

1-1-2017

## Constrained control allocation for nonlinear systems with actuator failures or faults

SAMAN EBRAHIMI BOUKANI

MOHAMMAD JAVAD KHOSROWJERDI

ROYA AMJADIFARD

Follow this and additional works at: <https://journals.tubitak.gov.tr/elektrik>



Part of the [Computer Engineering Commons](#), [Computer Sciences Commons](#), and the [Electrical and Computer Engineering Commons](#)

---

### Recommended Citation

BOUKANI, SAMAN EBRAHIMI; KHOSROWJERDI, MOHAMMAD JAVAD; and AMJADIFARD, ROYA (2017) "Constrained control allocation for nonlinear systems with actuator failures or faults," *Turkish Journal of Electrical Engineering and Computer Sciences*: Vol. 25: No. 4, Article 49. <https://doi.org/10.3906/elk-1604-149>

Available at: <https://journals.tubitak.gov.tr/elektrik/vol25/iss4/49>

This Article is brought to you for free and open access by TÜBİTAK Academic Journals. It has been accepted for inclusion in Turkish Journal of Electrical Engineering and Computer Sciences by an authorized editor of TÜBİTAK Academic Journals. For more information, please contact [academic.publications@tubitak.gov.tr](mailto:academic.publications@tubitak.gov.tr).

## Constrained control allocation for nonlinear systems with actuator failures or faults

Saman EBRAHIMI BOUKANI<sup>1,\*</sup>, Mohammad Javad KHOSROWJERDI<sup>2</sup>, Roya AMJADIFARD<sup>3</sup>

<sup>1</sup>Department of Electrical Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran

<sup>2</sup>Department of Electrical Engineering, Sahand University of Technology, Sahand, Tabriz, Iran

<sup>3</sup>Engineering Faculty, Kharazmi University, Tehran, Iran

Received: 12.04.2016

Accepted/Published Online: 23.12.2016

Final Version: 30.07.2017

**Abstract:** In this paper, a combination of dynamic constrained control allocation with terminal sliding mode control is proposed for a general class of overactuated nonlinear systems with actuator faults/failures. First, the terminal sliding mode control is designed to converge the system tracking error to zero in a finite-time. Then a control allocation strategy is developed and will be solved by a Lyapunov method, which leads to a dynamic update law with finite-time convergence. This strategy satisfies input limits and when faults/failures occur in some of the actuators, the control signals are automatically redistributed among the healthy actuators. Simulation results on a near space vehicle show the effectiveness of the proposed approach.

**Key words:** Overactuated systems, Lyapunov method, Terminal sliding mode control, Dynamic control allocation

### 1. Introduction

Actuator faults and failures might have serious damaging effects on the performance of engineering systems if they are not well handled. To maintain specified safety performance of these systems when unexpected faults/failures occur, fault-tolerant control (FTC) is a very valuable control technology. Many FTC methods have been introduced in recent years. Liao et al. [1] used a linear matrix inequality method to design a robust FTC and Yang et al. [2] designed an  $H_\infty$  controller for linear systems with sensor and actuator failures. Moreover, adaptive feedback control schemes were developed in [3,4] for linear systems with actuator failures. Adaptive fault tolerant controllers were presented in [5,6] for nonlinear systems with actuator failures. However, in [5,6], input limits were not taken into account, whereas if the actuators reached their constraints, every effort to increase the actuators' output would create no variation in the output and would result in system instability.

To reduce the effects of faults/failures and to achieve FTC, an important factor is the availability of redundant actuators. This is because of more freedom regarding controller design. Control allocation (CA) is an approach that can effectively manage overactuated systems, especially when some of the actuators become damaged and CA redistributes the control signals into the healthy actuators. There has been much study of CA problems, and methods like pseudoinverse [7], linear programming [8], direct CA [8], daisy chain [9], constrained quadratic programming [10], and constrained nonlinear programming [11] have been previously introduced. They all, independently of the dynamic control problem, solve the control problem as a static optimization problem. Unlike the above approaches, Johansen [12] designed an optimal CA scheme in the shape

\*Correspondence: [khosrowjerdi@sut.ac.ir](mailto:khosrowjerdi@sut.ac.ir)

of a dynamic update law with infinite-time convergence for nonlinear systems, and Benosman [13] designed an optimal CA for a class of nonlinear systems with unstable internal dynamics, but methods in [12,13] do not consider actuator faults/failures.

Sliding mode control (SMC) schemes have an inherent capability to reject matched disturbances and uncertainties. Therefore, their combination with CA seems to have great potential for the development of FTC [14]. However, robustness in conventional SMC schemes can only be achieved after the sliding phase [14]. To eliminate the reaching phase, an integral sliding mode control (ISMC) was proposed in [14]. To deal with fault, combinations of the CA with ISM are used in [15,16] for linear systems. The integral action has deficiencies, like large overshoot and long settling time. Terminal sliding mode control (TSMC) has improved characteristics such as finite-time convergence and higher control accuracy compared with ISMC [17,18].

This paper investigates a combination of dynamic CA and TSMC according to Figure 1 for a general class of nonlinear systems. The main contributions of this paper are as follows:

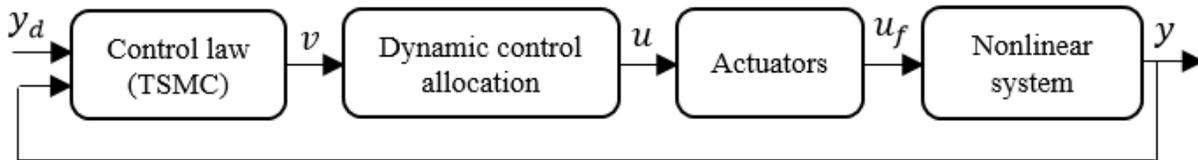


Figure 1. Block diagram of the proposed method.

- The CA problem will be solved by a Lyapunov method, which leads to a dynamic update law with finite-time convergence.
- The proposed CA method can satisfy control constraints in the presence of faults/failures in actuators.
- A TSMC with nonlinear sliding surface will be proposed that can guarantee the finite-time convergence of the system tracking error to zero.

2. Problem formulation

This paper focuses on a class of MIMO nonlinear systems represented by

$$\dot{x} = f(x) + g(x) u_f(t)$$

$$y = h(x), \tag{1}$$

where  $x \in R^n$  is the state vector;  $f(x) \in R^n$  and  $g(x) \in R^{n \times n_a}$  are smooth nonlinear functions of  $x$ ;  $u_f(t) = [u_{f_1}, \dots, u_{f_{n_a}}]^T \in R^{n_a}$  is the control signal whose components may be failed during operation; and  $y \in R^m$  is the output vector.

**Assumption 1** Eq. (1) has a relative degree  $r_1, \dots, r_m$  such that  $\sum_{i=1}^{i=m} r_i = n$ .

**Assumption 2** Eq. (1) has more actuators than controlled outputs, i.e.  $n_a > m$ .

At the beginning, the system is healthy and devised to track a desirable output  $y_d$ . Afterward, faults or failures might occur in some of the actuators in  $t_f > 0$ . In this case, output of the actuator  $\rho$ ,  $\rho \in \{1, \dots, n_a\}$  can be modeled as follows [6]:

$$u_{f_\rho}(t) = (1 - \sigma_\rho) \delta_\rho u_\rho(t) + \sigma_\rho \tilde{u}_\rho, \tag{2}$$

where  $\tilde{u}_\rho$  shows the failure in the  $\rho$ th actuator,  $0 \leq \delta_\rho \leq 1$  shows the amount of the remaining effective section of the actuator  $\rho$ , and

$$\sigma_\rho = \begin{cases} 1 & \text{if the } \rho\text{th actuator is failed} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

If the actuator  $\rho$  is healthy,  $u_{f_\rho}(t) = u_\rho(t)$ . When a fault or failure happens at the actuator  $\rho$  at  $t \geq t_f$ , multiple usual fault/failure models could exist [5].

- I. Loss of effectiveness: in this case,  $u_{f_\rho} = \delta_\rho u_\rho$ .
- II. Lock in place or stuck-type: in this case,  $u_{f_\rho} = \tilde{u}_\rho = \text{constant}$ .
- III. Float fault: in this case,  $u_{f_\rho} = 0$ .
- IV. Hard over fault: in this case,  $u_{f_\rho} = \bar{u}_\rho$  or  $u_{f_\rho} = \underline{u}_\rho$ .

Now we can express the input vector  $u_f(t)$  in Eq. (1) as follows [6]:

$$u_f(t) = (I - \sigma) \delta u(t) + \sigma \tilde{u}, \tag{4}$$

where  $u(t)$  is the applied control vector and

$$\begin{cases} \sigma = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_{n_a}) \\ \delta = \text{diag}(\delta_1, \delta_2, \dots, \delta_{n_a}) \end{cases} \tag{5}$$

Here our aim is to develop a combination of TSMC and CA based on the Lyapunov method for Eq. (1) with the actuator faults from Eq. (4) such that the output  $y(t)$  tracks a desired smooth trajectory  $y_d(t)$ , and at the same time all actuators satisfy control constraints:

$$u(t) \in G = \left\{ u = [u_1 \dots u_{n_a}]^T \mid \underline{u}_\rho \leq u_\rho \leq \bar{u}_\rho, \rho = 1, \dots, n_a \right\}, \tag{6}$$

where  $\underline{u} = [\underline{u}_1 \dots \underline{u}_{n_a}]^T$  and  $\bar{u} = [\bar{u}_1 \dots \bar{u}_{n_a}]^T$  are lower and upper control constraints.

**Assumption 3** *There exists  $B_{di} > 0$  such that  $\left\| \left( y_{di}, y_{di}^{(1)}, \dots, y_{di}^{(r_i)} \right) \right\| \leq B_{di}$ .*

Under assumption 1, there exists a local coordinate transformation:

$$z = T(x) = [z_1(x)^T, \dots, z_m(x)^T]^T \tag{7}$$

$$z_i(x) = [z_{i1}(x), \dots, z_{ir_i}(x)]^T = [h_i(x), \dots, L_f^{r_i-1} h_i(x)]^T \tag{8}$$

that converts Eq. (1) to the following form [19]:

$$\begin{aligned} \dot{z}_{i1} &= z_{i2} \\ &\vdots \\ \dot{z}_{i(r_i-1)} &= z_{ir_i} \\ \dot{z}_{ir_i} &= v_i(z, t) \\ y_i &= h_i = z_{i1} \quad (i= 1, \dots, m), \end{aligned} \tag{9}$$

where

$$v_i(z, t) = b_i(z) + \sum_{\rho=1}^{n_a} a_{i\rho}(z) [(1-\sigma_\rho) \delta_\rho u_\rho(t) + \sigma_\rho \tilde{u}_\rho] \tag{10}$$

and where  $b_i(z) = L_f^{r_i} h_i$  and  $a_{i\rho}(z) = L_{g_\rho} L_f^{r_i-1} h_i$ . Eq. (10) can be expressed as follows:

$$v(z, t) = B(z) + A(z) [(1-\sigma)] \delta u(t) + \sigma \tilde{u} \tag{11}$$

where  $B(z) = [b_1 \dots b_m]^T$  and  $A(z) = \begin{bmatrix} a_{11} & \dots & a_{1n_a} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn_a} \end{bmatrix}$ ;  $u(t) = [u_1 \dots u_{n_a}]^T \in R^{n_a}$  is the real control input;

and  $v(z, t) = [v_1 \dots v_m]^T \in R^m$  is the virtual control effort produced by the TSMC law, which will be designed in the next section. By considering  $n_a > m$ , we will not have a unique solution (i.e.  $u$ ) for Eq. (11). In Section 4, a CA problem is formulated to find the best solution for Eq. (11) while satisfying the control constraints in Eq. (6).

### 3. TSMC

In this section, in order to find virtual control  $v(z, t) \in R^m$  for Eq. (11), a TSMC is proposed that guarantees the output tracking problem in a finite time. From Eq. (9), the system outputs and its derivatives can be rewritten as

$$\begin{aligned} y_i &= h_i = z_{i1} \\ \dot{y}_i &= \dot{z}_{i1} = z_{i2} \\ &\vdots \\ y_i^{(r_i)} &= v_i(z, t) \quad (i= 1, \dots, m) \end{aligned} \tag{12}$$

By defining the  $i$ th element of tracking error as  $e_{i1} = y_i - y_{di}$ , a nonlinear terminal sliding surface is set to [20]

$$\begin{aligned} e_{i2} &= \dot{e}_{i1} + \alpha_{i1} e_{i1} + \beta_{i1} e_{i1}^{\frac{p_{i1}}{q_{i1}}} \\ &\vdots \\ e_{i(r_i-1)} &= \dot{e}_{i(r_i-2)} + \alpha_{i(r_i-2)} e_{i(r_i-2)} + \beta_{i(r_i-2)} e_{i(r_i-2)}^{\frac{p_{i(r_i-2)}}{q_{i(r_i-2)}}} \\ s_i &= \dot{e}_{i(r_i-1)} + \alpha_{i(r_i-1)} e_{i(r_i-1)} + \beta_{i(r_i-1)} e_{i(r_i-1)}^{\frac{p_{i(r_i-1)}}{q_{i(r_i-1)}}} \quad , \end{aligned} \tag{13}$$

where  $i=1, \dots, m$  and  $j=1, \dots, r_i-1$ ,  $\alpha_{ij} > 0$  and  $\beta_{ij} > 0$ ;  $p_{ij} < q_{ij}$  are positive odd constants and  $s_i$  is a nonlinear terminal sliding surface for  $e_{i1}$ .

The  $r_i$ th derivative of  $e_{i1}$  becomes

$$e_{i1}^{(r_i)} = v_i(z, t) - y_{di}^{(r_i)} \tag{14}$$

and the  $l$ th time derivative of  $e_{ij}$  is given by

$$e_{ij}^{(l)} = e_{i(j-1)}^{(l+1)} + \frac{d^{(l)}}{dt^{(l)}} \left( \alpha_{i(j-1)} e_{i(j-1)} + \beta_{i(j-1)} e_{i(j-1)}^{\frac{p_{i(j-1)}}{q_{i(j-1)}}} \right) \tag{15}$$

In accordance with Eqs. (13) – (15), the time derivative of  $s_i$  becomes

$$\dot{s}_i = v_i(z, t) - y_{di}^{(r_i)} + \sum_{j=1}^{r_i-1} \left[ \alpha_{ij} e_{ij} + \beta_{ij} e_{ij}^{\frac{p_{ij}}{q_{ij}}} \right]^{(r_i-j)} \tag{16}$$

Therefore, the TSMC law is set to

$$v_i(z, t) = y_{di}^{(r_i)} - \sum_{j=1}^{r_i-1} \left[ \alpha_{ij} e_{ij} + \beta_{ij} e_{ij}^{\frac{p_{ij}}{q_{ij}}} \right]^{(r_i-j)} - \gamma_i s_i - K_i \text{sign}(s_i) \tag{17}$$

where  $\gamma_i > 0$  and  $K_i > 0$  ( $i=1, \dots, m$ ).

**Lemma 1** ([18,21]): *If there are positive definite functions  $V(x)$ ,  $\lambda_1 > 0$ ,  $\lambda_2 > 0$  and  $0 < r < 1$  such that  $\dot{V}(x) \leq -\lambda_1 V(x) - \lambda_2 V^k(x)$ , then the equilibrium point  $x=0$  is finite-time stable and the convergence time  $t_s$  satisfies*

$$t_s \leq \frac{1}{\lambda_2(1-r)} \ln \frac{\lambda_2 V^{1-r}(x_0) + \lambda_1}{\lambda_1} \quad x_0 = x(0) \tag{18}$$

**Theorem 1** *For the nonlinear sliding surface (Eq. (13)) and the control law (Eq. (17)), the overactuated system (Eq. (1)) will be stable and closed-loop signals will converge to the equilibrium point in a finite-time.*

**Proof** By substituting Eq. (17) into Eq. (16), we have

$$\dot{s}_i = -\gamma_i s_i - K_i \text{sign}(s_i) \tag{19}$$

To prove stability of Eq. (1), we choose the Lyapunov function as

$$V_1 = \frac{1}{2} S^T S, \tag{20}$$

where  $S = [s_1, \dots, s_m]^T$ . Differentiating  $V_1$  and using Eq. (19) yields

$$\dot{V}_1 = \sum_{i=1}^m s_i \dot{s}_i = - \sum_{i=1}^m K_i s_i \text{sign}(s_i) - \sum_{i=1}^m \gamma_i s_i^2 \leq -\underline{K} \sum_{i=1}^m |s_i| - \underline{\gamma} \sum_{i=1}^m s_i^2, \tag{21}$$

where  $\underline{\gamma} = \{\gamma_i\} > 0$  and  $\underline{K} = \{K_i\} > 0$ . Eq. (21) can be expressed as

$$\dot{V}_1 \leq -\sqrt{2\underline{K}}V_1^{\frac{1}{2}} - 2\underline{\gamma}V_1 \tag{22}$$

According to Eq. (22) and Lemma 1, we know that  $V_1$  converges to the equilibrium point in the finite-time

$$t_{s_1} \leq \frac{1}{\underline{\gamma}} \ln \frac{\sqrt{2\underline{\gamma}}V_1^{\frac{1}{2}}(S_0) + \underline{K}}{\underline{K}}. \tag{23}$$

#### 4. CA

After finding the virtual control vector  $v(z, t) \in R^m$  in the previous section, we intend to find the law control  $u(t) \in R^{n_a}$  from Eq. (11) so as to satisfy the control constraints in Eq. (6). However, considering  $n_a > m$ , we will not have a unique solution for Eq. (11). Therefore, a secondary objective is added to minimize the magnitude of the control vector, or its distance from a preferred control value,  $u_0$ . Based on the above-mentioned, the CA problem can be formulated as follows:

$$J(z, u, t) \tag{23}$$

$$\text{subject to } \underline{u} \leq u \leq \bar{u}$$

where  $J = J_1(z, u, t) + J_2(u)$  and

$$J_1(z, u, t) = \|c_1(B(z) + A(z)[(I - \sigma)\delta u + \sigma \tilde{u}] - v(z, t)\|_2^2 \tag{24}$$

$$J_2(u) = \|c_2(u - u_0)\|_2^2 \tag{25}$$

where  $c_1 \in R^{m \times m}$  and  $c_2 \in R^{n_a \times n_a}$  are positive definite diagonal weighting matrices. The cost function  $J_1$  minimizes the allocation error of the virtual control  $v$  into  $u$  in Eq. (11) and  $J_2$  minimizes the magnitude of the control input  $u$ .

Now the CA problem (Eq. (23)) is converted to a quadratic problem (QP). First, let

$$X_0 = u_0 - \underline{u}, \quad X = u - \underline{u}, \quad X_{max} = \bar{u} - \underline{u}$$

$$a_p(z, t) = -[B(z) + A(z)[(I - \sigma)\delta u + \sigma \tilde{u}] - v(z, t) \tag{26}$$

By substituting Eqs. (24) – (26) into Eq. (23), we have

$$J(z, u, t) = \|c_1[A(z)(I - \sigma)\delta X - a_p(z, u, t)]\|_2^2 + \|c_2(X - X_0)\|_2^2 \tag{27}$$

$$\text{subject to } 0 \leq X \leq X_{max}$$

By extending Eq. (27), we obtain

$$\begin{aligned} J(z, u, t) = & X^T \delta^T (I - \sigma)^T A^T(z) c_1^T c_1 A(z) (I - \sigma) \delta X \\ & - X^T \delta^T (I - \sigma)^T A^T(z) c_1^T c_1 a_p(z, t) - a_p^T(z, t) c_1^T c_1 A(z) (I - \sigma) \delta X \\ & + a_p^T(z, t) c_1^T c_1 a_p(z, t) + X^T c_2^T c_2 X - X^T c_2^T c_2 X_0 \\ & - X_0^T c_2^T c_2 X + X_0^T c_2^T c_2 X_0 \end{aligned} \tag{28}$$

If we define  $H_1 = c_1^T c_1, H_2 = c_2^T c_2, C^T(z, t) = -2 [a_p^T(z, t)H_1 A(z) (I-\sigma) \delta + X_0^T H_2]$ ,  $R(z, t) = a_p^T(z, t)H_1 a_p(z, t) + X_0^T H_2 X_0$ , and  $H(z) = 2 [\delta^T (I-\sigma)^T A^T(z)H_1 A(z) (I-\sigma) \delta + H_2]$ , then Eq. (28) can be rewritten as follows:

$$J(z, u, t) = \frac{1}{2} X^T H(z) X + C^T(z, t) X + R(z, t) \tag{29}$$

$$\text{subject to } 0 \leq X \leq X_{max}$$

**Remark 1**  $R(z, t)$  is a constant matrix and does not affect the optimal solution (Eq. (29)).

By defining  $E = \begin{bmatrix} E_1 \\ \vdots \\ E_{2n_a} \end{bmatrix} = \begin{bmatrix} I_{n_a} \\ -I_{n_a} \end{bmatrix}$  and  $F = \begin{bmatrix} F_1 \\ \vdots \\ F_{2n_a} \end{bmatrix} = \begin{bmatrix} X_{max} \\ 0_{n_a \times 1} \end{bmatrix}$ , the optimal problem Eq.

(29) becomes a QP problem, as follows:

$$J(z, u, t) = \frac{1}{2} X^T H(z) X + C^T(z, t) X \tag{30}$$

$$\text{subject to } EX - F \leq 0$$

For solving the QP problem (Eq. (30)), we consider a Lagrange function:

$$L(X, \lambda, z, t) = \frac{1}{2} X^T H(z) X + C^T(z, t) X + \lambda^T (EX - F) \tag{31}$$

where  $\lambda \in R^{2n_a}$  is the Lagrange multiplier. Now we define the limiting optimal set  $E^*$  as

$$E^* = \left\{ (X, \lambda, z) \mid \frac{\partial L}{\partial X} = 0, \frac{\partial L}{\partial \lambda} = 0 \right\} \tag{32}$$

In theorem 2, a dynamic update law is designed so as to attract  $(X, \lambda, z)$  to  $E^*$ . However, before this theorem, an important lemma is presented.

**Lemma 2** ([18,21]): Suppose there are positive definite functions  $V(x), \lambda_1 > 0$  and  $0 < r < 1$  such that  $\dot{V}(x) + \lambda_1 V^k(x) \leq 0$ , then the equilibrium point  $x = 0$  is finite-time stable. The convergence time  $t_s$  satisfies

$$t_s \leq \frac{V^{1-r}(x_0)}{\lambda_1(1-r)} \quad x_0 = x(0) \tag{33}$$

**Theorem 2**  $(X, \lambda, z)$  tend to  $E^*$  in a finite-time if the following dynamic update laws are adopted:

$$\dot{X} = -\Gamma_1 \tau_1 \tag{34}$$

$$\dot{\lambda} = -\Gamma_2 \tau_2 - \tau_2 (\tau_2^T \tau_2)^{-1} (\Delta + \omega V_2^a) \tag{35}$$

where  $\Gamma_1 \in R^{n_a \times n_a}, \Gamma_2 \in R^{2n_a \times 2n_a}$  are positive definite matrices,  $\omega > 0, 0 < a < 1$  and

$$\tau_1 = H(z) (H(z) X + C(z, t) + E^T \lambda) + E^T (EX - F) \tag{36}$$

$$\tau_2 = E (H(z) X + C(z,t) + E^T \lambda) \quad (37)$$

$$\Delta = \left( \frac{\partial L^T}{\partial X} \frac{\partial^2 L}{\partial z \partial X} \right) \dot{z} + \frac{\partial L^T}{\partial X} \frac{\partial^2 L}{\partial t \partial X} + \frac{\partial L^T}{\partial \lambda} \frac{\partial^2 L}{\partial t \partial \lambda} \quad (38)$$

**Proof** Consider the following Lyapunov function:

$$V_2(X, \lambda, z, t) = \frac{1}{2} \left( \frac{\partial L^T}{\partial X} \frac{\partial L}{\partial X} + \frac{\partial L^T}{\partial \lambda} \frac{\partial L}{\partial \lambda} \right) \quad (39)$$

The time derivative of  $V_2$  will be as follows:

$$\begin{aligned} \dot{V}_2 &= \frac{\partial V_2}{\partial X} \dot{X} + \frac{\partial V_2}{\partial \lambda} \dot{\lambda} + \frac{\partial V_2}{\partial z} \dot{z} + \frac{\partial V_2}{\partial t} \\ &= \left[ (H(z) X + C(z,t) + E^T \lambda)^T H(z) + (EX - F)^T E \right] \dot{X} \\ &\quad + \left[ (H(z) X + C(z,t) + E^T \lambda)^T E^T \right] \dot{\lambda} + \left( \frac{\partial L^T}{\partial X} \frac{\partial^2 L}{\partial z \partial X} \right) \dot{z} + \frac{\partial L^T}{\partial X} \frac{\partial^2 L}{\partial t \partial X} + \frac{\partial L^T}{\partial \lambda} \frac{\partial^2 L}{\partial t \partial \lambda} \end{aligned} \quad (40)$$

By substituting Eqs. (36) – (38) into Eq. (40), we have

$$\dot{V}_2(X, \lambda, z, t) = \tau_1^T \dot{X} + \tau_2^T \dot{\lambda} + \Delta \quad (41)$$

Now by choosing  $\dot{X}$  and  $\dot{\lambda}$  according to Eqs. (34) and (35), Eq. (41) becomes

$$\dot{V}_2(X, \lambda, z, t) = -\tau_1^T \Gamma_1 \tau_1^T - \tau_2^T \Gamma_2 \tau_2^T - \omega V_2^a \leq -\omega V_2^a \quad (42)$$

From Eq. (42) and lemma 2, we know that  $(X \lambda z)$  converge to  $E^*$  in finite-time as

$$t_{s_2} \leq \frac{V_2^{1-a}(x_0, \lambda_0, z_0, t_0)}{\omega(1-a)} \quad (43)$$

The design procedure of terminal sliding mode control allocation can be summarized in the following algorithm:

□

---

### Algorithm 1

---

Given the nonlinear overactuated system (Eq. (1)) with the actuator faults/failures (Eq. (4)) and the input limits (Eq. (6)), design FTC by performing the following steps:

Step 1. Given the desired trajectories  $y_{di}$ , obtain the virtual control using Eqs. (13) and (17).

Step 2. Given the weighting matrices  $c_1, c_2, \Gamma_1$  and  $\Gamma_2$ , obtain the CA result  $\dot{X}$  using Eq. (34).

Step 3. According to Eq. (26), the derivative of control inputs will be  $\dot{u} = \dot{X} + \dot{\underline{u}} = \dot{X}$ .

---

**5. Simulation results**

The attitude dynamics of a near space vehicle (NSV) is described by [22]

$$\dot{\gamma} = \Xi(\gamma)\omega$$

$$\dot{\omega} = J^{-1}\Omega(\omega)J\omega + Gu_f(t), \tag{44}$$

where  $\gamma = [\mu, \beta, \alpha]^T$  is the attitude angle vector,  $\omega = [p, q, r]^T$  is the angular rate vector, and  $u_f(t) = [u_{f_1}, u_{f_2}, u_{f_3}, u_{f_4}, u_{f_5}, u_{f_6}]^T$  is the input vector. The matrices  $\Xi(\gamma)$ ,  $\Omega(\omega)$ ,  $G$ , and  $J$  are given by

$$G = \begin{bmatrix} -0.288 & 0.288 & -0.865 & 0.865 & 0 & 0.002 \\ -0.085 & -0.085 & -0.526 & -0.526 & 0.009 & -0.007 \\ -0.095 & 0.095 & 0.207 & -0.207 & -0.004 & -0.005 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} 554486 & 0 & -23002 \\ 0 & 1136949 & 0 \\ -23002 & 0 & 1376852 \end{bmatrix}, \quad \Xi(\gamma) = \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ \sin(\alpha) & 0 & -\cos(\alpha) \\ 0 & 1 & 0 \end{bmatrix} \tag{45}$$

If we define the state vector of Eq. (44) as  $x = [\gamma^T, \omega^T]^T$ , then Eq. (44) converts to Eq. (1), and

$$f(x) = \begin{bmatrix} \Xi(\gamma)\omega \\ J^{-1}\Omega(\omega)J\omega \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0_{3 \times 6} \\ G \end{bmatrix} \quad \text{and} \quad y = \gamma = [\mu, \beta, \alpha]^T.$$

The initial state and the desired output are chosen as  $x_0 = [3.5, 5, 1, 0, 0, 0]$  and  $y_d = [3 \cos(t) + 0.5, 5, 1 + 4 \sin(0.5t)]^T$ . The bounds of control input are

$$u_\rho \in [-30^\circ, 30^\circ]^T \quad (\rho = 1, \dots, 6) \tag{46}$$

Based on Section 4, the CA parameters are  $H_1 = c_1^T c_1 = \text{diag}(100, 70, 40)$ ,  $H_2 = c_2^T c_2 = 0.1I_{6 \times 6}$ ,  $\Gamma_1 = 3I_{6 \times 6}$ ,  $\Gamma_2 = 10I_{12 \times 12}$ ,  $\omega = 100$ , and  $a = 0.5$ .

To proceed the design of the terminal sliding mode control according to Eq. (17), the design parameters are  $\alpha_{ij} = 40$ ,  $\beta_{ij} = 1$ ,  $p_{ij} = 5$ ,  $q_{ij} = 7$ ,  $\gamma_i = 60$ , and  $K_i = 20$  ( $i = 1, 2, 3$  and  $j = 1$ ).

The following cases are simulated to demonstrate the effectiveness of our method:

Case 1: Consider input limits (Eq. (46)) and all actuators fault/failure-free (i.e.  $u_{f_\rho} = u_\rho$ ).

Case 2: Consider all actuators fault-free and without input limits (Eq. (46)).

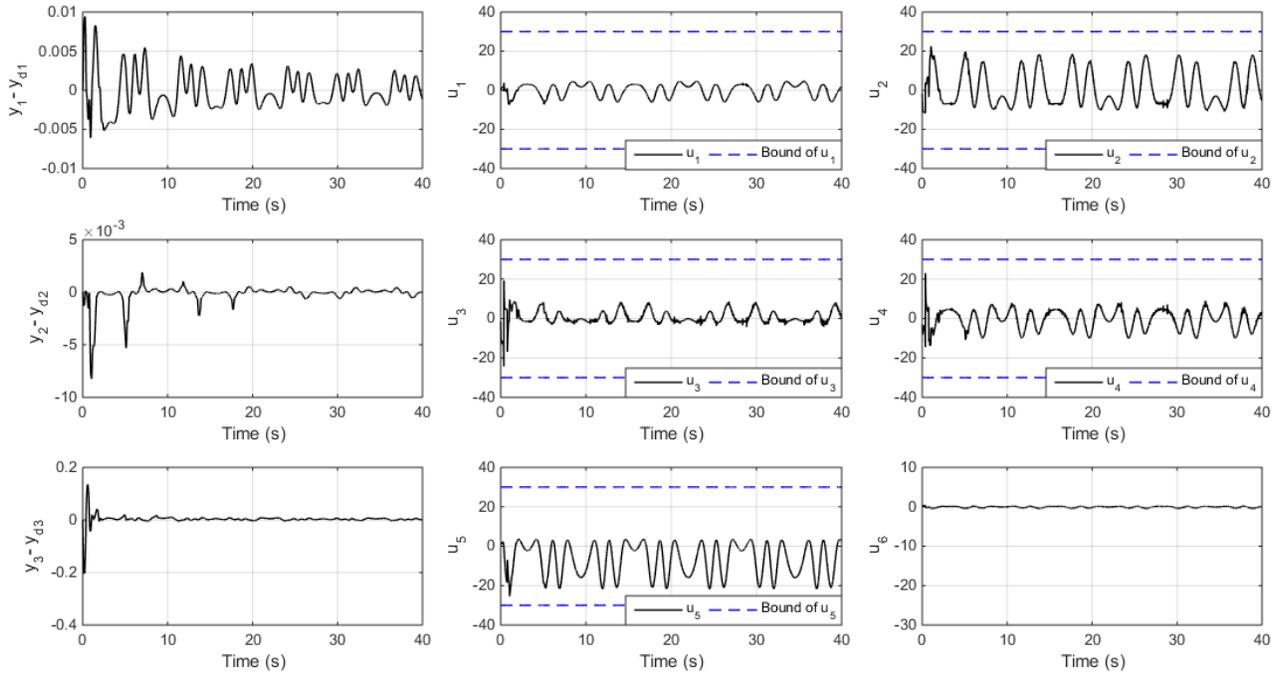
Case 3: Consider control constraints (Eq. (46)) and

- I. For  $0 \leq t < 5$ , all actuators are fault-free.
- II. For  $5 \leq t < 10$ , a lock in place fault happens on  $u_1$  ( $u_{f_1} = -20$ ).
- III. For  $10 \leq t < 15$ , a hard-over fault occurs on  $u_6$  ( $u_{f_6} = -30$ ) and loss of effectiveness fault occurs on  $u_3$  and  $u_4$  ( $u_{f_3} = 0.6u_3$  and  $u_{f_4} = 0.7u_4$ ).
- IV. For  $15 \leq t < 20$ , a float fault occurs on  $u_2$  ( $u_{f_2} = 0$ ).
- V. For  $20 \leq t < 25$ , a lock in place fault happens on  $u_5$  ( $u_{f_5} = 10$ ).

VI. For  $25 \leq t < 35$ , float fault occurs on  $u_1$  and  $u_5$  ( $u_{f_1} = u_{f_5} = 0$ ),  $u_{f_6} = 10$  and loss of effectiveness fault occurs on  $u_3$  and  $u_4$  ( $u_{f_3} = 0.5u_3$  and  $u_{f_4} = 0.6u_4$ ).

VII. For  $35 \leq t \leq 40$ , all actuators are fault/failure-free.

For case 1, the results of the simulation are illustrated in Figure 2. As seen in Figure 2a, the outputs track all the desired signals precisely. It is obvious from Figure 2b that all controls  $u_\rho$  ( $\rho = 1, \dots, 6$ ) remain within the control constraints (Eq. 46)).



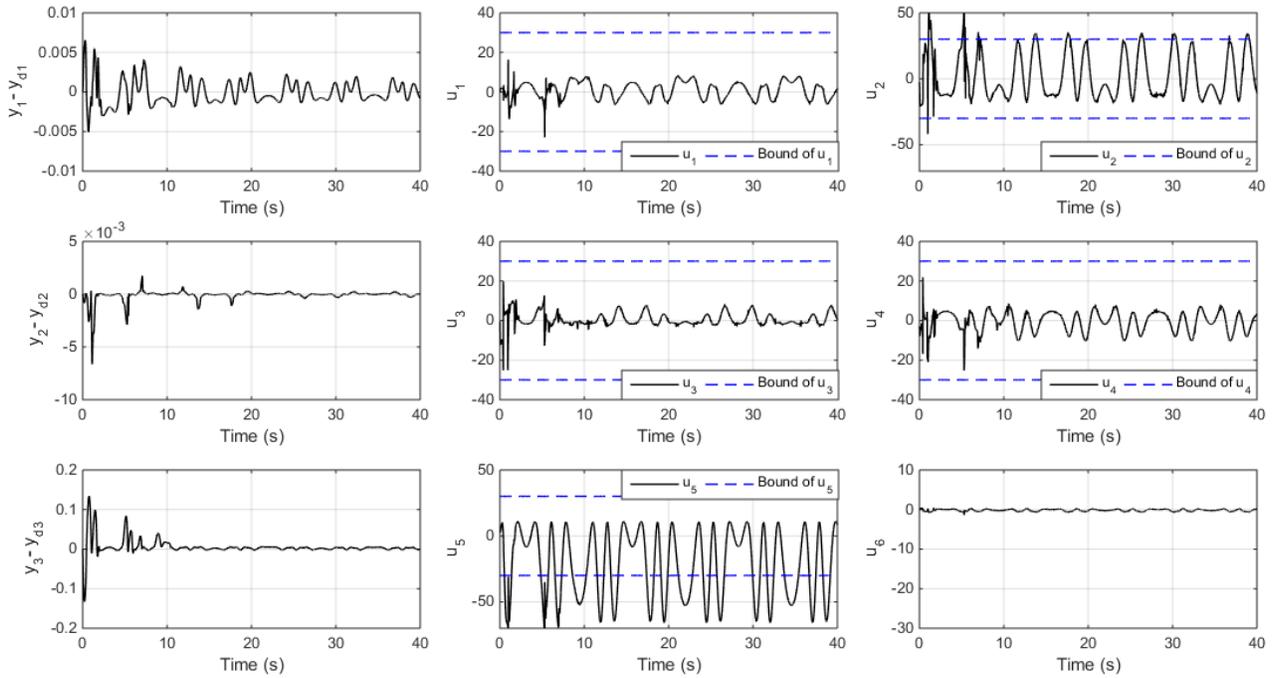
**Figure 2.** NSV response for case 1. (a) Tracking performance. (b) CA outputs ( $u$ ).

For case 2, the results of the simulation are illustrated in Figure 3. By increasing the input limits (Eq. (46)), it is obvious from Figure 3b that the inputs  $u_2$  and  $u_5$  indeed violate their bounds. This shows that the proposed CA effectively forces the input signals to stay within their bounds. Of course, according to Figure 3a, tracking is achieved perfectly.

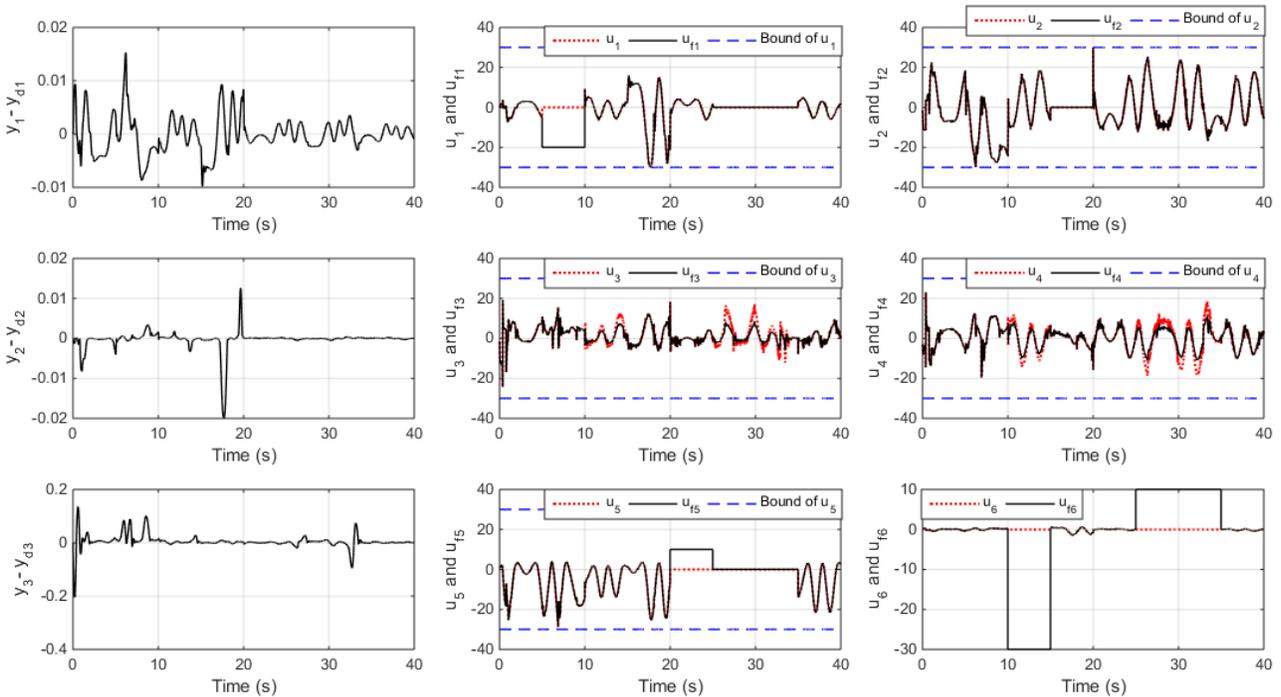
Finally, the simulation results for case 3 are shown in Figure 4. In the presence of the actuator faults/failures (I–VII), the input saturation constraints are still satisfied, as seen in Figure 4b. According to Figure 4b, when an actuator is damaged, control allocation cuts the related control signal of this actuator and redistributes control signals among the healthy actuators. In addition, good tracking is obtained when our system is under actuator faults/failures (Figure 4a).

### 6. Conclusion

In this paper, a new TSMC allocation algorithm has been successfully designed for overactuated nonlinear systems. The TSMC has a nonlinear sliding surface, which ensures the output tracking performance at finite-time in the presence of actuator faults or failures. The CA problem has been converted to a QP optimization problem and solved by a Lyapunov design approach. The derived CA law is in the form of a finite-time convergent



**Figure 3.** NSV response for case 2. (a) Tracking performance. (b) CA outputs ( $u$ ).



**Figure 4.** NSV response for case 3. (a) Tracking performance. (b) Plant inputs ( $u_f$ ) and CA outputs ( $u$ ).

dynamic update law, which satisfies control constraints and manages the actuators when faults/failures occur. The effectiveness of the proposed method has been demonstrated by applying it to an NSV example. The

simulation results show that in the presence of actuator faults/failures, good tracking performance is obtained and CA redistributes control signals among the healthy actuators.

### References

- [1] Liao F, Wang JL, Yang GH. Reliable robust flight tracking control: an LMI approach. *IEEE T Contr Syst T* 2002; 10: 76-89.
- [2] Yang GH, Ye D. Adaptive fault-tolerant H-infinity control against sensor failures. *IET Control Theory A* 2008; 2: 95-107.
- [3] Li XJ, Yang GH. Robust adaptive fault tolerant control for uncertain linear systems with actuator failures. *IET Control Theory A* 2012; 6: 1544-1551.
- [4] Wang J, Pei HL, Wang NZ. Adaptive output feedback control using fault compensation and fault estimation for linear system with actuator failure. *Int J Automat Comput* 2013; 10: 463-471.
- [5] Yang Q, Ge SS, Sun Y. Adaptive actuator fault tolerant control for uncertain nonlinear systems with multiple actuators. *Automatica* 2015; 60: 92-99.
- [6] Wang H, Liu X, Liu PX, Li S. Robust adaptive fuzzy fault-tolerant control for a class of non-lower-triangular nonlinear systems with actuator failures. *Inform Sciences* 2016; 336: 60-74.
- [7] Jin J. Modified pseudoinverse redistribution methods for redundant control allocation. *J Guid Control Dynam* 2005; 28: 1076-1079.
- [8] Bodson M. Evaluation of optimization methods for control allocation. *J Guid Control Dynam* 2002; 25: 703-711.
- [9] Buffington JM, Enns DF, Teel AR. Control allocation and zero dynamics. *J Guid Control Dynam* 1998; 21: 458-464.
- [10] Petersen JAM, Bodson M. Constrained quadratic programming techniques for control allocation. *IEEE T Contr Syst T* 2006; 14: 91-98.
- [11] Poonamallee VL, Doman DB. A nonlinear programming approach for control allocation. In: *Proceedings of the American Control Conference*; 30 June–2 July 2004; Boston, MA, USA. IEEE. pp. 1689-1694.
- [12] Johansen TA. Optimizing nonlinear control allocation. In: *Proceedings of the 43rd IEEE Conference on Decision and Control*; 2004; Atlantis, Bahamas. pp. 3435-3440.
- [13] Benosman M, Liao F, Lum KY. Nonlinear control allocation for non-minimum phase systems. *IEEE T Contr Syst T* 2009; 17: 394-404.
- [14] Edwards C, Spurgeon SK. *Sliding Mode Control: Theory and Applications*. London, UK: Taylor & Francis, 1998.
- [15] Hamayun MT, Edwards C, Alwi H. A fault tolerant control allocation scheme with output integral sliding modes. *Automatica* 2013; 49: 1830-1837.
- [16] Alwi H, Edwards C, Stroosma O, Mulder JA, Hamayun MT. Real-time implementation of an ISM fault-tolerant control scheme for LPV plants. *IEEE T Ind Electron* 2015; 62: 3896-3905.
- [17] Du H, Li S, Qian C. Finite-time attitude tracking control of spacecraft with application to attitude synchronization. *IEEE T Automat Contr* 2011; 56: 2711-2717.
- [18] Song ZH, Li H, Sun K. Finite-time control for nonlinear spacecraft attitude based on terminal sliding mode technique. *ISA T* 2014; 53: 117-124.
- [19] Isidori A. *Nonlinear Control Systems*. 2nd ed. New York, NY, USA: Springer-Verlag, 1989.
- [20] Chen M, Wu QX, Cui RX. Terminal sliding mode tracking control for a class of SISO uncertain nonlinear systems. *ISA T* 2013; 52: 198-206.
- [21] Miao P, Shen Y, Xia X. Finite time dual neural networks with a tunable activation function for solving quadratic programming problems and its application. *Neurocomputing* 2014; 143: 80-89.
- [22] Chen M. Constrained control allocation for over-actuated aircraft using a neurodynamic model. *IEEE T Syst Man Cy B* 2016; 46: 1630-1641.