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Unknown input observer based on LMI for robust generation residuals

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Abstract: In this paper, a method of generating robust residuals of a linear system, subject to unknown inputs, is proposed. The impact of disturbances and uncertainty may create difficulties at the decision stage of diagnosis (false alarm); this has resulted in the use of a robust observer for the unknown inputs to ensure the robustness of the system based on the unknown input observer with an optimal decoupling approach, which has a sensitivity that is minimal to unknown inputs and maximal to faults. A generation of robust residuals is then transformed into a problem of robustness/sensitivity constraints (H_∞, H_-) and then solved via a linear matrix inequality formulation by the solver CVX. An application for the method performance is also given.

Key words: Unknown input observer, residual generation, robustness, linear matrix inequality formulation

1. Introduction

In the context of linear systems, the generation of residuals and the detection of faults based on observers of states are effective. Observer − based residual generation is a technique that is well developed. Based on a good operating system model, this technique consists of performing a states estimation given the inputs and outputs of the system. The residual vector is then constructed as the difference between the estimated output and the measured output, using the output error estimation. This residual is sensitive to faults $f(t)$ and to unknown inputs $d(t)$, as well. The observers were created for purely technological and commercial reasons (cost minimization), as hardware sensors are replaced by software sensors that allow reconstructing internal information (states, unknown inputs, unknown parameters) of the system from a model that involves unknown inputs.

The unknown input observer (UIO), with approximate decoupling, could solve the problem of adjusting the sensitivity to various faults and disturbances, as well as the problem of optimization. by introducing their state matrices into the equations of observer synthesis for residual generation, whose decision-making requires comparing the indicator of faults with the empirical or theoretical threshold obtained. Robustness is the main element in the synthesis of this observer in model − based diagnosis, which means determining the ability of such a method by detecting faults with few false alarms (no fault alarm).

The literature offers several works in this field. Wang et al. [1] were the first to use the UIO design problem in systems with some unknown inputs. In the mid-1980s, Viswanadham and Srichander [2] introduced observers to detect faults.

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Golub and Van Loan [3] and Rambeaux et al. [4] introduced the standard $H_\infty H_-$, which reflects the maximum and minimum gain values, respectively, between signals in the optimization of residuals.

Moreover, Ding et al. [5] worked on H_∞ techniques; Wang and Lum [6] used techniques that are based on the linear matrix inequality (LMI) method, which has become an active theme.

Other methods have been developed in this context by Ding and Frank [7], Jiang and Chowdhury [8], Johansson et al. [9], Meseguer et al. [10], Khan and Ding [11], Chen and Patton [12], Mangoubi [13], Rank and Niemann [14], and Henry and Zolghadri [15,16].

In this context, the generation of residual systems based on linear models has been the subject of several research studies using the UIO; among these are the works of Kiyak et al. [17,18], Hajiyev and Caliskan [19], and Patton et al. [20]. Observer-based approaches have become the most popular and important methods for model-based fault detection and isolation, as in the studies of Fonod et al. [21], Bagherpour and Hairi-Yazdi [22], and Hamdaoui et al. [23].

Toscano [24] proposed a new structure for generating robust residuals based on LMI. This technique was resolved by the solver CVX. Based on the aforementioned application, we conducted a detailed investigation on the detection of faults.

2. Problem description

The problem is to generate a residual such that

$$\begin{cases} \sup\limits_{\omega} \bar{\sigma}(Gd(j\omega)) < \gamma \\ \inf\limits_{\omega} \underline{\sigma}\,(Gf\,(j\omega)) > \beta \end{cases}. \tag{1}$$

$\overline{\sigma}$ and $\underline{\sigma}$ are the largest and the smallest singular values, respectively. $G_f\,(j\omega)$ *and* $G_d(j\omega)$ are transfer matrices that link residuals to faults and unknown inputs, respectively.

α and β are the levels of sensitivity to disturbances d and faults f.

γ must take the smallest singular value of G_d and β the largest singular value of G_f.

The detection problem can be redefined as a problem of approximate decoupling [25].

$$\begin{cases} \|\psi(d\,(t)\,,0)\|_n < \gamma \\ \|\Psi(d\,(t)\,,f\,(t)\,)\|_n > \beta \end{cases} \tag{2}$$

$\|.\|_n$ with $n = \{1,2,\infty\}$ is a standard.

After reformulation of Eq. (2), we have

$$J_{+/-} = \frac{\|G_d(p)\|_\infty}{\|G_f(p)\|_-}. \tag{3}$$

$\|G_d\|_\infty$ is the standard H_∞ of transfer between the indicator signal r and the disturbances d.

$\|G_f\|_-$ is the index H_- of transfer between the indicator signal r and the faults f to be detected.

The index H_- is defined on a specified frequency range within which we want to reach the desired sensitivity.

It is assumed that the residual r depends only on disturbances d and faults f, via a vector function ψ such that

$$r\,(t) = \psi\,(d\,(t)\,,f\,(t))\,. \tag{4}$$

The residual vector should be almost zero in normal operation and nonzero in the presence of a fault.

The uniqueness of a solution of Eq. (3) was demonstrated by Ding et al. [1], when only additive uncertainty is considered.

Of course, it is necessary to use the matrices V and K in the context of solving the optimization problem.

$$min_{K,V}\frac{\|G_d(p,K,V)\|_\infty}{\|G_f(p,K,V)\|_-} \tag{5}$$

$G_f\,(p,K,V)$ *and* $G_d\,(p,K,V)$ are transfer matrices that link residuals to faults and unknown inputs, respectively; K and V are adjustment matrices.

We will later present the approach that solves this min/max problem in the form of very simplified LMIs; this allows a rapid and accurate numerical resolution [25].

3. Designing an UIO with approximate optimal decoupling

The perfect decoupling can only be implemented if the number of independent measurements is greater than the number of unknown inputs we want to decouple. Practically, this decoupling condition is not always satisfied.

Imperfect modeling of the system can greatly influence the detection and location of faults; this means that we need to have a new model that takes into account the modeling uncertainties and faults' effects on the behavior of the nominal system.

The system to monitor is supposed to be correctly described by the following state representation:

$$\begin{cases} \dot{x}\,(t)=Ax\,(t)+Bu\,(t)+F_xf\,(t)+D_xd(t) \\ y\,(t)=Cx\,(t)+Du\,(t)+F_yf\,(t)+D_yd\,(t) \end{cases}. \tag{6}$$

$$x\,(t)\in R^n u\,(t)\,\epsilon R^m d\,(t)\in R^{nd} f(t)\in R^{nf}$$

D_x and D_y are action matrices of disturbances $d(t)$, and F_x and F_y are action matrices of faults $f(t)$ to be detected.

The structure of the UIO adopted with approximate optimal decoupling is as follows [24]:

$$\begin{cases} \dot{\hat{x}}\,(t)=A\hat{x}\,(t)+Bu\,(t)+F_xf\,(t)+D_xd(t) \\ \hat{y}\,(t)=C\hat{x}\,(t)+Du\,(t) \\ r\,(t)=V\,(y\,(t)-\hat{y}\,(t)) \end{cases}. \tag{7}$$

$r(t)$ is the residual vector of Eq. (5).

The block diagram of the UIO is shown in Figure 1.

Let $e_x\,(t)=x\,(t)-\hat{x}\,(t)$ be the state estimation error.

$$\begin{cases} \dot{e_x}\,(t)=(A-KC)\,e_x+(F_x-KF_y)\,f\,(t)+(D_x-KD_y)\,d(t) \\ r\,(t)=VCe_x\,(t)+VF_yf\,(t)+VD_yd\,(t) \end{cases} \tag{8}$$

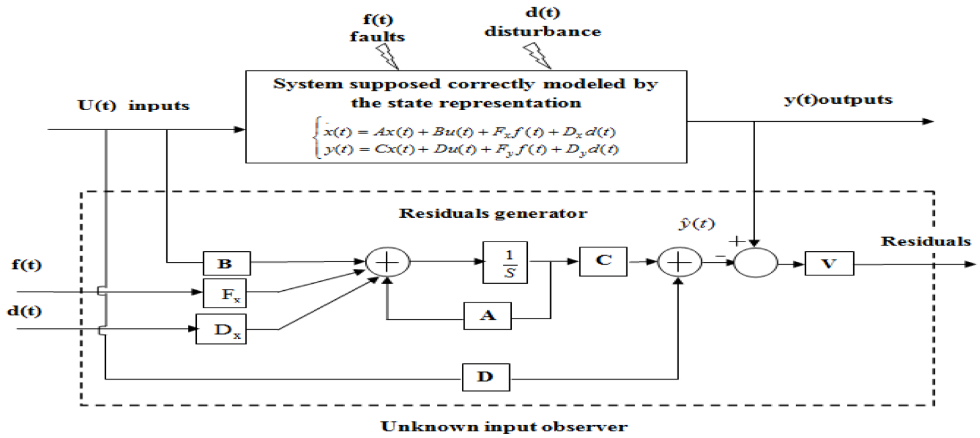

Figure 1. Block diagram of UIO.

Using Laplace transform of $r(t)$ (zero initial conditions), we get

$$r(p) = G_f(p,K,V)\, f(p) + G_d(p,K,V)\, d(p). \tag{9}$$

The transfer matrices G_f and G_d are given by

$$G_f(p,K,V) = VC[pI-(A-KC)]^{-1}(F_x - KF_y) + VF_y, \tag{10}$$

$$G_d(p,K,V) = VC[pI-(A-KC)]^{-1}(D_x - KD_y) + VD_y. \tag{11}$$

In order to achieve an effective resolution, we introduce an LMI formulation of the optimization problem that satisfies the two conditions of robustness and sensitivity.

4. Joint synthesis H$_\infty$ / H$_-$ of a residual generator

In order to solve the optimization problem, the two constraints H$_\infty$ / H$_-$ are checked in the form of an LMI, using the conditions of robustness and sensitivity, respectively.

4.1. The robustness condition

The purpose is to make the residual generator less sensitive to unknown inputs. This condition can be satisfied by imposing $\|G_d\|_\infty < \gamma$.

If we can find three matrices $P = P^T > 0$, K, and V, the following matrix inequality is satisfied:

$$\begin{bmatrix} (A-KC)^T P + P(A-KC) + C^T V^T VC & P(D_x - KD_y) + C^T V^T V D_y \\ (P(D_x - KD_y) + C^T V^T V D_y)^T & D_y^T V^T V D_y - \gamma^2 I \end{bmatrix} < 0. \tag{12}$$

Note that the above inequality of Eq. (12) has the disadvantage of being nonlinear with respect to the variables K, P, V. A resolution method can be used for a change of variables; it allows us to have a linear LMI.

Put $Q = V^T V L = PK$, and hence

$$\begin{bmatrix} (A^T P + PA - LC - C^T L^T + C^T QC & PD_x - LD_y + C^T Q D_y \\ (PD_x - LD_y + C^T Q D_y)^T & D_y^T Q D_y - \gamma^2 I \end{bmatrix} < 0. \tag{13}$$

Therefore, Eq. (13) is an LMI problem, which is to be solved with respect to $L, P,$ and Q, for a given value of γ.

4.2. The condition of sensitivity to faults

The purpose is not only to make the residual generator insensitive to unknown inputs but also to make it as sensitive as possible to faults. This condition can be satisfied as we consider $\|G_f\|_- > \beta$.

If $P = P^T > 0$, then K and V will allow satisfying this matrix inequality, which is not linear.

$$\begin{bmatrix} C^TV^TVC-(A-KC)^TP-P(A-KC) & C^TV^TVF_y-P(F_x-KF_y) \\ (C^TV^TVF_y-P(F_x-KF_y))^T & F_y^TV^TVF_y-\beta^2 I \end{bmatrix} > 0 \tag{14}$$

We obtain Eq. (15) where the change of variables is performed to pass to the next linearity: $Q{=}V^TVL{=}PK$.

$$\begin{bmatrix} C^TQC - A^TP - PA + C^TL^T + LC & C^TQF_y - PF_x + LF_y \\ (C^TQF_y - PF_x + LF_y)^T & F_y^TQF_y - \beta^2 I \end{bmatrix} > 0 \tag{15}$$

The joint synthesis is then introduced, with $g = \gamma^2$ and $b = \beta^2$, $P = P^T > 0$.

P is a symmetric positive definite matrix.

If $P{=}P^T{>}$,

$$\begin{bmatrix} (A^TP+PA-LC-C^TL^T+C^TQC & PD_x-LD_y+C^TQD_y \\ (PD_x-LD_y+C^TQD_y)^T & D_y^TQD_y-\gamma^2 I \end{bmatrix} <0, \tag{16}$$

$$\begin{bmatrix} C^TQC-A^TP-PA+C^TL^T+LC & C^TQF_y-PF_x+LF_y \\ (C^TQF_y-PF_x+LF_y)^T & F_y^TQF_y-\beta^2 I \end{bmatrix} >0. \tag{17}$$

P_{opt}, L_{opt}, g_{opt} give the solution of the optimization problem.

$$K_{opt}{=}P_{opt}^{-1}L_{opt}, V_{opt}{=}Q_{opt}^{\frac{1}{2}}$$

K_{opt}, V_{opt} are the matrices of the robust residual generator.

5. Fault detection

According to Ding and Frank [26], the detection threshold is defined as the maximum value of the function $J(r)$ in the absence of faults, or in normal operation.

$$J(r,\varepsilon) = [\frac{1}{2\Pi\varepsilon} \int_{\omega_1}^{\omega_1+\varepsilon} r^*(j\omega)\, r(j\omega)\, d\omega]^{1/2} \tag{18}$$

$J^2(r,\varepsilon)$ is the average power of residual $r(j\omega)$ on the window of frequencies $[\omega_1\omega_1 + \varepsilon]$.

In approximate decoupling, the thresholds are calculated using the two optimization conditions.

6. Application

Consider the model under representation in Eq. (6), with two outputs and four states, given by the following states form.

$$A=\begin{bmatrix} -2.121 & -0.5624 & -0.2651 & -0.25 \\ 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \end{bmatrix} B=\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} F_x=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} D=0 D_x=\begin{bmatrix} 0.02 & -0.02 & 0 \\ 0.02 & 0.1 & 0 \\ 0.02 & -0.02 & 0 \\ 0.02 & 0.1 & 0 \end{bmatrix}$$

$$C=\begin{bmatrix} -1.414 & -0.4374 & -0.1768 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad F_y=\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad D_y=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In this application, the output $y(t)$ and the internal states support unknown additive inputs $d(t)$.

The MATLAB software program is used to solve the convex optimization. The program developed is based on commands of the solver CVX (20) (semidefinite program (SDP) - LMI). This solver is known for high-speed digital resolution, accuracy, and robustness.

Variation matrices V and K are the solutions of the optimization problem.

For $b = 0.1$, $\beta=\sqrt{b}= 0.3162$, $\gamma_{opti}= 1.1385$, we have:

$$V=\begin{bmatrix} 0.4125 & -0.2617 \\ -0.2617 & 0.6622 \end{bmatrix}, \quad K=\begin{bmatrix} 0.4278 & 0.0313 \\ 0.0919 & 0.3649 \\ -0.1896 & 0.7829 \\ -0.0613 & 0.6027 \end{bmatrix},$$

$$r(p)=G_f(p,K,V)\, f(p)+G_d(p,K,V)\, d(p).$$

$G_{f_{ij}}$ is defined as follows.

$$G_{f11}=\frac{0.825P^4+1.744P^3+1.992P^2+1.522P+0.8463}{P^4+2.229P^3+2.61P^2+1.962P+1.133}$$

$$G_{f12}=\frac{-0.5234P^4-1.091P^3-1.271P^2-0.998P-0.5492}{P^4+2.229P^3+2.61P^2+1.962P+1.133}$$

$$G_{f21}=\frac{-0.5234P^4-1.046P^3-1.122P^2-0.8317P-0.3929}{P^4+2.229P^3+2.61P^2+1.962P+1.133}$$

$$G_{f22}=\frac{1.324P^4+2.802P^3+3.162P^2+2.352P+1.317}{P^4+2.229P^3+2.61P^2+1.962P+1.133}$$

The transfer matrix G_d that relates the unknown input to the residual vector is

$$G_d(p,K,V)=VC\left[pI-(A-KC)\right]^{-1}(D_x-KD_y)+VD_y,$$

$$G_d(p,K,V)=\begin{bmatrix} G_{d11} & G_{d12} \\ G_{d21} & G_{d22} \end{bmatrix}.$$

$G_{d_{ij}}$ is the transfer function of the transfer matrix G_d.

$$G_{d11}=\frac{-0.01971P^3-0.03173P^2-0.02286P-0.01694}{P^4+2.229P^3+2.61P^2+1.962P+1.133}$$

$$G_{d12}=\frac{0.4125P^4+1.074P^3+1.466P^2+1.133P+0.71}{P^4+2.229P^3+2.61P^2+1.962P+1.133}$$

$$G_{d21}=\frac{0.02243P^3+0.0376P^2+0.03341P+0.03458}{P^4+2.229P^3+2.61P^2+1.962P+1.133}$$

$$G_{d22}=\frac{-0.2617P^4-0.6014P^3-0.7642P^2-0.5314P-0.5584}{P^4+2.229P^3+2.61P^2+1.962P+1.133}$$

$$r\left(p\right)=\begin{bmatrix} G_{f11} & G_{f12} \\ G_{f21} & G_{f22} \end{bmatrix} f\left(p\right)+\begin{bmatrix} G_{d11} & G_{d12} \\ G_{d21} & G_{d22} \end{bmatrix} d(p)$$

Then a table of theoretical signatures, generated by the set of signals r_i defined as

$$r_i\left(p\right)=\begin{cases} 1 \text{ if the residue is sensitive to } f_i \\ 0 \text{ if the residue is insensitive to } f_i \end{cases},$$

is drawn up. In the Table , "1" means that fault f_i will certainly affect residual r_i, while "0" means that the residual is insensitive to the fault.

Table. Table of theoretical fault signatures.

	f_1	f_2	d_1	d_2
Residual 1	1	1	0	1
Residual 2	1	1	0	1

According to $r(p)$, when the transfer matrices G_f and G_d are evaluated for $p\rightarrow\infty$, we verify that the zero elements correspond to 0, and nonzero elements to 1, in the table of signatures [27].

The structure of the residual generator adopted is (Eq. (7)) as follows.

$$\begin{cases} \hat{\dot{x}}\left(t\right)=\begin{bmatrix} -2.121 & -0.5624 & -0.2651 & -0.25 \\ 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \end{bmatrix} x\left(t\right)+\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u\left(t\right)+\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} f\left(t\right) \\ +\begin{bmatrix} 0.2 & -0.2 & 0 \\ 0.2 & 0.1 & 0 \\ 0.2 & -0.2 & 0 \\ 0.2 & 0.1 & 0 \end{bmatrix} d\left(t\right) \\ \hat{y}=\begin{bmatrix} -1.41 & -0.4374 & -0.1768 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \hat{x} \\ r\left(t\right)=V\left(y\left(t\right)-\hat{y}\left(t\right)\right) \end{cases}$$

$$V=\begin{bmatrix} 0.4125 & -0.2617 \\ -0.2617 & 0.6622 \end{bmatrix}$$

We have the state representation of the residual generator as follows.

$$\begin{cases} \hat{\dot{x}}(t)=\begin{bmatrix} -2.121 & -0.5624 & -0.2651 & -0.25 \\ 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \end{bmatrix} x(t) \\ +\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)+\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} f(t)+\begin{bmatrix} 0.2 & -0.2 & 0 \\ 0.2 & 0.1 & 0 \\ 0.2 & -0.2 & 0 \\ 0.2 & 0.1 & 0 \end{bmatrix} d(t) \\ \hat{y}=\begin{bmatrix} -1.41 & -0.4374 & -0.1768 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\hat{x} \\ r(t)=\begin{bmatrix} 0.4125 & -0.2617 \\ -0.2617 & 0.6622 \end{bmatrix}(y(t)-\hat{y}(t)) \end{cases}$$

$r(t)$ is the residual vector. The signature table is established from the following reasoning: the observer builds residuals 1 and 2 of the system; if the output gives a fault, then it will be evaluated and will directly present the fault. Therefore, if residuals r_1 and r_2 deviate from the threshold interval, fault f_1 or fault f_2 will certainly appear. Thus, with this observer, we have a good level of fault detection.

In the application, faults f_1 and f_2 are supposed to be defined for $t \geq 3s$.

The detection threshold is determined by simulations, with no faults, of the residual generator obtained in normal operation. It is set to $\pm\gamma > 1.1385$ (the highest singular value of the transfer function of unknown inputs G_d).

7. Results and discussion

The use of the proposed observer allows designing a residual generator to achieve a good level of fault detection. The strategy used here is to design an observer with minimum sensitivity to disturbance and maximum sensitivity to the faults of the system to be monitored. As an application example, our system has two outputs.

Simulating the system presented in the previous section allows finding the residuals shown in the figures, with the detection threshold determined in normal system operation. The fault affecting the two residuals is an amplitude bias between 3 and 4, occurring at time $t \geq 3s$. The analysis of residuals 1 and 2 by the proposed observer allows concluding that there is a fault, indeed.

Figures 2–5 give the theoretical results; Figures 6–9 indicate the two residuals associated with the observer in the absence or presence of faults and disturbances.

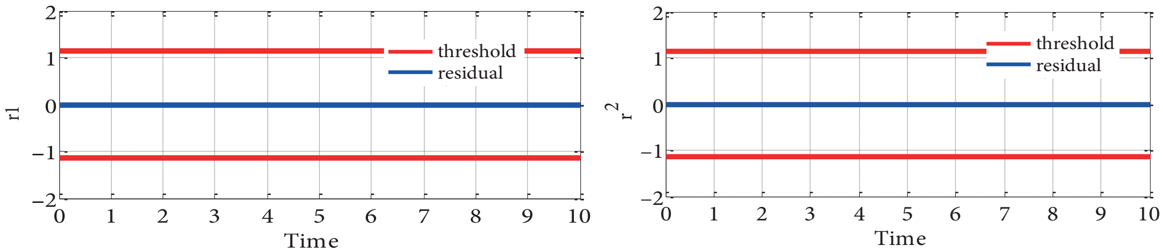

Figure 2. Residuals r_1 and r_2 with no disturbance and no fault.

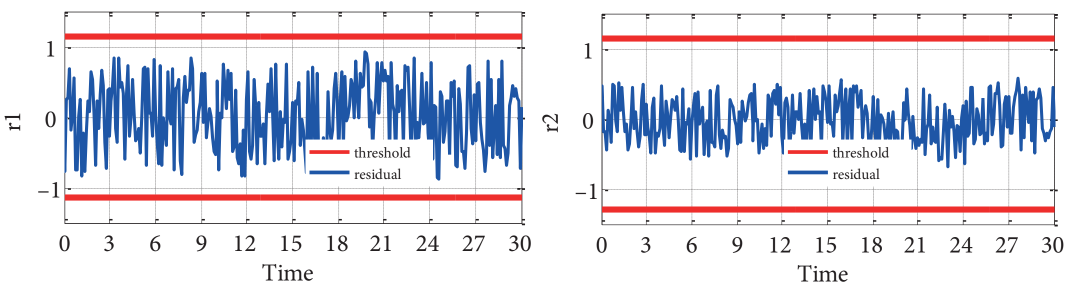

Figure 3. Residuals r_1 and r_2 with disturbance and no fault.

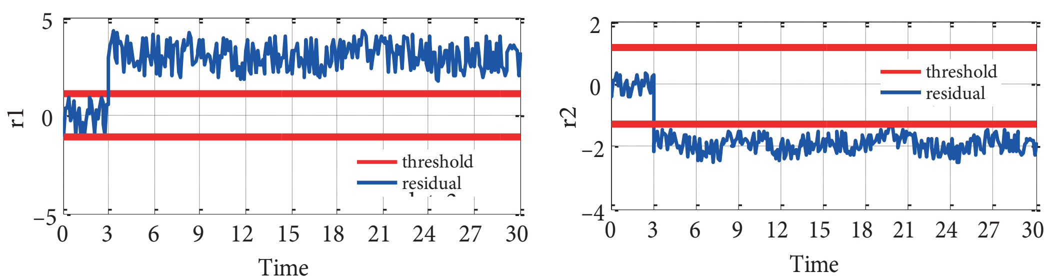

Figure 4. Residuals r_1 and r_2 with disturbance and fault f_1.

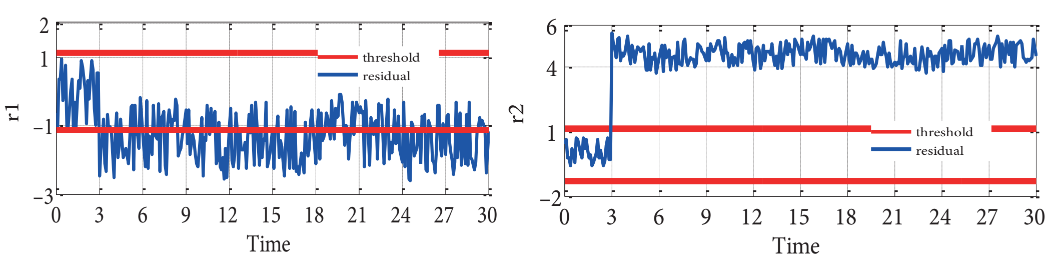

Figure 5. Residuals r_1 and r_2 with disturbance and with fault f_2.

Simulation results of residual generation with a UIO approximately decoupled with unknown inputs and the theoretical results of the residual vector $r(p)$ are well correlated, except for residual $r_1(p)$, which is almost decoupled from perturbations according to the signature table; this seems to lead to a better robustness of this solution for modeling errors.

The observer provides residuals r_1 and r_2, respectively, in the absence of faults and disturbances, as illustrated in Figures 2 and 6.

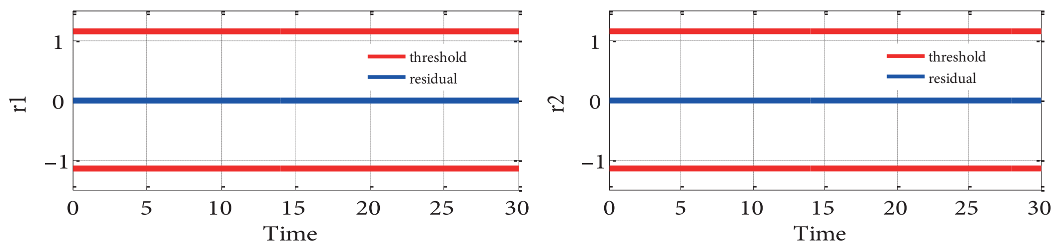

Figure 6. Residuals r_1 and r_2 with no disturbance and no fault.

The observer provides residuals r_1 and r_2, respectively, in the absence of faults and in the presence of disturbances, as illustrated in Figures 3 and 7.

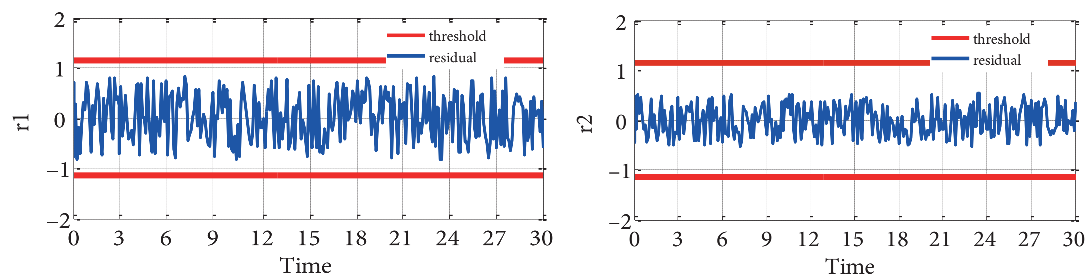

Figure 7. Residuals r_1 and r_2 with disturbance and no fault.

The residuals r_1 and r_2 generated by the observer indicate that there is a fault at a time $t \geq 3s$, which corresponds to a fault f_1, as illustrated in Figures 4 and 8.

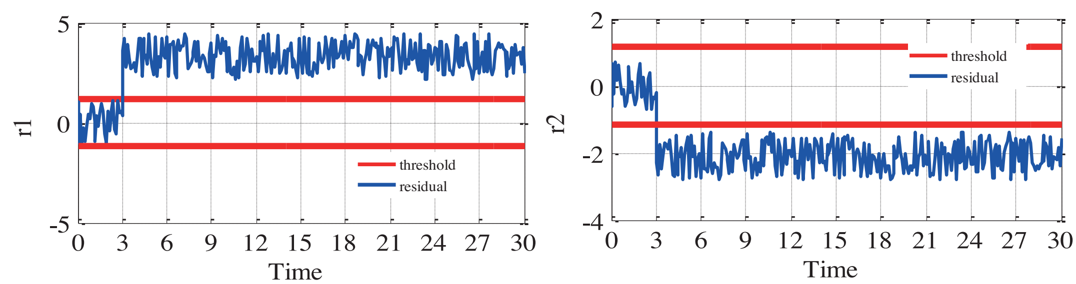

Figure 8. Residuals r_1 and r_2 with disturbance and with fault f_1.

However, fault f_2 appears on residuals r_1 and r_2, shown in Figures 5 and 9.

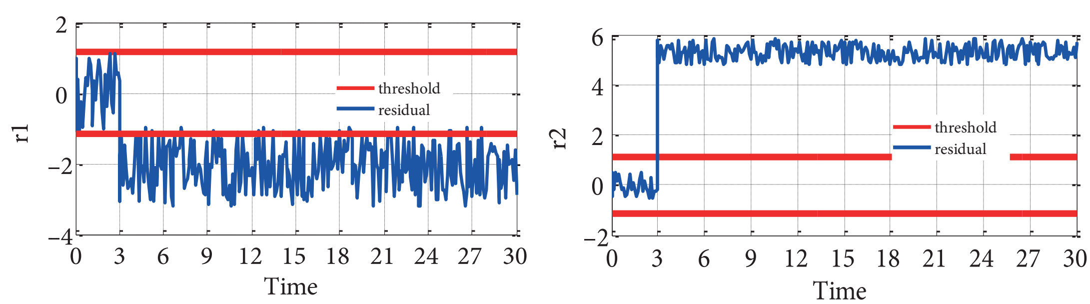

Figure 9. Residuals r_1 and r_2 with disturbance and with fault f_2.

From these simulations, some interesting points can be mentioned:

- Simulation results of Figures 2–5 correspond to the table of theoretical signatures.
- Using this dedicated observer to estimate each one of faults f_1 and f_2 allows for a good detection level. We equally note that false alarms are avoided.
- The structure of the obtained Table is not a localizing one (same signature for f_1 and f_2), because the number of faults exceeds the number of residuals. In this example, there are two actuator faults and two sensor faults according to F_x and F_y. This means that the two residuals have identical signatures (two sensor and two actuator faults). The maximum number of localizable faults is conditioned by the number q of residuals, which is equal to $2^q - 1$; this is not our case. This situation is due to the nonlocalization of faults in the signature table.

8. Conclusion

In this paper, a strategy for generating robust residuals for linear systems is presented. A UIO with approximate decoupling is used to generate robust residuals capable of detecting faults. These residuals are represented by the observer, using the design conditions (robustness and sensitivity constraints) under the LMI formalism, when the faults to be detected affect the system. These conditions are established using the Lyapunov method under the LMI form in order to highlight the presence of faults despite the presence of disturbances. The resolution of these LMI constraints is carried out using a method based on a variable change, which is considered as a global method that allows an easier determination of matrices describing the UIO observer. Such a variable change is not always possible, as it depends on the structure of the initial nonlinear inequality that can be easily solved with the latest digital SDP tools of the CVX (20) solver.

We have shown, through an application, how the proposed technique of generating robust residuals can be exploited in a diagnostic context of linear systems and can have a good level of fault detection (minimizing the number of false alarms).

Nomenclature

UIO	Unknown input observer
LMI	Linear matrix inequality
H_-	The index norm
H_∞	H_∞ norm
CVX	Convex programming
SDP	Semidefinite program - LMI
$\overline{\sigma}\,\underline{\sigma}$	The largest and the smallest singular values
$d(t)$	Disturbances (unknown inputs)
$f(t)$	Faults signal
$G_f(j\omega)$, $G_d(j\omega)$	Transfer matrices that link residuals to faults and unknown inputs
$\|G_f\|_-$	The index H_- of transfer between the indicator signal r and the faults f to be detected
$\|G_d\|_\infty$	The standard H_∞ of transfer between the indicator signal r and the disturbances d
F_xF_y	Action matrices of faults $f(t)$
D_x, D_y	Action matrices of disturbances $d(t)$
$r(t)$	The residual vector
KV	Adjustment matrices

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