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Abstract

Using analytical properties of a 1-loop open parabosonic M-point transition amplitude, we show that the space-time critical dimension depends on the order of the paraquantization.

1. Introduction

One of the main goals of quantum mechanics (QM) is to provide a consistent and unified description of the so-called wave-particle duality which is a direct consequence of the Heisenberg equations of motion. It turns out that the canonical commutation relations - which guarantee the Heisenberg equations - are not unique [1]. The general framework in which the canonical commutation relations are generalized is called paraquantization and characterized by an order parameter Q [2-9]. Although it is, in principle, possible to study the paraquantum observables within the usual Hilbert space, it is often convenient to use a larger Hilbert space in which the operators satisfy simple bilinear relations [2], [10-12]. Traditionally, for Focks-type irreducible representation of paraquantum theories

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with a unique vacuum state, this is done by means of the Green decomposition [2], [10-12]:

$$a_n = \sum_{\beta=1}^{Q} a_n^{(\beta)} \tag{1}$$

where Q is the order of the paraquantization, β the Green index and $a_n^{(\beta)}$ is the bosonic annihilation operators with Green components satisfying the following bilinear but anomalous commutation relations:

$$\left[a_n^{(\beta)}, a_m^{+(\alpha)}\right]_+ = 0 \quad \alpha \neq \beta$$
$$\left[a_n^{(\alpha)}, a_m^{+(\alpha)}\right]_- = \delta_{mn}. \tag{2}$$

The purpose of this paper is to derive the space-time critical dimension for an open parabosonic string by using the meromorphic properties of the M-point transition amplitude. In Section 2, we describe the formalism and in Section 3 we derive the critical dimension and finally in Section 4 we draw our conclusions.

2. Formalism

The Nambu-Goto classical action of a free relativistic open bosonic string is given by [13]:

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma [(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2]^{1/2}, \tag{3}$$

where τ and σ are dimensionless word-sheet parameters and α' is the string tension (here, "' " as in x' and "·" as in $\dot{x}$ denotes $\frac{\partial}{\partial\sigma}$ and $\frac{\partial}{\partial\tau}$, respectively). The general solution of the equations of motion in the light cone gauge is [13]:

$$x^i(\sigma,\tau) = q^i + 2\alpha' p^i + 2\alpha' \frac{1}{\sqrt{2}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} [a_n^i e^{-in\tau} + a_n^{+i} e^{in\tau}] \cos n\sigma, \tag{4}$$

where q^i and p^i are the string centre of mass coordinates and momentum, respectively.

After quantization the physical states $|\Psi\rangle_{phy}$ are subject to the Virasoro conditions:

$$L_n |\Psi\rangle_{phy} = 0 \qquad n > 1$$
$$\text{and}$$
$$[L_0 - \alpha(0)] |\Psi\rangle_{phy} = 0, \tag{5}$$

(here, $\alpha(0) = 1$) where the Virasoro generators L_n and L_0 are given by:

$$L_n = \frac{1}{2\alpha'} \sum_{m=1}^{\infty} : \alpha_{n-m}^i \alpha_m^i :$$
$$L_0 = \frac{1}{2\alpha'} \sum_{m=1}^{\infty} \alpha_{-m}^i \alpha_m^i \tag{6}$$

with

$$\alpha_0^i = 2\alpha' p^i, \quad \alpha_{-n}^i = \sqrt{2\alpha' n} a_n^{+i}, \quad \alpha_n^i = \sqrt{2\alpha' n} a_n^i.$$

It is to be noted that the string dynamical variables q^i, p^i, q^-, p^+, a^i and a^{+i} verify the following non vanishing canonical commutation relations:

$$\begin{aligned} [q^i, p^i] &= i\delta^{ij} \\ [q^-, p^+] &= -i \\ [a_n^i, a_m^{+j}] &= \delta_{mn}\delta^{ij}. \end{aligned} \tag{7}$$

Now, for the paraquantization the commutation relations (2.5) become:

$$\begin{aligned} [q^i, p^i] &= i\delta^{ij} \\ [q^-, p^+] &= -i \\ [a_n^{i(\beta)}, a_m^{+j(\alpha)}]_+ &= 0, \quad \alpha \neq \beta \\ [a_n^{i(\alpha)}, a_m^{+j(\alpha)}]_- &= \delta_{mn}\delta^{ij}, \end{aligned} \tag{8}$$

where we have used the Green decomposition (1.1) for a_n^i and a_m^{+j} and the fact that the observables, like q^i, p^i, q^-, p^+, which describe the center of mass coordinates and momentum of the string, should not be affected by the paraquantization [14-17]. In other words, the space-time properties of the string remain unchanged. This can be achieved by choosing a specific direction in the Green para-space-like relations [14-17]:

$$q^{i(\alpha)} = q^i \delta_{\alpha 1}, \quad p^{i(\alpha)} = p^i \delta_{\alpha 1}, \quad q^{-(\alpha)} = q^- \delta_{\alpha 1}, \quad p^{+(\alpha)} = p^+ \delta_{\alpha 1}. \tag{9}$$

3. M-Point Transition Amplitude

The 1-loop open parabosonic string M-point transition amplitude for a planar diagrams with M external tachyons, which is topologically equivalent to a disk with a hole quenched in the interior and external lines located on the exterior edge, can be written as:

$$A(1, 2, \ldots, M) = \int \prod_{\beta=1}^{Q} d^D p^{(\beta)} Tr[\Delta V(k_1, 1)\Delta V(k_2, 1) \cdots \Delta V(k_M, M)]. \tag{10}$$

(Here, $k_j = \overline{1, M}$ is the jth external tachyon momentum and propagator Δ has is expressed as

$$\Delta = (L_0 - \alpha(0))^{-1} \tag{11}$$

with L_0 as the paraquantum Virasoro operator [15-17] given by:

$$L_0 = -\sum_{\beta=1}^{Q} \sum_{m=1}^{\infty} : \alpha_{-m}^{i(\beta)} \alpha_m^{i(\beta)} : \tag{12}$$

(we take $2\alpha' = 1$) and

$$\alpha(0) = Q(D-2)/24.$$

(": :" means normal ordering). The paraquantum vertex operator $V(K_r, 1)$ has the expression

$$V(k_r, 1) = e^{iL_0} \ V(k_r, 0)e^{iL_0},$$

where

$$V(k_r, 0) = g : \exp\left[\frac{i}{2}\sum_{\gamma=1}^{Q}\sum_{i=1}^{D-2} K^{i(\gamma)}q^{i(\gamma)}\right], \tag{13}$$

where g is the coupling.

It is to be noted that the propagator Δ has the following useful integral representation:

$$\Delta = \int dx\ x^{L_0-\alpha(0)-1}. \tag{14}$$

Now, using the integral representation (3.5) and the fact that

$$x^{L_0}V(k_r, 1) = V(k_r, x)x^{L_0}, \tag{15}$$

where

$$V(k_r, x) = e^{i\times L_0}V_0(k_r, 0)e^{-i\times L_0}, \tag{16}$$

where

$$V(k_r, x) = e^{i\times L_0}V_0(k_r, 0)e^{-i\times L_0}, \tag{17}$$

the transition amplitude (3.1) can be rewritten as:

$$A(1, 2, \ldots, M) = \int \prod_{i=1}^{M} dx_i \int \prod_{\beta=1}^{Q} d^D p^{(\beta)} Tr\left[V_0(k_1, x_1)\cdots V_0(k_M, x_1\cdots x_M)w^{L_0-1-\alpha(0)}\right] \tag{18}$$

with

$$w = x_1x_2\cdots x_M. \tag{19}$$

Noticing that

$$\prod_{i=1}^{M} dx_i = dw \prod_{i=1}^{M-1}\frac{d\rho_i}{\rho_i}, \tag{20}$$

where

$$\rho_i = x_1x_2\cdots x_i, \tag{21}$$

Eq. (3.8) can be simplified to:

$$A(1, 2, \ldots, M) = \int \frac{dw}{w^{1+Q(D-2)/24}} \int \prod_{r=1}^{M-1}\frac{d\rho_r}{\rho_r}\vartheta(\rho_r - \rho_{r+1})I(1, 2, \ldots, M) \tag{22}$$

with

$$I(1,2,\ldots,M)=\int\frac{dw}{\beta=1}d^Dp^{(\beta)}Tr\left[V_0(k_1,\rho_1)V_0(k_2,\rho_2)\cdots V_0(k_M,\rho_M)w^{L_0}\right]. \tag{23}$$

The trace (3.13) can be easily calculated by using the paraquantum coherent state method [2]. In fact, using the identity

$$TrM=\sum_{\beta=1}^{Q}\frac{1}{\pi}\int d\lambda_n^{(\beta)}d\lambda_n^{(\beta)}e^{-|\lambda_n^{(\beta)}|^2}\langle\lambda_n^{(\beta)}|M|\lambda_n^{(\beta=}\rangle, \tag{24}$$

where

$$|\lambda_n^{(\beta)}\rangle=\exp\left[\lambda_n^{(\beta)}a_n^{+(\beta)}\right]|0\rangle \tag{25}$$

and

$$a_n^{(\alpha)}|\lambda_m^{(\beta)}\rangle=\delta_{\alpha\beta}\delta_{nm}\lambda_n^{(\beta)}|\lambda_m^{(\beta)}\rangle \tag{26}$$

$$\langle\mu_n^{(\alpha)}|\lambda_m^{(\beta)}\rangle=\exp\left[\mu_n^{*(\alpha)}\lambda_m^{(\beta)}\right]\delta_{\alpha\beta}\delta_{nm} \tag{27}$$

($\mu_n^{(\alpha)}$ and $\lambda_m^{(\beta)}$ are arbitrary complex numbers) and the fact that

$$x^{\sum_{i=1}^{D-2}a_n^{+i(\beta)}a_n^{i(\beta)}}|\lambda_m^{(\beta)}\rangle=\delta_{nm}|\lambda_m^{(\beta)}x\rangle \tag{28}$$

and

$$\langle 0|\exp\left(-K_I\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}a_n^{i(\beta)}\right)\quad+\sum_{k=1}^{D-2}\sum_{m=1}ma_m^{k+(\beta)}a_m^{k(\beta)}\exp\left(K_J\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}a_n^{j(\beta)}\right)|0\rangle$$
$$=(1-x)^{K_IK_J}\delta_{ij}, \tag{29}$$

straightforward calculations give:

$$I(1,2,\ldots,M)=Q[f(w)]^{-Q(D-2)}\left(-\frac{2\pi}{Lnw}\right)^{Q(D-2)/2}\prod_{I<J}[\Psi_{IJ}]^{k_Ik_J}, \tag{30}$$

where

$$f(w)=\prod_{n=1}(1-w^n) \tag{31}$$

and

$$\Psi_{IJ}=-2\pi i\ \exp\left[\frac{\ln^2 C_{JI}}{2\ln w}\right]\vartheta_1\left(\frac{\ln C_{JI}}{2\pi i}|\frac{\ln w}{2\pi i}\right)/\vartheta_1'\left(0|\frac{\ln w}{2\pi i}\right) \tag{32}$$

with

$$C_{JI}=\rho_J/\rho_I \tag{33}$$

and ϑ_1 (resp ϑ_1') being Jacobi function (resp. its derivative). Now, introducing new variables

$$\nu_r = \frac{\ln \rho_r}{\ln w} \tag{34}$$

and

$$q = \exp\left(\frac{2\pi^2}{\ln w}\right), \tag{35}$$

and using the identities

$$\frac{dw}{w}\prod_{r=1}^{M-1}\frac{d\rho_r}{\rho_r}\vartheta(\rho_r - \rho_{r+1}) = \frac{1}{2\pi^2}(-\ln w)^{M+1}\frac{dq}{q}\prod_{r=1}^{M-1}\vartheta(\nu_{r+1}-\nu_r)d\nu_r \tag{36}$$

and

$$\frac{1}{w^{Q(D-2)/24}}[f(w)]^{-Q(D-2)} = \left(-\frac{\pi}{\ln q}\right)^{Q(D-2)/2}\frac{1}{q^{Q(D-2)/12}}[f(q^2)]^{-Q(D-2)}, \tag{37}$$

the transition amplitude (3.12) takes the form:

$$\begin{aligned} A(1,2,\ldots,M) = \frac{Q}{\pi}g^M \int_0^1 \prod_{i=1}^{M-1}\vartheta(\nu_{i+1}-\nu_i)d\nu_i \int_0^1 dq q^{-1+Q(2-D)/12}W^{-1-Q(2-D)/24} \\ \times\left(-\frac{2\pi^2}{\ln q}\right)^M [f(q^2)]^{-Q(D-2)}\prod_{I<J}[\Psi_{IJ}]^{k_I k_J}. \end{aligned} \tag{38}$$

Now, by extracting the $\ln q$ factor from (3.22) and using the kinematical relation

$$\sum_{I<J}K_I K_J = -1/2\sum_I K_I^2 = -M, \tag{39}$$

and in order that the integrand in (3.28) can be a meromorphic function, i.e., the only existent singularities are a finite number of poles, the power of the W factor must vanish. Consequently, one deduces that the space time critical dimension must verify the relation

$$D = \frac{24}{Q} + 2.$$

4. Conclusion

We conclude that the meromorphic property of the M-point transition amplitude with external tachyons and the generalization of the quantization procedure are strongly related to the critical space-time dimension of the parabosonic string $D = \frac{24}{Q} + 2$. More details are being investigated [19].

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References

[1] E.P. Wigner, Phys. Rev., 77, 711 (1950).

[2] N. Mebarki, N. Belaloui, M.H. Traikia, Tr. Journ. of Phys., 19, 1101 (1995).

[3] Y. Ohnuki, T. Kashiwa, Prog. Theor. Phys. (Kyoto), 60, 548 (1978).

[4] Y. Ohnuki, S. Kamefuchi, Phys. Rev., 170, 1279 (1968).

[5] M. Omote, Y. Ohnuki, S. Kamefuchi, Prog. Theor. Phys. (Kyoto), 56, 1984 (1976).

[6] K.V. Kadenova, M.N. Karaev, Nucl. Phys., B26, 326 (1971); Phys. Lett., B34, 147 (1971).

[7] O. Klein, Journ. Phys. Radium, 9, 1 (1938).

[8] N. Mebarki, N. Belaloui, M.H. Traikia, Acta Physica Pol., B23, 1203 (1992).

[9] H.S. Green, Phys. Rev., 90, 270 (1953).

[10] N. Mebarki, N. Belaloui, M.H. Traikia in "Topics in Theoretical Physics", Second Autumn School on Theoretical Physics, November 9-21 (1991), Constantine University, Algeria.

[11] K. Druhl, R. Haag, J.E. Roberts, Commun. Math. Phys., 18, 204 (1970).

[12] J. Sherk, Rev. Mod. Phys., 47, 123 (1975).

[13] N. Mebarki, M. Haouchine, N. Belaloui in "Open parabosonic string BRST transformations and the critical dimension", Constantine University, preprint 19, 1996 (submitted for publication in Acta. Phys. Pol.B).

[14] N. Mebarki, M. Haouchine, N. Belaloui in "Modified Virasoro and super-Virasoro algebra", Constantine University, preprint 12, 1996 (submitted for publication in Czech. J. of Physics).

[15] N. Mebarki, M. Haouchine, N. Belaloui in "No-Ghost theorem for an open parabosonic string", Constantine University, preprint 14, 1996 (submitted for publication in Acta.Phys.Pol.B).

[16] N. Mebarki, M. Haouchine, N. Belaloui in "New critical dimensions for parabosonic string and superstrings", Constantine University, preprint 16, 1996 (submitted for publication in Acta.Phys.Pol.B).

[17] M. Haouchine, Magister thesis (1995), Constantine University, (unpublished).

[18] N. Mebarki, N. Belaloui (in preparation).