(m-j)} \right) x, x \right) = \int_{-\infty}^{+\infty} \left( (\xi^2 + \mu^2)^m - \beta \xi^{2j} \mu^{2(m-j)} \right) (dE_\mu x, x) = \\ & = \int_{\mu_0}^{\infty} \left( 1 - \beta \frac{\xi^{2j} \mu^{2(m-j)}}{(\xi^2 + \mu^2)^\mu} \right) (\xi^2 + \mu^2) (dE_\mu x, x) \geq \int_{\mu_0}^{\infty} (1 - \beta b_j^2) (\xi^2 + \mu^2) (dE_\mu x, x); \end{aligned}$$

then equality (6) yields

$$\|u\|_{W_2^m(R;H)}^2 \geq \beta \left\| A^{m-j} u^{(j)} \right\|_{L_2(R;H)}^2, \quad (8)$$

for all  $\beta \in [0, b_j^{-2})$  and  $u(t) \in D(R; H)$ . Passing to the limit as  $\beta \rightarrow b_j^{-2}$  we get

$$\|u\|_{W_2^m(R;H)}^2 \geq d_{m,j}^{-m/2} \left\| A^{m-j} u^{(j)} \right\|_{L_2(R;H)}^2.$$

Hence it follows

$$\left\| A^{m-j} u^{(j)} \right\|_{L_2(R;H)}^2 \leq d_{m,j}^{m/2} \|u\|_{W_2^m(R;H)}^2, \quad (j = \overline{0, m}). \quad (9)$$

Show that inequalities (9) are exact. To this end, for the given  $\varepsilon > 0$  we show the existence of the vector-function  $u_\varepsilon(t) \in W_2^m(R; H)$ , such that

$$E(u_\varepsilon) = \|u\|_{W_2^m(R;H)}^2 - (d_{m,j}^{-m} + \varepsilon) \left\| A^{m-j} u^{(j)} \right\|_{L_2(R;H)}^2 < 0. \quad (10)$$

We'll look for  $u_\varepsilon(t)$  in the form  $u_\varepsilon(t) = g_\varepsilon(t) \varphi_\varepsilon(t)$ , where  $g_\varepsilon(t)$  is a scalar function from the space  $W_2^m(R)$  and  $\varphi_\varepsilon \in H_{2m}$ , where  $\|\varphi_\varepsilon\| = 1$ . Using the Plancherel theorem, we write  $E(u_\varepsilon)$  in the equivalent form

$$E(u_\varepsilon) = \int_{-\infty}^{+\infty} \left( \left( (\xi^2 E + A^2)^m - (d_{m,j}^{-m} + \varepsilon) \xi^{2j} A^{2m-2j} \right) \varphi_\varepsilon, \varphi_\varepsilon \right) |\hat{g}_\varepsilon(\xi)|^2 d\xi.$$

Note that  $\hat{u}(\xi)$  and  $\hat{g}_\varepsilon(\xi)$  are the Fourier transformations of the vector-functions  $u(t)$  and  $g_\varepsilon(t)$ , respectively.

Notice that if  $A$  has even if one even value  $\mu$ , then for  $\varphi_\varepsilon$  we can choose appropriate eigen-vector  $\varphi_\varepsilon = \varphi (\|\varphi\| = 1)$ . Indeed, then at the point  $\xi_0 = (j/m)^{1/2} \cdot \mu$

$$\begin{aligned} & \left( \left( (\xi_0^2 E + A^2)^m - (d_{m,j}^{-m} + \varepsilon) \xi_0^{2j} A^{2m-2j} \right) \varphi_\varepsilon, \varphi_\varepsilon \right) = (\xi_0^2 + \mu^2)^m - \\ & - (d_{m,j}^{-m} + \varepsilon) \xi_0^{2j} \mu^{2m-2j} = (\xi_0^2 + \mu^2)^m \left[ 1 - (d_{m,j}^{-m} + \varepsilon) \frac{\xi_0^{2j} \mu^{2m-2j}}{(\xi_0^2 + \mu^2)^m} \right] < 0. \end{aligned} \quad (11)$$

If the operator  $A$  has no eigenvalue, for  $\mu \in \sigma(A)$  and for any  $\delta > 0$  we can construct a vector  $\varphi_\delta$ ,  $\|\varphi_\delta\| = 1$  such that

$$A^\delta \varphi_\delta = \mu^m \varphi_\delta + 0(1, \delta), \quad \delta \rightarrow 0, \quad m = 1, 2, \dots$$

In this case, and for  $\xi_0 = (j/m)^{1/2} \mu$  and

$$\begin{aligned} & \left( \left( (\xi^2 E + A^2)^m - (d_{m,j}^{-m} + \varepsilon) \xi^{2j} A^{2m-2j} \right) \varphi_\delta, \varphi_\delta \right) = \\ & = ((\xi_0^2 + \mu^2) - (d_{m,j}^{-m} + \varepsilon) \xi^{2j} \mu^{2m-2j}) + 0(1, \delta). \end{aligned}$$

Thus for small  $\delta > 0$  it holds inequality (11). Consequently, for any  $\varepsilon > 0$  there will be found a vector  $\varphi_\varepsilon$  ( $\|\varphi_\varepsilon\| = 1$ ), for which

$$\left( \left( (\xi^2 E + A^2)^m - (d_{m,j}^{-m} + \varepsilon) \xi^{2j} A^{2m-2j} \right) \varphi_\varepsilon, \varphi_\varepsilon \right) < 0 \quad (12)$$

Now for  $\xi = \xi_0$  we construct  $g_\varepsilon(t)$ . Since the left hand side of inequality (12) is a continuous function from the argument  $\xi$ , it is true at some vicinity of the point  $\xi_0$ . Assume that inequality (12) holds in the vicinity  $(\eta_1, \eta_2)$ . Then we construct  $\hat{g}(\xi)$ -infinitely-differentiable function of argument  $\xi$  with support in  $(\eta_1, \eta_2)$  and denote it by

$$g_\varepsilon(t) = \frac{1}{\sqrt{2\pi}} \int_{\eta_1}^{\eta_2} \hat{g}(\xi) e^{i\xi t} d\xi.$$

It follows from the Paley-Wienere theorem that  $g_\varepsilon(t) \in W_2^m(R)$  and obviously



$$\begin{aligned}
 E(u_\varepsilon) &= E(g_\varepsilon, \varphi_\varepsilon) = \\
 &= \int_{\eta_1}^{\eta_2} \left( \left( (\xi^2 E + A^2)^m - (d_{m,j}^{-m} + \varepsilon) \xi^{2j} A^{2m-2j} \right) \varphi_\varepsilon, \varphi_\varepsilon \right) |\hat{g}_\varepsilon(\xi)|^2 d\xi < 0,
 \end{aligned}$$

i.e. inequalities (9) are exact. The lemma is proved.  $\square$

### The Main Result

Now let's prove the main theorem.

**Theorem 3.1.** *Let  $A$  be a positive-definite self-adjoint operator in  $H$ , the operators  $B_j = A_j \cdot A^{-j}$  ( $j = \overline{0, m}$ ) and  $D_j = A^{-m} A_j A^{m-j}$  ( $j = \overline{m+1, 2m}$ ) be bounded in  $H$  and it holds the inequality*

$$\gamma = \sum_{j=0}^m d_{m,j}^{m/2} \|B_j\| + \sum_{j=m+1}^{2m} d_{m,2m-j}^{m/2} \|D_j\| < 1, \tag{13}$$

where the numbers  $d_{m,j}$  are determined from lemma 2.

Then equation (1) has a unique generalized solution and the inequality

$$\|u\|_{W_2^m(R;H)} \leq \text{const} \|f\|_{L_2(R;H)}$$

holds.

**Proof.** Show that for  $\gamma < 1$  for all vector-functions  $\psi \in W_2^m(R;H)$  it holds the inequality

$$\begin{aligned}
 (P(d/dt)\psi, \psi)_{L_2(R;H)} &\equiv \\
 &\equiv \|\psi\|_{W_2^m(R;H)}^2 + \sum_{k=1}^{2m} C_{2m}^k \left\| A^{m-k} \psi^{(k)} \right\|_{L_2(R;H)}^2 + L(\psi, \psi) \geq C \|\psi\|_{W_2^m(R;H)}^2,
 \end{aligned}$$

where  $C > 0$  is a constant number.

Obviously,

$$\left| (P(d/dt)\psi, \psi)_{L_2(R;H)} \right| \geq \|\psi\|_{W_2^m(R;H)}^2 - |L(\psi, \psi)|. \tag{14}$$

Since

$$|L(\psi, \psi)| < \sum_{j=0}^m \left| \left( A_j \psi^{(m-j)}, \psi^{(m)} \right)_{L_2(R;H)} \right| + \sum_{j=m+1}^{2m} \left| \left( A_j \psi^{(2m-j)}, \psi \right)_{L_2(R;H)} \right|,$$

then we use lemma 2 and get

$$\begin{aligned} |L(\psi, \psi)| &\leq \left( \sum_{j=0}^m \|B_j\| d_{m,j}^{m/2} + \sum_{j=m+1}^{2m} \|D_j\| d_{m,2m-j}^{m/2} \right) \|\psi\|_{W_2^m(R;H)}^2 = \\ &= \gamma \|\psi\|_{W_2^m(R;H)}. \end{aligned} \tag{15}$$

Allowing for inequality (15) in (14), we get

$$\left| (P(d/dt)\psi, \psi)_{L_2(R;H)} \right| \geq (1 - \gamma) \|\psi\|_{W_2^m(R;H)}^2. \tag{16}$$

Further, we consider the problem

$$P_0(d/dt)u(t) = f(t),$$

where  $f(t) \in L_2(R;H)$ . It is easy to see that the vector function

$$u_0(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} (\xi^2 + A^2)^m \int_{-\infty}^{+\infty} f(s) e^{-2(s-\xi)} ds d\xi \tag{17}$$

belongs to the space  $W_2^m(R;H)$  and satisfies the condition

$$(u_0, \psi) = (f, \psi).$$

Now we'll look for the generalized solution of equation (1) in the form  $u = u_0 + \xi_0$ , where  $\xi_0 \in W_2^m(R;H)$ . Putting this expression into equality (5), we get

$$(P(d/dt)u, \psi)_{L_2(R;H)} = -L(u_0, \psi), \quad \psi \in W_2^m(R;H). \tag{18}$$

The right hand-side is a continuous functional in  $W_2^m(R;H)$ , the left hand-side satisfies Lax-Milgram [6] theorem's conditions by inequality (16). Therefore, there exists

a unique vector function  $u(t) \in W_2^m(R; H)$  satisfying equality (18). On the other hand, for  $\psi = u$  it follows from inequality (16) that

$$\left| (P(d/dt)u, u)_{L_2(R; H)} \right| = \left| (f, u)_{L_2(R; H)} \right| \geq C \|u\|_{W_2^m(R; H)}^2 \geq C \|u\|_{W_2^m(R; H)}^2,$$

then hence it follows

$$\|u\|_{W_2^m(R; H)} \leq \text{const} \|f\|_{L_2(R; H)}.$$

The theorem is proved. □

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Rovshan Z. HUMBATALIYEV  
 Institute of Mathematics  
 and Mechanics of NAS of  
 Azerbaijan, Baku, AZ 1141,  
 F.Agaev str.9. AZERBAIJAN  
 e-mail: rovshangumbataliev@rambler.ru

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