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## Direct radio frequency sampling methodology for multiple signals in an energy-sparse spectrum

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**Abstract:** In software defined radio (SDR) systems, it is desirable to down-convert multiple RF signals simultaneously by placing an analogue-to-digital converter (ADC) as near the antenna as possible. Radio frequency (RF) subsampling is one of the established methodologies used by radio receivers to directly down-convert and digitize RF signals. In this paper, we propose a novel direct RF sampling method to find out the minimum sampling frequency for an evenly spaced spectrum comprising multiband RF signals. However, it can also be applied to nonuniformly spaced signals with slight modification. The proposed methodology describes a set of rules to achieve the lowest possible sampling frequency rates without the compromise of spectrum folding or overlapping of aliases in the baseband after down-conversion. Moreover, the proposed formula is general, and flexible to the number of input signals or bands and to their positions in the desired spectrum. Simulations verify that by using the offered minimum sampling rates, the desired signal is extracted without the mentioned constraints.

**Key words:** Band-pass sampling, direct sampling, frequency-sparse spectrum, optimum sampling, wideband receiver

### 1. Introduction

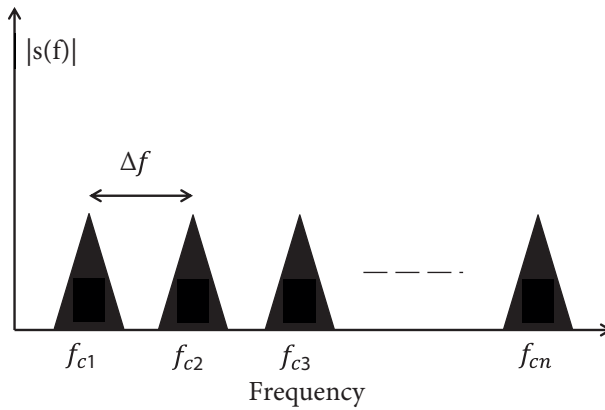
Bandpass sampling (BPS) is one of the recognized techniques to reduce the high sampling rates required for frequency translation and digitization of the analogue spectrum. Analytical methods exist that permit minimum bandpass sampling frequencies to be computed in the case of a single-band spectrum [1–3]. However, BPS has its issues for multiband signal spectrum due to certain constraints like aliasing, spectral folding, and reconstruction of information band [4–7]. In addition, there are no analogous equation sets available for a general multiband bandpass system consisting of signals with arbitrary center frequencies and bandwidths. In the last decade, BPS for multiband signals has gained significant attention, and efforts have been made to find valid band-pass sampling frequency ranges for direct down-conversion of multiband RF signals [8–9]. In [10,11], the focus shifted to finding a single minimum sampling frequency for multiband signals. However, minimum sampling frequency cannot be acquired in a closed form due to the nonlinear nature of spectrum folding in the process of sampling. In order to find a minimum useable sampling frequency, the literature [10–12] highlights the need to address certain BPS limitations like sparse spectrum, fixed bandwidth (BW), and unavailability of a universal rule. In [13], minimum sampling rate was achieved; however, it does not hold true for all sparsity ratios between information bands and null bands. In the proposed work, our objective is to set a universal subsampling rule for direct digitization and down-conversion of multiple signals using minimum sampling rate. Furthermore, there is no restriction on bandwidth or intersignal space in the spectrum of interest. The proposed method is applicable

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for all sparsity ratios, and it is very simple to find out the exact minimum subsampling frequency without many iterations. Initially, all the calculations are performed with the major assumption of uniformly spaced signals in the spectrum of interest. This paper is organized in the following manner: Section 2 explains the model, numerous terms, and already ascertained constraints. Optimized sampling rates with reference to sparsity ratio and central frequency of the lowest band or signal in the spectrum are derived in Section 3. By carrying out some improvisation, the methodology is also applied to unevenly spaced nonuniform signals in Section 4. Simulations of the theoretical work based on MATLAB examples are presented in Section 5, where sampling rates and their usability to sparsity ratios are compared with those produced in [13]. Finally, conclusions are given in Section 6.

**2. System model**

Consider a frequency spectrum having  $N$  information signals, each of bandwidth  $B$ , which are separated by space  $\Delta f$ . Assume that the spectrum is sparse, such that the total bandwidth contained by information signals  $NB$  is much less than the total bandwidth of spectrum  $S_{BW}$ . Figure 1 shows the layout of our spectrum. Assuming  $f_{\min}$  as minimum frequency in the spectrum of interest, the central frequency  $f_{cn}$  of each signal present in the spectrum is given by



**Figure 1.** Spectral layout of  $N$  information signals separated by  $\Delta f$ .

$$f_{cn} = f_{\min} + (n - 1) \Delta f + B/2. \tag{1}$$

In the case of multiple bands being present in the spectrum of interest, [7] describes a relation between sampling frequency  $f_s$  and aliased frequency  $f_a$  to avert the aliased bands from overlapping with zero and the Nyquist frequency  $f_s/2$ . This relation is limited to 2 input signals only, and no algorithm is given which is applicable to  $N > 2$ . Two basic constraints described by [7] are given as

$$0 < f_a - B/2, \tag{2}$$

$$f_a + B/2 < f_s/2. \tag{3}$$

However, for the multiband spectrum, the constraints in (2) and (3) are not sufficient. Therefore, [14,15] introduced an additional constraint and extended the relationship for  $N$  bands. Later, [16] applied the same for quadrature bandpass sampling for single and multiband communications and satellite navigational receivers. In addition, to prevent overlapping with baseband boundaries, this constraint prevents the bands from overlapping

each other as well. Considering  $B_x$  and  $B_y$  as information bandwidths of 2 neighbouring bands in the baseband region with their intermediate frequencies as  $f_{ax}$  and  $f_{ay}$ , the constraint is given as

$$|f_{ax} - f_{ay}| \geq (B_x + B_y)/2. \tag{4}$$

Here  $y = 2 \dots N$  and  $x = 1 \dots y$ . However, it is observed that the sampling frequency used for such cases is not the lowest useable sampling frequency, and can be reduced further. Replacing  $f_{c1} = f_{\min} + B/2$  in (1), the general expression for  $f_{cn}$  can be written as

$$f_{cn} = f_{c1} + (n - 1)\Delta f. \tag{5}$$

Now if the complete spectrum is sampled with  $f_s$ , the replication  $f_{an}$  of nonzero energy contents can be obtained as

$$f_{an} = |\pm f_{cn} \pm m f_s|, n = 1, 2, 3, \dots, \tag{6}$$

where  $m$  is an arbitrary positive integer such that  $f_s/2 \geq f_{an} \geq 0$ . In the context of aliasing, it can be seen that the left-hand side of Eq. (5) is dependent on the central frequency of the first RF carrier and the interband space  $\Delta f$ . Here the central frequency of the first carrier plays a vital role in defining the sampling frequency. By choosing  $f_{\min}$  or  $f_{c1}$  analytically, we can avoid overlap between down-converted aliases irrespective of the ratio between  $B$  and  $\Delta f$ ; however, selection of  $f_{\min}$  is not a trivial issue and requires deliberate calculations.

Since there is no formula that universally provides a minimum sampling frequency for multiple input signals in the spectrum [13], in such cases spectral intervals are derived that contain the permissible sampling frequencies. These frequencies are later obtained through search algorithms [7–9]. Thereafter, selecting an appropriate  $f_{\min}$  makes it possible to write a standard expression for minimum sampling frequency. For this purpose, we are analyzing the central frequency of the lowest signal in the spectrum of interest and also the separations between the signals, which are assumed to be evenly spaced. In the context of an evenly spaced equal-bandwidth multiband spectrum, [13] proposes that the minimum sub-Nyquist sampling frequency does not imply any aliasing with the constraint on the lowest frequency in the band of interest. However, the proposed  $f_{s,\min}$  (minimum sampling frequency) and  $f_{\min}$  (smallest frequency of the lowest band in the spectrum of interest) in [13] do not hold true for all ratios of  $\Delta f$  and  $B$ . In contrast, our proposed method is applicable to all ratios of  $\Delta f$  and  $B$ . Using the proposed  $f_s$  with limitations on  $f_{\min}$  aliasing may be avoided for multiple signals present in the desired spectrum.

### 3. Proposed methodology

In our proposed method, the minimum sampling frequency depends on 2 factors. The first is the sparsity ratio  $\gamma$  and the second is  $f_{c1}$ .

#### 3.1. Sparsity ratio $\gamma$

Let us represent the complete spectrum in the form of a column matrix:

$$F = [ f_{c1} \quad f_{c2} \quad f_{c3} \dots f_{cn} ]^T, \tag{7}$$

where  $F$  represents carrier positions in compact form as a single column matrix. Eqs. (8) and (9) show the carrier frequency of each band in the RF spectrum and at the baseband level, respectively.

$$[ f_{c1} \quad f_{c2} \quad f_{c3} \quad \dots \quad f_{cn} ]^T = [ B/2 \quad (2\gamma + 1)B/2 \quad (4\gamma + 1)B/2 \quad \dots \quad (1 + 2\gamma(n - 1)B/2 ]^T; \tag{8}$$

$$[ f_{a1} \ f_{a2} \dots \ f_{an} ]^T = [ B/2 \pm 2mNB \ (2\gamma + 1)B/2 \pm 2mNB \dots \ (1/2 - \gamma + n\gamma)B \pm 2mNB ]^T. \quad (9)$$

For ease of comprehension, if we assume  $\gamma = 2$  and  $N = 3$ , carrier locations after down-conversion can be detected at the following frequencies:

$$f_{a1} = B/2 \pm 2mNB; \quad (10)$$

$$f_{a2} = 5B/2 \pm 2mNB; \quad (11)$$

$$f_{a3} = 9B/2 \pm 2mNB. \quad (12)$$

Now considering the constraints mentioned in (2–4), for  $f_{a1}$  and  $f_{a2}$  value of  $m$  cannot exceed zero. Similarly,  $m = 1$  satisfies the required aliasing conditions for  $f_{a3}$ , and finally the down-converted replications are received at  $B/2, 5B/2, 9B/2$ . Using a sparsity ratio equal to 2 and 4, Figure 2 shows this process, where received signals do not overlap in the first Nyquist zone. However, when  $\gamma = 3$ ,  $N_3$  completely overlaps with  $N_1$  after down-conversion. To analyze the problem, it can be observed that the difference between central frequencies of  $N_1$  and  $N_3$  is 6 times bandwidth  $B$ , which is equal to the sampling frequency ( $2NB$ ). Obviously, whenever 2 signals are spaced at a gap that is equal to or is a multiple of the sampling frequency, their replications will always overlap in the first Nyquist zone. This can be avoided by changing the sampling frequency. Since any sampling rate less than  $2NB$  creates an aliasing problem, it is reasonable to increment the sampling rate by an integer multiple of the bandwidth. This implies that, besides  $N$  and  $B$ , the minimum sampling frequency depends on  $\gamma$  also. To elaborate further, an algorithm to compute the sampling frequency based on center frequency is given as follows:

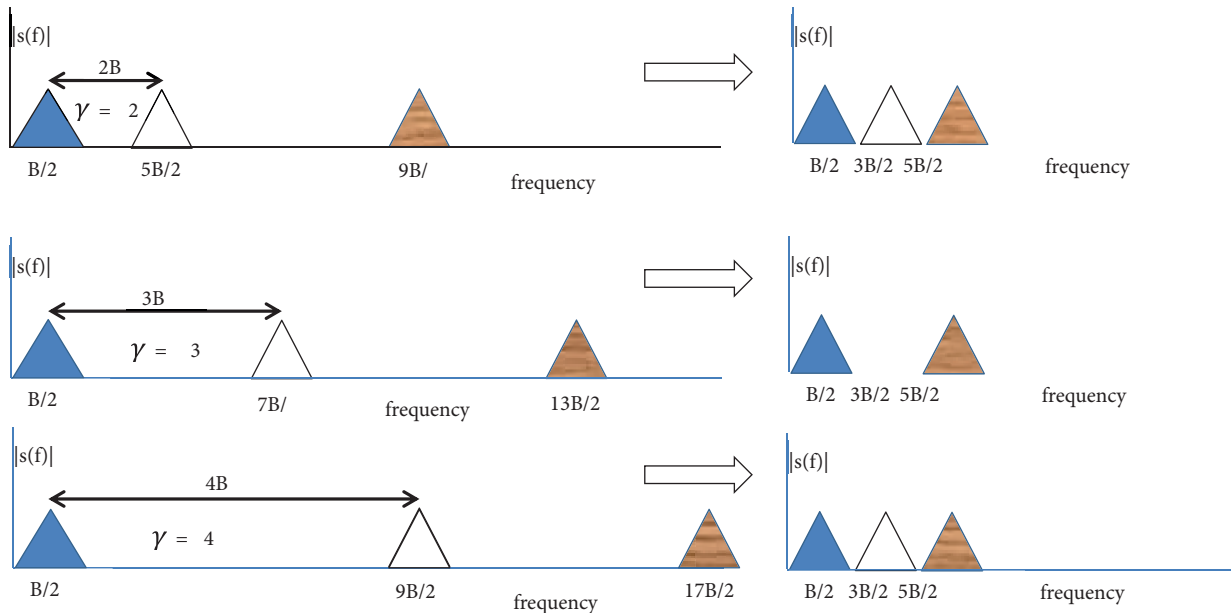


Figure 2. Direct down-conversion of a tri-band RF spectrum.

*input:*  $N, B, \gamma$ ;  
 $f_{cn} = (1/2 - \gamma + n\gamma)B$   
*process:*  $f_s = 2NB$ ;  
 if  $(f_j - f_i) \bmod(f_s) = 0$  for  $j > i$ ;  
 $f_s = (2N + 1)B$   
*output:*  $f_s$ .

It has been observed during our simulations that for most cases, the minimum useable sampling is equal to  $2NB$  and can be calculated in a single cycle of the given algorithm. Similarly, in more than 99% of cases, the process does not take more than 2 to 3 iterations. No comparison is drawn with other existing iterative algorithms, as this was not the objective of this paper. However, the above-described algorithm is fast and simple in terms of iterations to reach optimal sampling frequency. This gives it an edge over [10,11,17] in terms of reduced number of iterations to calculate minimum sampling frequency.

### 3.2. Central frequency of lowest carrier

After ascertaining the separation between carriers and fixation of optimal sampling frequency, it was observed that there exists some pitfall that does not allow elimination of the aliasing problem. Keeping in view the prescribed conditions, (13) defines the lowest possible passband positions of  $N$  bands depending on sparse value  $\gamma$ . Our next step is to find out the positions of the carriers in the passband, which prevents alias overlaps when sampled at the optimum sampling rates derived in the previous section. Since  $f_{c1}$  can never be less than  $B/2$ , after a little manipulation Eq. (5) can be written as

$$f_{cn} = (1 - 2\gamma + 2n\gamma)B/2. \quad (13)$$

This indicates that the position of  $f_c$  is limited to the set of frequencies defined by (13) such that  $f_{cn} \leq S_{BW} - B/2$ . To improve the illustration, let us keep  $N = 2$  and vary sparsity ratio  $\gamma$  only. Obviously, in this case,  $f_{c1}$  will be at  $B/2$ , and the position of the second carrier will be at  $(1 + 2\gamma)B/2$ . This simple spectrum does not pose any problem when sampled at the rate equal to  $2NB$ . However, if both signals are shifted towards the right (to higher frequencies), overlapping of aliases begins appearing until the time  $f_{c1} = 5B/2$ , despite having the same sampling rate. After shifting the lowest carrier at  $5B/2$ , there is no overlap due to aliasing or spectral folding. By continuing the process, it can soon be observed that the positions of  $f_{c1}$  for which there is no overlap on down-conversion follow certain patterns. These patterns depend on the sparsity ratio  $\gamma$  and separation between any of the 2 carriers  $f_j - f_i$  and can be expressed by arithmetic sequences as follows.

#### 3.2.1. When $\text{mod}((f_j - f_i), 2NB) \neq 0$ .

When the separation between any 2 signals is not a multiple of  $2NB$ , then the position of the lowest carrier depends on whether the sparsity ratio  $\gamma$  is odd or even. For odd values of  $\gamma$ , to circumvent the overlap between aliases in the first Nyquist zone, the lowest carrier position should be a member of the frequency set expressed in (14).

$$f_{c1} = (\gamma + 2(k - 1)N)B/2, \forall \text{mod}(\gamma, 2) \neq 0, k = 1, 2, 3, \dots \quad (14)$$

Putting  $\phi_k = (\gamma + 2(k - 1)N)$ , we can further simplify  $f_{c1}$  as

$$f_{c1} = \phi_k B/2. \quad (15)$$

Similarly for even values of  $\gamma$ , we have

$$f_{c1} = (2k - 1)B/2. \tag{16}$$

**3.2.2. When  $\text{mod}((f_j - f_i), 2NB) = 0$**

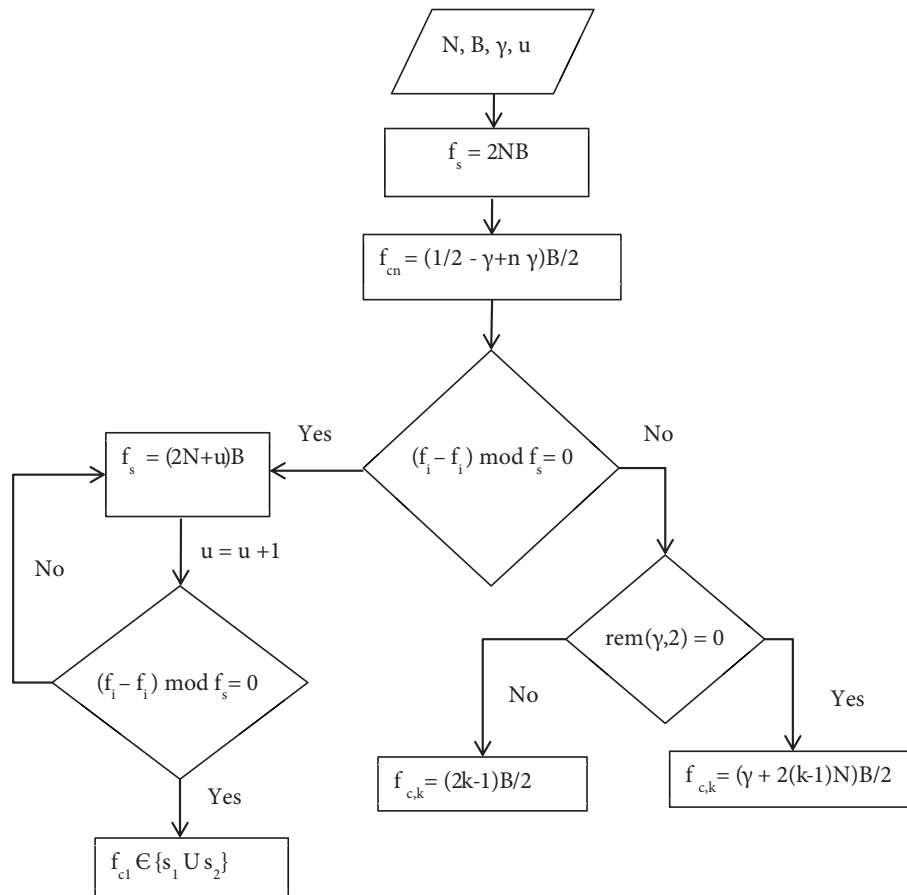
As described in Section 2, when any of the 2 or more carriers have their frequency separation  $(f_j - f_i)$  as a multiple of  $2NB$ , there will always be loss of information due to overlapping or spectral folding. This issue can be resolved by increasing the sampling frequency. For all such cases, the recommended positions of RF carriers can be found by choosing  $\varphi_k \in \{s_1 \cup s_2\}$  in (15) for all positive values of  $f_{c1}$ . Here sequences  $s_1$  and  $s_2$  are as follows:

$$s_0 = \gamma \pm k(2N + 1), \forall s_1 > 0, \tag{17}$$

where  $k \geq 0$  and

$$s_2 = 2\gamma \pm k(2N + 1), \forall s_2 > 0. \tag{18}$$

The results obtained in Section 2 are summarized with the help of the flowchart shown in Figure 3. The given flowchart describes the complete process for computing the optimal sampling frequency, carrier separation, and central frequency of the lowest signal,  $f_{c1}$ .



**Figure 3.** Flowchart to calculate sampling frequency and carrier positions in passband.

**4. Application to nonuniform signals**

Although the described methodology was developed for equally spaced signals, it can also be applied to nonuniformly spaced signals with a little manipulation and a tradeoff in the cost of hardware complexity. In the manipulation part, we reorganize the nonuniformly spaced spectrum into an equally spaced pattern; the hardware complexity is just the use of an additional multiplier for up- or down-conversion of the signals or bands that are out of pattern. Let us take the example of GSM900 (935 MHz–960 MHz) and WCDMA (2119 MHz–2124 MHz), which are spaced nonuniformly, covering a combined bandwidth of 30 MHz. A contiguous uniform spectrum can be formed by the down-conversion of the WCDMA band in the neighborhood of GSM900 from 930 MHz–935 MHz. For simplicity in the calculations, let us assume that there are 30 channels of 1-MHz bandwidth each with  $\gamma = 1$ . The only thing that needs to be ensured before down-conversion is that  $f_{c1}$  (in this case  $f_{c1} = 930.5\text{MHz}$ ) should be an element of the set proposed in (14). This can be achieved by putting  $k = 32$  for  $N = 30$ . Selecting  $f_{c1}$  helps to acquire a minimum sampling frequency equal to  $2NB$ , which is 60 MHz in our case. It may be noticed that the acquired sampling rate is less than that achieved in [11,17], which was 64.3636 MHz. Moreover, the minimum sampling rate is achieved in a single iteration without using any search algorithm.

**5. Simulation results and analysis**

In order to verify the expressions derived in the previous sections, MATLAB-based simulations are carried out. In the analysis part, 2 parameters—i.e. optimal sampling frequency and recommended layout of the passband spectrum—are analyzed for various values of  $\gamma$ . Although simulations are performed for a large number of input signals ( $N$ ), in the presented scenario, the maximum number of inputs is kept at 6. This scenario mainly pertains to the position of the first carrier in the passband and separation of RF carriers. To circumvent aliasing overlaps, the required minimum sampling frequency should be greater than or equal to twice the occupied information bandwidth. Therefore, we start with  $f_s = 2NB$ . As mentioned above, since the carrier is assumed to be located centrally in each signal, let us initially place our first carrier,  $f_{c1}$ , at  $B/2$ ; the rest of the carriers are separated as given in (5). Now, by choosing  $\gamma = 1$ , it implies that there are no gaps between information bands. In such a case, the lowest carrier may be placed at  $B/2$  and subsequent signals are separated by  $B$  Hz. The recommended positions of the first (the lowest) carrier are  $(1, 13, 25, 37\dots)B/2$ . The same are listed in the last column of Table 1. It can be verified that if we assign a frequency to the lowest carrier other than that recommended in the Table 1, there will always be aliasing overlap or spectrum folding, which will cause loss of information.

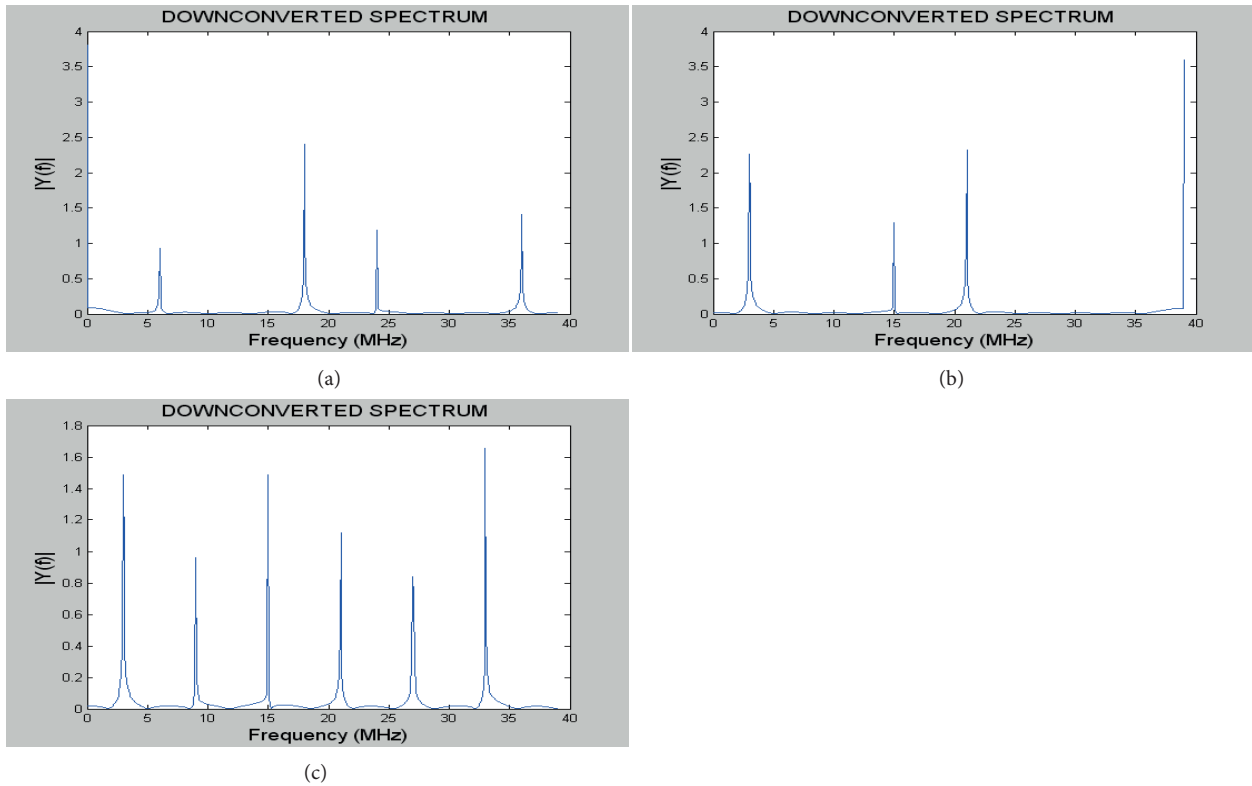
**Table 1.** Recommended suitable positions are listed in last column for a spectrum composed of bands and varying sparsity ratio.

$\Gamma$	Initial Positions $B/2, (1 + 2\gamma) B/2$	Recommended Positions $f_{c1} = \varphi_k B/2$
1	$B/2, 3B/2$	$[\gamma + 2(k - 1)N] B/2$
2	$B/2, 5B/2$	$(2k - 1) B/2$
3	$B/2, 7B/2$	$[\gamma + 2(k - 1)N] B/2$
4	$B/2, 9B/2$	$X$
5	$B/2, 11B/2$	$[\gamma + 2(k - 1)N] B/2$
6	$B/2, 13B/2$	$(2k - 1)B/2$

To reconstruct these overlapping signals requires extra processing power, which in turn increases the size, cost, and power requirements. There may be another important case when the sampling frequency,  $2NB$ , is a



multiple of the difference between 2 carriers within the range of the first Nyquist zone. In such cases, on the next iteration, sampling frequency is incremented by the addition of  $B$  Hz. In the present scenario, as listed in Table 1, the ultimate sampling frequencies are computed within 2 iterations. Table 1 also shows the values of  $f_s$  and  $\varphi_k$  for  $\gamma$  changing from 1 to 6. It can be noted that for  $\gamma = 3, 4$  or  $6$  separations between any 2 carriers becomes a multiple of  $2NB$ . This causes overlapping in aliased replications; therefore, the sampling rate is increased to  $(2N + 1)B$ . Consider Table 1, for a sparsity ratio  $\gamma = 3$ . Figure 4 shows the down-converted spectrum for  $\varphi_k = 1, 2$  and  $3$ . It can be seen that when  $\varphi_k = 1$  or  $2$ , there is overlapping or spectral folding, which causes loss of information as shown in Figures 4(a) and 4(b), respectively. However, in Figure 4(c), there is no overlapping for  $\varphi_k = 3$ . The last column of Table 1 contains the sequences for each sampling frequency.



**Figure 4.** In (a) and (b) there is overlapping and spectrum folding for  $\phi_k = 1$  and  $2$  respectively, whereas in (c), all 6 signals are visible.

Realization of the minimum sampling rate heavily depends on the minimum frequency in the band of interest. However, for any deterministic number of input signals in the desired spectrum, it is not permissible to plan  $f_{\min}$  at any arbitrary frequency. This is the reason that the suggested  $f_{\min}$  (shown as  $f_{L1}$  by Muñoz-Ferreras et al. in [13]) is restricted to a set of frequencies  $\in 2kNB$ . Consider the example given in [13] for  $N = 4$  and  $\Delta f = 2B$ . It can be seen from Table 2 that  $f_{\min}$  is restricted to the set of frequencies. This imposes the necessary condition that the lowest frequency in the spectrum of interest should either be 0 or a multiple of 8, i.e.  $\text{mod}(f_{\min}, 8) = 0$ ; otherwise there will be overlapping. In other words, it can be concluded that it has a frequency selection flexibility of 1 to  $8B$ . On the other hand, in our proposed technique,  $f_{\min}$  can be placed at any frequency that is an even multiple of bandwidth  $B$ . Hence, it has a frequency selection flexibility of 1 to  $2B$ , which is 4 times greater than in [13].

**Table 2.** A spectrum of nonzero energy contents with sparsity ratio 1 to 6. The last column lists values of  $\varphi$  to avoid aliasing for given minimum sampling frequencies.

$\gamma$	No. of iterations	$f_s$	$\varphi_k$
1	1	$2NB$	1, 13, 25, 37, . .
2	1	$2NB$	1, 3, 5, 7. . .
3	2	$(2N + 1)B$	3, 6, 16, 19, 29, 32. . .
4	2	$(2N + 1)B$	4, 8, 17, 21, 30, 34. . .
5	1	$2NB$	5,17, 29. . .
6	2	$(2N + 1)B$	6, 12, 19, 25, 32, 38. . .

Another advantage of our proposed method is its flexibility in terms of  $\gamma$ . Flexibility in terms of sparsity ratio  $\gamma$  offered by [13] as shown in Table 3 is applicable to 37.5% of the values of  $\gamma$ . On the other hand, our proposal is applicable to 100% of the values of  $\gamma$ . This clearly indicates that the proposed method is more universal in terms of sparseness in the spectrum of interest, and is also applicable to a greater number of signal locations. Finally, to make the advantages more evident, some statistics for different numbers of input bands ranging from  $N = 2$  to  $N = 12$  are shown in Table 4. This clearly shows that the useable average value of  $\gamma$  for [13] is 0.404 (40.4%). On the other hand, our proposed technique is applicable for all (100%) integer values of  $\gamma$ .

**Table 3.** For  $N = 4$ , the proposed technique is more flexible and versatile to apply over a wide variety of sparse spectrums.

$\gamma = \Delta f/B$	$f_{\min}$		Spectrum usage flexibility	
	Muñoz-Ferreras et al.	Proposed	Muñoz-Ferreras et al.	Proposed
2	0, 8, 16, 24. . .	0, 1, 2, 3. . .	1/8B	1/2B
3	None	2, 10, 18. . .	-	7/50B
4	None	3, 7, 12, 16, 21. . .	-	11/48B
5	None	4, 12, 20. . .	-	7/52B
6	0, 8, 16, 24. . .	0, 1, 2, 3. . .	1/8B	1/2B
7	None	6, 14, 22. . .	-	7/54B
8	None	7, 8, 16, 17, 25, 26. . .	-	3/13B
9	0, 8, 16, 24. . .	0, 1, 2, 3. . .	1/8B	7/50B

**Table 4.** A comparison of the number of ratios  $\Delta f/B$  where  $f_s$  is applicable.

N	Values of $\Delta f/B$		Normalized sparsity ratio	
	Muñoz-Ferreras et al. [13]	Proposed	[13]	Proposed
2	1, 2, 5, 6, 9, 10, 13, 14, 17, 18, 21, 22, 25, 26, 29, 30. . .	1, 2, 3. . .	.48	1
3	1, 2, 4, 7, 8, 10, 13, 14, 16, 19, 20, 22, 25. . .	1, 2, 3. . .	.52	1
4	1, 2, 6, 9, 10, 14, 17, 18, 22, 25. . .	1, 2, 3. . .	.4	1
5	1, 2, 4, 6, 8, 11, 12, 14, 16, 18, 21, 22, 24. . .	1, 2, 3. . .	.52	1
6	1, 2, 10, 13, 14, 22, 25, 26. . .	1, 2, 3. . .	.28	1
7	1, 2, 4, 6, 8, 10, 12, 15, 16, 18, 20, 22, 24. . .	1, 2, 3. . .	.52	1
8	1, 2, 6, 10, 14, 17, 18, 22, 26, 30. . .	1, 2, 3. . .	.32	1
9	1, 2, 4, 8, 10, 14, 16, 19, 20, 22. . .	1, 2, 3. . .	.40	1
10	1, 2, 6, 14, 18, 21, 22, 26. . .	1, 2, 3. . .	.28	1
11	1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 23. . .	1, 2, 3. . .	.48	1
12	1, 2, 10, 14, 22, 25, 26. . .	1, 2, 3. . .	.24	1

In the case of nonuniform signal spacing, the results are shown in terms of computation complexity and the number of iterations required to determine sampling frequency. Table 5 shows the results for different combinations of multiband bandpass signals without any ordering constraint. It is evident from all 3 cases that we can successfully calculate sampling frequency without much computational complexity.

**Table 5.** Complexity comparison for finding the minimum sampling frequency without an ordering constraint for various combinations of bandpass signals.

Bands	Method in [17]			Method in [11]			Proposed		
	ADD	MUL	iter	ADD	MUL	iter	ADD	MUL	iter
GSM900, GSM1800	38	50	27	17	28	7	6	11	1
GSM900, WCDMA	101	176	90	21	29	7	6	11	1
DAB, WCDMA	452	878	441	77	141	34	6	11	1

## 6. Conclusion

The proposed direct RF sampling methodology achieves an absolute freedom from aliasing overlap and spectrum folding for any number of input bands using a simple low-cost ADC. The proffered relaxation to analogue-to-digital converter (ADC) technology in terms of sampling rates is highly advantageous in the ultra-high-frequency (UHF) band for wideband receivers. This also includes the associated advantages like size, cost, and power consumption. In addition, these advantages are applicable in the design of SDRs and reducing the detection time for cognitive radios. That there are no intermediate stages involved in translation of signals down to baseband is a great advantage. The given algorithm also leads to computation of the optimal sampling frequency with the least possible number of iterations, giving it an edge over contemporary iterative algorithms. Although the prescribed methodology is developed for equally spaced bands, it can also be extended to nonuniformly spaced energy contents, and can be effectively used for real applications with some modifications. In case of asymmetric composition, RF spectrum may be translated in a contiguous symmetric band to carry out further signal processing.

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