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The self-adaptive alternating direction method for the multiarea economic dispatch problem

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Abstract: The alternating direction method has been used widely in the power systems field for solving the multiarea dispatch problem. However, experience with applications has shown that the convergence rate of the alternating direction method depends significantly on the selection of the penalty parameter of the linear consistency constraint. Typically, it is difficult to obtain the optimal penalty parameter in advance. In this paper, for the purpose of solving this problem, we propose centralized and distributed self-adaptive penalty parameter strategies that allow the value of the penalty parameter to increase or decrease based on the information from each iteration. Simulation results illustrate that the proposed centralized and distributed self-adaptive methods are superior to the traditional alternating direction method in terms of robustness and convergence rate.

Key words: Multiarea economic dispatch, alternating direction method, variational inequality, self-adaptive

1. Introduction

The objective of economic dispatch (ED) is to allocate generator output economically while meeting various physical constraints, such as power balance and limits on variables [1,2]. The modern power system is composed of distributed subnetworks interconnected by tie-lines. Generally, each subnetwork has an independent energy management system (EMS), so it is necessary to propose effective strategies to solve the multiarea economic dispatch (MAED) problem in distributed computing environments [3,4].

The MAED problem can be described as a separable convex programming problem with linear consistency constraints. The augmented Lagrangian relaxation (ALR) method is widely used to cope with linear consistency constraints. Compared with the classic Lagrangian relaxation (CLR) method, a significant advantage of ALR is that it ensures the convex property of the objective function and has better convergence performance. However, ALR also brings some new challenges, for instance destroying the separable property of the objective function.

Plenty of methods have been proposed to solve the nonseparable quadratic term that was introduced by the ALR method. In 1980, Cohen proposed the auxiliary problem principle (APP) to decompose a centralized problem into subproblems and coordinate these subproblems [5]. In 1992 the APP was first employed to deal with the daily generation scheme optimization problem [6]. Since then, the APP has been widely applied to the multiarea dispatch problem [7,8]. Another powerful decomposition method, the alternating direction method (ADM), which was proposed by Gabay and Mercier, has been widely used to solve the multiarea dispatch problem [9,10].

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Roughly speaking, the APP is a nonlinear Jacobi iterative method, and ADM is a nonlinear Gauss–Seidel iterative method. This means that the computation of the APP for the current iteration only takes advantage of the information of the last iteration. By contrast, the ADM method incorporates the new iterative information that has been generated in the current iteration. Therefore, the ADM method is expected to have a better convergence performance [11].

In addition, experience with applications illustrates that the choice of the penalty parameter for linear consistency constraints has a significant influence on the convergence performance of the APP and ADM. In this paper, taking advantage of the concept of balance that was proposed by He et al. [12], we propose two novel self-adaptive penalty parameter strategies for the ADM method. For different test systems with a variety of penalty parameters, simulation results testify that the proposed strategies are correct and effective, in addition to having strong robustness in terms of the selection of penalty parameter.

2. Problem formulation

2.1. Centralized economic dispatch formulation

The aim of ED is to allocate generator output economically while satisfying various physical constraints. The corresponding mathematical formulation for a centralized ED problem can be expressed as follows [13]:

$$\begin{aligned} &\min\ F(x) \\ &subjectto: g(x) \leq 0 \\ &h(x) = 0 \end{aligned} \tag{1}$$

where $F(x)$ is an objective function and denotes total fuel cost; x is the vector of control and state variables, including the real power output of the generating unit and tie-line power flow between different areas. The function $h(x)$ represents power balance constraints; $g(x)$ denotes tie-line power flow constraints and generator capacity constraints.

2.2. Duplication of variables

First let us start with the simplest case, a two-area ED problem, and then Eq. (1) can be expressed as follows:

$$\begin{aligned} &\min\ f_1(x_{I1}, y) + f_2(x_{I2}, y) \\ &subjectto: g_1(x_{I1}, y) \leq 0 \\ &h_1(x_{I1}, y) = 0 \\ &g_2(x_{I2}, y) \leq 0 \\ &h_2(x_{I2}, y) = 0 \end{aligned} \tag{2}$$

where f_1 and $f_2$represent the total fuel cost in area 1 and area 2, respectively. x_{Ii}denotes real power output of generating units in area i; $y$denotes tie-line power flow between two areas. Then $\{g_i,\ h_i\}$represents the corresponding constraints for area i.

It is clear that $\{g_1,\ h_1\}$and $\{g_2,\ h_2\}$are coupled. Taking advantage of the concept of duplication of variables [14], y can be duplicated as y_{12} and y_{21} and a new consistency constraint, $y_{12} - y_{21} = 0$, is introduced to the problem of Eq. (2). Then an equivalent form of the problem of Eq. (2) can be expressed as follows:

$$\begin{aligned} &\min \ f_1(x_1) + f_2(x_2) \\ &subjectto : x_1 \in \Omega_1, x_2 \in \Omega_2 \\ &A_{12}x_1 + B_{12}x_2 = 0 \end{aligned} \tag{3}$$

where

$$\begin{aligned} &x_1 = (x_{I1}, y_{12}), x_2 = (x_{I2}, y_{21}) \\ &\Omega_1 = \{x_1 \,| h_1(x_1) = 0, g_1(x_1) \le 0\} \\ &\Omega_2 = \{x_2 \,| h_2(x_2) = 0, g_2(x_2) \le 0\} \\ &A_{12}x_1 + B_{12}x_2 = y_{12} - y_{21} \end{aligned} \tag{4}$$

A_{12} and B_{12} are given constant matrices.

Similarly, by denoting $A_{ij}x_i + B_{ij}x_j = 0$ for the consistency constraint between area i and area j, and using the concept of duplication of variables, a general formulation for the N-area economic dispatch problem can be expressed as follows:

$$\begin{aligned} &\min \ \sum_{i=1}^{n} f_i(x_i) \\ &subjectto : x_i \in \Omega_i, i = 1, 2, \cdots, n \\ &A_{ij}x_i + B_{ij}x_j = 0, i, j = 1, 2, \cdots, nandj > i \end{aligned} \tag{5}$$

2.3. Traditional alternating direction method for MAED problem

In this section, the traditional ADM method for solving the MAED problem is presented. The key concept is addressing the consistency constraint $A_{ij}x_i + B_{ij}x_j = 0$. Taking advantage of the ALR method, the MAED problem shown in Eq. (5) can be transformed into the following optimization problem:

$$\begin{aligned} &\min \ \sum_{i=1}^{n} f_i(x_i) + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(-\lambda_{ij}^T (A_{ij}x_i + B_{ij}x_j) + \tfrac{c_{ij}}{2}\|A_{ij}x_i + B_{ij}x_j\|^2\right) \\ &subjectto : x_i \in \Omega_i i = 1, 2, \cdots, n \end{aligned} \tag{6}$$

where λ_{ij} and c_{ij} are the Lagrangian multiplier and the penalty parameter for consistency constraint $A_{ij}x_i + B_{ij}x_j = 0$. The Euclidean norm of vector x will be denoted with $\|x\|$, i.e. $\|x\| = \sqrt{x^T x}$; the superscript T denotes the transposition of corresponding vector.

In addition, the optimization problem of Eq. (6) is equivalent to solving a saddle-point problem via the following iterative scheme:

$$\left(x_1^{k+1}, \cdots, x_n^{k+1}\right) = \arg\min \left\{ \sum_{i=1}^{n} f_i(x_i) + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(-\lambda_{ij}^{T,k}(A_{ij}x_i + B_{ij}x_j) + \frac{c_{ij}}{2}\|A_{ij}x_i + B_{ij}x_j\|^2\right) | x_i \in \Omega_i, i = 1, 2, \cdots, n \right\} \tag{7}$$

$$\lambda_{ij}^{k+1} = \lambda_{ij}^{k} - c_{ij}\left(A_{ij}x_i^{k+1} + B_{ij}x_j^{k+1}\right), i, j = 1, 2, \cdots, nandj > i \tag{8}$$

where the superscript k is the iteration index.

Now, using the ADM method to cope with the nonseparable quadratic term $\frac{c_{ij}}{2}\left\|A_{ij}x_i+B_{ij}x_j\right\|^2$, which is described in Eq. (7), the iterative scheme of ADM for solving the saddle-point problem of Eqs. (7) and (8) can be expressed as follows [9]:

Step 1. Compute

$$\begin{aligned}x_m^{k+1}=\min\Bigg\{&f_m\left(x_m\right)+\sum_{i=m}\sum_{j=m+1}^{n}\left(-\lambda_{ij}^{T,k}A_{ij}x_i+\frac{c_{ij}}{2}\left\|A_{ij}x_i+B_{ij}x_j^k\right\|^2\right)\\&+\sum_{i=1}^{m-1}\sum_{j=m}\left(-\lambda_{ij}^{T,k}B_{ij}x_j+\frac{c_{ij}}{2}\left\|A_{ij}x_i^{k+1}+B_{ij}x_j\right\|^2\right)|x_m\in\Omega_m\Bigg\},m=1,2,\cdots,n\end{aligned}\tag{9}$$

Step 2. Lagrangian multiplier updating

$$\lambda_{ij}^{k+1}=\lambda_{ij}^k-c_{ij}\left(A_{ij}x_i^{k+1}+B_{ij}x_j^{k+1}\right),i,j=1,2,\cdots,nandj>i\tag{10}$$

Step 3. Check the stop criterion. If

$$\left\|B_{ij}x_j^{k+1}-B_{ij}x_j^k\right\|<\eta,\left\|\lambda_{ij}^{k+1}-\lambda_{ij}^k\right\|<\eta i,j=1,2,\cdots,nandj>i\tag{11}$$

then stop; $\left(x_1^{k+1},x_2^{k+1},\cdots,x_n^{k+1}\right)$ is the solution of the problem of Eq. (6). If not, $k=k+1$. If $k>k_{max}$, the ADM iterative scheme fails to converge with maximum iteration k_{max}, then stop. If not, go to Step 1.

3. Self-adaptive ADM (SADM) for MAED problem

3.1. Centralized self-adaptive ADM (CSADM)

A great number of applications have demonstrated that if the chosen penalty parameter for the consistency constraint is too small or too large, then the ADM iterative scheme needs more iterations to reach the optimum solution. Meanwhile, it is difficult to obtain a proper penalty parameter in advance. Fortunately, inspired by the concept of balance, He et al. proposed a modified alternating direction method for adjusting the penalty parameter [12].

According to He et al. [12], the stop criterion for the problem of Eq. (3) can be expressed as follows:

$$\begin{bmatrix}e_{x_1}^k\\e_{x_2}^k\\e_\lambda^k\end{bmatrix}=\begin{bmatrix}x_1^k-P_{\Omega 1}\left\{x_1^k-\left[\nabla f\left(x_1^k\right)-A_{12}^T\lambda^k\right]\right\}\\x_2^k-P_{\Omega 2}\left\{x_2^k-\left[\nabla g\left(x_2^k\right)-B_{12}^T\lambda^k\right]\right\}\\Ax_1^k+Bx_2^k\end{bmatrix}=0\tag{12}$$

where $P_\Omega\left(\cdot\right)$ is the projection on Ω. $\nabla f\left(\cdot\right)$ and $\nabla g\left(\cdot\right)$ denote the gradient of $f\left(\cdot\right)$ and $g\left(\cdot\right)$, respectively.

In addition, according to the iterative scheme of ADM, we can get

$$e_{x_2}^k=x_2^k-P_{\Omega 2}\left\{x_2^k-\left[\nabla g\left(x_2^k\right)-B_{12}^T\lambda^k\right]\right\}=0\tag{13}$$

Then the error between current iteration $\left(x_1^k,x_2^k,\lambda^k\right)$ generated by the ADM and $\left(x_1^*,x_2^*,\lambda^*\right)$ can be expressed as $\left\|e_{x_1}^k\right\|^2+\left\|e_\lambda^k\right\|^2$. For the sake of balance, we should adjust the penalty factor in order to ensure $\left\|e_{x_1}^k\right\|\approx\left\|e_\lambda^k\right\|$ [12].

Similarly, for the N-area ED problem, the error between the current iteration generated by the ADM and the corresponding optimal value can be expressed as follows:

$$\sum_{m=1}^{n} \left\| e_{x_m}^k \right\|^2 + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left\| e_{\lambda_{ij}}^k \right\|^2 \tag{14}$$

where

$$e_{x_m}^k = x_m^k - P_{\Omega m} \left\{ x_m^k - \left[\nabla f_m \left(x_m^k \right) - \sum_{i=m} \sum_{j=m+1}^{n} A_{ij}^T \lambda_{ij}^k - \sum_{i=1}^{m-1} \sum_{j=m} B_{ij}^T \lambda_{ij}^k \right] \right\}, m = 1, 2, \cdots, n \tag{15}$$

$$e_{\lambda_{ij}}^k = A_{ij} x_i^k + B_{ij} x_j^k, i, j = 1, 2, \cdots, n; j > i \tag{16}$$

For the sake of balance, we should adjust the penalty factor so that $\sum\limits_{m=1}^{n} \left\| e_{x_m}^k \right\|^2 \approx \sum\limits_{i=1}^{n} \sum\limits_{j=i+1}^{n} \left\| e_{\lambda_{ij}}^k \right\|^2$. The corresponding CSADM for adjusting the penalty parameter can be expressed as follows:

$$c_{ij} = \begin{cases} 0.5 c_{ij} if r^{k+1} > 10 \\ 2 c_{ij} if r^{k+1} < 0.1 \\ c_{ij} otherwise \end{cases} \tag{17}$$

where

$$r^{k+1} = \frac{\sqrt{\sum\limits_{m=1}^{n} \left\| e_{x_m}^k \right\|^2}}{\sqrt{\sum\limits_{i=1}^{n} \sum\limits_{j=i+1}^{n} \left\| e_{\lambda_{ij}}^k \right\|^2}} \tag{18}$$

3.2. Distributed self-adaptive ADM (DSADM)

Lemma 1. If sequence $\left\{ v_{ij}^k \right\}$ is generated by the ADM iterative scheme and v_{ij}^* is the optimum solution for corresponding variables v_{ij}, we get

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n} \left\| v_{ij}^k - v_{ij}^* \right\|_{N_{ij}}^2 - \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left\| v_{ij}^{k+1} - v_{ij}^* \right\|_{N_{ij}}^2 > 0 \tag{19}$$

where

$$w_{ij} = \begin{pmatrix} x_i \\ x_j \\ \lambda_{ij} \end{pmatrix}, v_{ij} = \begin{pmatrix} B_{ij} x_j \\ \lambda_{ij} \end{pmatrix}, N_{ij} = \begin{pmatrix} c_{ij} I & 0 \\ 0 & \frac{1}{c_{ij}} I \end{pmatrix} \tag{20}$$

The N_{ij}-norm of vector x is denoted by $\|x\|_{N_{ij}}$, i.e. $\|x\|_{N_{ij}} = \sqrt{x^T N_{ij} x}$.

4. Proof

Using the concept of variational inequality, for the kth iteration, solving the optimization problem of Eq. (9) is equivalent to solving x_m^{k+1}, which satisfies [15]

$$\begin{array}{l} f_m(x_m) - f_m\left(x_m^{k+1}\right) + \left(x_m - x_m^{k+1}\right)^T \\ \left\{ \sum_{i=m} \sum_{j=m+1}^{n} \left(-A_{ij}^T \lambda_{ij}^k + c_{ij} A_{ij}^T \left(A_{ij} x_i^{k+1} + B_{ij} x_j^k\right)\right) + \sum_{i=1}^{m-1} \sum_{j=m} \left(-B_{ij}^T \lambda_{ij}^k + c_{ij} B_{ij}^T \left(A_{ij} x_i^{k+1} + B_{ij} x_j^{k+1}\right)\right) \right\} \geq 0, \\ \forall x_m \in \Omega_m; m = 1, 2, \cdots, n \end{array} \tag{21}$$

and using $\lambda_{ij}^{k+1} = \lambda_{ij}^k - c_{ij}\left(A_{ij} x_i^{k+1} + B_{ij} x_j^{k+1}\right)$, we get

$$\begin{array}{l} \sum_{i=1}^{n} f_i(x_i) - \sum_{i=1}^{n} f_i\left(x_i^{k+1}\right) \\ + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left(w_{ij} - w_{ij}^{k+1}\right)^T \left\{ \begin{pmatrix} -A_{ij}^T \lambda_{ij}^{k+1} + cA^T\left(A_{ij} x_i^{k+1} + B_{ij} x_j^{k+1}\right) \\ -B_{ij}^T \lambda_{ij}^{k+1} + cB^T\left(A_{ij} x_i^{k+1} + B_{ij} x_j^{k+1}\right) \\ \left(A_{ij} x_i^{k+1} + B_{ij} x_j^{k}\right) \end{pmatrix} + \begin{pmatrix} 0 & -c_{ij} A_{ij}^T B_{ij} & A_{ij}^T \\ 0 & 0 & B_{ij}^T \\ 0 & 0 & \frac{1}{c_{ij}} I \end{pmatrix} \left(w_{ij}^{k+1} - w_{ij}^k\right) \right\} \geq 0, \\ \forall w_{ij} \in \Omega_i \times \Omega_j \times R^r \end{array} \tag{22}$$

Setting $w_{ij} = w_{ij}^*$ in Eq. (22), we get

$$\begin{array}{l} \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left(w_{ij}^* - w_{ij}^{k+1}\right)^T \begin{pmatrix} 0 & -c_{ij} A_{ij}^T B_{ij} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{c_{ij}} I \end{pmatrix} \left(w_{ij}^{k+1} - w_{ij}^k\right) \\ \geq \sum_{i=1}^{n} f_i\left(x_i^{k+1}\right) - \sum_{i=1}^{n} f_i\left(x_i^*\right) + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left(w_{ij}^{k+1} - w_{ij}^*\right)^T F\left(w_{ij}^{k+1}\right) \end{array} \tag{23}$$

where

$$F(w_{ij}) = \begin{pmatrix} -A_{ij}^T \lambda_{ij} \\ -B_{ij}^T \lambda_{ij} \\ A_{ij} x_i + B_{ij} x_j \end{pmatrix} \tag{24}$$

Using the concept of variational inequality to deal with the problem of Eq. (5) directly [15], we get

$$\sum_{i=1}^{n} f_i\left(x_i^{k+1}\right) - \sum_{i=1}^{n} f_i\left(x_i^*\right) + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left(w_{ij}^{k+1} - w_{ij}^*\right)^T F\left(w_{ij}^*\right) \geq 0 \tag{25}$$

In fact, F is a monotone operator [16]. We have

$$\left(w_{ij}^{k+1} - w_{ij}^*\right)^T \left(F\left(w_{ij}^{k+1}\right) - F\left(w_{ij}^*\right)\right) \geq 0 \tag{26}$$

where the equality holds up if and only if $w_i^{k+1} = w_{ij}^*$.

If $v_i^{k+1} \neq v_{ij}^*$, then we only can get

$$\left(w_{ij}^{k+1} - w_{ij}^*\right)^T \left(F\left(w_{ij}^{k+1}\right) - F\left(w_{ij}^*\right)\right) > 0 \tag{27}$$

Combining Eqs. (23), (25), and (27), if $v_i^{k+1} \neq v_{ij}^*$, we get

$$\begin{aligned}
&\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(w_{ij}^* - w_{ij}^{k+1}\right)^T \begin{pmatrix} 0 & -c_{ij}A_{ij}^TB_{ij} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{c_{ij}}I \end{pmatrix} \left(w_{ij}^{k+1} - w_{ij}^k\right) > 0 \\
&\Rightarrow \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(v_{ij}^* - v_{ij}^{k+1}\right)^T N_{ij}\left(v_{ij}^{k+1} - v_{ij}^k\right) + \sum_{i=1}^{n}\sum_{j=i+1}^{n} c_{ij}\left(A_{ij}x_i^{k+1} + B_{ij}x_{ij}^{k+1} - A_{ij}x_i^* - B_{ij}x_{ij}^*\right)^T B_{ij}\left(x_j^{k+1} - x_j^k\right) > 0 \\
&\Rightarrow \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(v_{ij}^* - v_{ij}^k + v_{ij}^k - v_{ij}^{k+1}\right)^T N_{ij}\left(v_{ij}^{k+1} - v_{ij}^k\right) - \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(\lambda_{ij}^{k+1} - \lambda_{ij}^k\right)^T B_{ij}\left(x_j^{k+1} - x_j^k\right) > 0 \\
&\Rightarrow \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(v_{ij}^* - v_{ij}^k\right)^T N_{ij}\left(v_{ij}^{k+1} - v_{ij}^k\right) > \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^{k+1} - v_{ij}^k\right\|_{N_{ij}}^2 + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(\lambda_{ij}^{k+1} - \lambda_{ij}^k\right)^T B_{ij}\left(x_j^{k+1} - x_j^k\right) \\
&\Rightarrow \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(v_{ij}^* - v_{ij}^k\right)^T N_{ij}\left(v_{ij}^{k+1} - v_{ij}^k\right) > \frac{1}{2}\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^{k+1} - v_{ij}^k\right\|_{N_{ij}}^2
\end{aligned} \tag{28}$$

Using Eq. (28), we get

$$\begin{aligned}
&\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^k - v_{ij}^*\right\|_{N_{ij}}^2 - \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^{k+1} - v_{ij}^*\right\|_{N_{ij}}^2 \\
&= \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^k - v_{ij}^*\right\|_{N_{ij}}^2 - \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^k - v_{ij}^* - (v_{ij}^k - v_{ij}^{k+1})\right\|_{N_{ij}}^2 \\
&= 2\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(v_{ij}^* - v_{ij}^k\right)^T N_{ij}\left(v_{ij}^{k+1} - v_{ij}^k\right) - \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^{k+1} - v_{ij}^k\right\|_{N_{ij}}^2 \\
&> 0
\end{aligned} \tag{29}$$

If $v_i^{k+1} = v_{ij}^*$, according the stop criterion described in Eq. (11), we get $v_i^k \neq v_{ij}^*$ and

$$\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^k - v_{ij}^*\right\|_{N_{ij}}^2 > 0, \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^{k+1} - v_{ij}^*\right\|_{N_{ij}}^2 = 0 \tag{30}$$

Thus, we get

$$\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^k - v_{ij}^*\right\|_{N_{ij}}^2 - \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^{k+1} - v_{ij}^*\right\|_{N_{ij}}^2 > 0 \tag{31}$$

Based on the above discussion, the proof of Lemma 1 is complete.

It is clear that Eq. (19) is Fejér monotone [17], and so we get

$$\lim_{k\to\infty}\left\|B_{ij}x_j^{k+1} - B_{ij}x_j^k\right\| = 0, \lim_{k\to\infty}\left\|\lambda_{ij}^{k+1} - \lambda_{ij}^k\right\| = 0, i, j = 1, 2, \cdots, nandj > i \tag{32}$$

Eq. (32) is consistent with the ADM's stop criterion. It is clear that $v_{ij}^k \to v_{ij}^*$ when $k \to \infty$. Hence, the magnitude of $\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^{k+1} - v_{ij}^k\right\|_{N_{ij}}^2$ can measure how v_{ij}^k fails to be close to v_{ij}^*. According to the description in Lemma 1, we get

$$\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|v_{ij}^{k+1} - v_{ij}^k\right\|_{N_{ij}}^2 = \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|B_{ij}x_j^{k+1} - B_{ij}x_j^k\right\|_{c_{ij}}^2 + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|\lambda_{ij}^{k+1} - \lambda_{ij}^k\right\|_{\frac{1}{c_{ij}}}^2 \tag{33}$$

For the sake of balance, we only need to adjust the penalty parameter so that

$$\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|B_{ij}x_j^{k+1}-B_{ij}x_j^{k}\right\|_{c_{ij}}^2 \approx \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left\|\lambda_{ij}^{k+1}-\lambda_{ij}^{k}\right\|_{\frac{1}{c_{ij}}}^2, i,j=1,2,\cdots,n and j>i \tag{34}$$

If we adjust the penalty parameter for the sake of balance in Eq. (34) directly, then a large amount of data communication between different areas is needed. For a distributed iterative scheme, a large amount of data communication will lead to communication bottlenecks and may not be suitable for practical applications. In fact, to achieve balance in Eq. (34), we just need to satisfy

$$\left\|B_{ij}x_j^{k+1}-B_{ij}x_j^{k}\right\|_{c_{ij}}^2 \approx \left\|\lambda_{ij}^{k+1}-\lambda_{ij}^{k}\right\|_{\frac{1}{c_{ij}}}^2, i,j=1,2,\cdots,n and j>i \tag{35}$$

The corresponding DSADM for adjusting the penalty parameter can be expressed as follows:

$$c_{ij}=\begin{cases} 0.5c_{ij} if r_{ij}^{k+1}>10 \\ 2c_{ij} if r_{ij}^{k+1}<0.1 \\ c_{ij} otherwise \end{cases} \tag{36}$$

where

$$r_{ij}^{k+1}=\frac{\left\|B_{ij}x_j^{k+1}-B_{ij}x_j^{k}\right\|_{c_{ij}}}{\left\|\lambda_{ij}^{k+1}-\lambda_{ij}^{k}\right\|_{\frac{1}{c_{ij}}}}, i,j=1,2,\cdots,n and j>i \tag{37}$$

5. Simulation

In this section, we employ a 40-unit system and a 10-unit system to demonstrate the convergence performance of the proposed methods. Throughout this paper, the stop criterion is set to be $\eta = 10^{-4}$, the initial Lagrangian multiplier is set to be zero, and the maximum iteration k_{max} is 100. In addition, the optimization problem for each area is solved by the *fmincon* code of the MATLAB optimization toolbox on a PC with Intel E7500 2.93 GHz CPU and 4 GB of RAM.

5.1. Case 1: 40-unit power system with two areas

A 40-unit power system consists of two areas, area 1 and area 2. In area 1, there are 25 units and the demand is set to be 8000 MW. In area 2, there are 15 units and the demand is set to be 2000 MW. The tie-line power flow limit between two areas is set to be 800 MW. Data related to the generator are from Chang et al. [18].

5.2. Case 2: 10-unit power system with three areas

This test system comprises three areas as shown in the Figure. Area 1 is made up of the first four units (P1, P2, P3, and P4). Area 2 is composed of the next three units (P5, P6, and P7). Area 3 consists of the remaining three units (P8, P9, and P10). The total demand is 2700 MW. The corresponding demand in area 1, area 2, and area 3 accounts for 50%, 25%, and 25% of the total demand, respectively. The tie-line power flow limits between any two areas are set to be 100 MW. All data about the generator are from [19]. To be more exact, each generator has three different fuel options. In this paper, we employ fuel option 1 as the fuel cost.

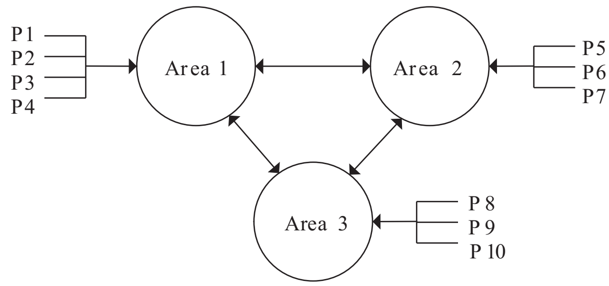

Figure. Three areas, 10-unit system.

5.3. Case 3: IEEE-118 power system with two areas

In this section, we divided the IEEE-118 power system into two areas, area 1 and area 2. Area 1 consists of the first 24 units and the demand is set to be 1883 MW. Area 2 consists of the next 30 units and the demand is set to be 2359 MW. The tie-line power flow limit between two areas is set to be 600 MW. Data related to the generator are from [3].

5.4. Analysis of simulation results

In this section, we employ three different strategies with different penalty factors to solve a multiarea ED problem.

1. Traditional alternating direct method (ADM)
2. Centralized self-adaptive alternating direct method (CSADM)
3. Distributed self-adaptive alternating direct method (DSADM)

The information presented in Tables 1–3 reflects the convergence performance of the three different iterative strategies with different starting penalty parameters. The word “FAIL” means that the corresponding method failed to converge with the maximum iteration number of 100. The lowercase letter “c” denotes an initial given value of a penalty parameter. The words “Iterations” and “CPU time (s)” represent the total number of iterations and CPU time (in seconds) when the stop criterion for the corresponding test systems is satisfied.

As shown in Tables 1–3, it is clear that the ADM is sensitive to the selection of the penalty parameter. In contrast, the proposed CSADM and DSADM methods are robust in terms of the choice of penalty parameter. Compared with the DSADM, the CSADM has two significant deficiencies: 1) CSADM needs a coordinator server that communicates with all areas to collect information. For a distributed iterative scheme, a large amount of data communication will lead to a communication bottleneck and is infeasible for practical applications. 2) Computational load for adjusting the penalty parameter in the CSADM is time-consuming, which means that CSADM needs more computing time for a single iteration. Based on the above discussion, we suggest using DSADM for solving the MAED problem in practice.

6. Conclusion

In this paper the CSADM and DSADM methods are proposed to solve the MAED problem. CSADM and DSADM take into consideration the balance of the stop criterion, and this gives us an insight into how to adjust the penalty parameter for consistency constraints between interconnected areas, that is, how to adopt

Table 1. Comparison of results for case 1.

c	ADM		CSADM		DSADM		Total cost ($/h)
	Iterations	CPU time (s)	Iterations	CPU time (s)	Iterations	CPU time (s)	
10^2	Fail	Fail	42	11.09	22	3.59	1.3602×10^5
10^1	Fail	Fail	53	12.50	22	3.81	
10^0	70	12.27	41	9.46	15	2.71	
10^{-1}	16	3.77	21	5.14	16	2.52	
10^{-2}	Fail	Fail	12	2.74	9	1.50	
10^{-3}	Fail	Fail	13	3.06	13	2.09	
10^{-4}	Fail	Fail	16	3.55	17	2.57	
10^{-5}	Fail	Fail	21	4.26	19	2.27	
10^{-6}	Fail	Fail	22	4.26	23	2.62	

Table 2. Comparison of results for case 2.

c	ADM		CSADM		DSADM		Total cost ($/h)
	Iterations	CPU time (s)	Iterations	CPU time (s)	Iterations	CPU time (s)	
10^2	Fail	Fail	35	10.01	48	7.57	718.0707
10^1	Fail	Fail	30	8.84	18	3.52	
10^0	Fail	Fail	30	8.95	28	4.51	
10^{-1}	Fail	Fail	32	9.08	29	4.66	
10^{-2}	76	11.96	28	8.14	37	6.44	
10^{-3}	25	4.95	34	9.70	22	4.24	
10^{-4}	Fail	Fail	44	12.09	31	5.73	
10^{-5}	Fail	Fail	43	12.23	36	6.36	
10^{-6}	Fail	Fail	47	13.12	32	5.69	

Table 3. Comparison of results for case 3.

c	ADM		CSADM		DSADM		Total cost ($/h)
	Iterations	CPU time (s)	Iterations	CPU time (s)	Iterations	CPU time (s)	
10^2	Fail	Fail	33	14.26	24	9.98	1.2595×10^5
10^1	Fail	Fail	33	13.28	25	9.06	
10^0	Fail	Fail	30	11.53	28	9.49	
10^{-1}	79	26.99	Fail	Fail	25	9.01	
10^{-2}	24	7.96	46	20.82	23	7.92	
10^{-3}	66	23.18	40	16.86	28	9.71	
10^{-4}	Fail	Fail	52	21.51	35	10.82	
10^{-5}	Fail	Fail	49	19.85	34	10.32	
10^{-6}	Fail	Fail	46	17.55	39	11.58	

a self-adaptive penalty parameter strategy instead of a fixed penalty parameter. Simulation results illustrated that the CSADM and DSADM are superior to the ADM in terms of robustness and convergence rate. Moreover, unlike the CSADM, the DSADM does not need to create a coordinator server to exchange data from all areas, and it just uses the exiting data in each area to update the penalty parameter, so the DSADM is more suitable for implementation in distributed environment.

Nomenclature

ED	Economic dispatch
MAED	Multiarea economic dispatch
EMS	Energy management system
ALR	Augmented Lagrangian relaxation
CLR	Classic Lagrangian relaxation
APP	Auxiliary problem principle
ADM	Alternating direction method
CSADM	Centralized self-adaptive alternating direction method
DSADM	Distributed self-adaptive alternating direction method
g_i	The equality constraints for area i
h_i	The inequality constraints for area i
$f_i(x_i)$	The total fuel cost for area i
λ_{ij}	The Lagrangian multiplier for consistency constraint between area i and area j
c_{ij}	The penalty parameter for consistency constraint between area i and area j

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