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Angular Momentum Evolution in Close Late-type Binaries

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Abstract

Accurate monitoring of the orbital period of late-type close binaries can provide important information on their secular evolution and on the structural changes of their components on very short timescales (i.e. decades). The orbital period turns out to be a powerful tool of investigation: it can be measured to great accuracy even with simple equipments and its change produces an easily detectable cumulative effect over time.

This paper focuses on two physical mechanisms that cause orbital periods changes in late type binaries with negligible mass loss from the system: the dynamical evolution due to magnetic braking in tidally locked systems, and the transfer of angular momentum among the outer layers of their components. We present a review of the current theories and discuss the observability of these phenomena on the typical time span of the currently available records.

1. Introduction

The orbital period of binary systems is by far their most accurately known property: for short period systems a current figure of its accuracy is a part on 10^7 , while in favourable circumstances it can be as small as one part in 10^9 (eclipsing short period systems with sharp minima). The measurement of the period has always been a primary activity of variable star observers, therefore, even if we restrict ourselves to the modern times, we find that data extending over one century are quite common, the record holder (with ~ 200 years) being Algol (β Per).

Many physical processes affect the orbital period, producing a time cumulative effect on the observed quantity, the time of occurrence of minima. Thanks to the fact that one deals with a time cumulative effect, to the high precision of the time measurements

and to the extended records at disposal, one can hope to detect even very small period variations. These can yield precious indications on mechanisms acting on scales that are very short (10-100 y) when compared to the evolutionary ones.

We will not discuss here the best known phenomenon producing period changes in close binaries – the mass exchange between the components because of Roche lobe overflow. This paper mainly deals with the case where the angular momentum loss (AML) takes place with no mass exchange and with a negligible amount of mass loss, i.e. that of tidally locked binaries with at least one late-type component. In this case the period variation can take place even if the components are both inside the Roche lobes. This configuration offers the rare opportunity of studying the physics of stellar interiors by means of a simple kinematical quantity.

The term “tidally locked” indicates binaries whose orbital and rotational (spin) motions have been synchronized by the tidal forces. For late type field stars the spin-orbit synchronization is observed at orbital periods of a few days, for systems with dwarf components [1], and of some tens of days for those with giant ones [2], but see as well [3]. These figures agree fairly well with the classical model of tidal interaction by Zahn [4,5].

As a consequence of the tidal action, the synchronized components spin usually much faster than coeval late type - single stars. The latter, even when starting the main sequence life as fast rotators, are rapidly spun-down by the mechanism of magnetic braking, first introduced by Schatzman [6] to explain the slow rotation of the Sun.

The late-type dwarf components of tidally locked binaries can therefore be fast rotators, independently of their age; on the other hand they have as well the physical properties necessary for efficient angular momentum loss by magnetic braking, according to Schatzman’s model: a convective envelope hosting a dynamo and a magnetized stellar wind. In their case, however, thanks to the efficient synchronization mechanisms, the angular momentum taken away from the outgoing wind is actually subtracted from the orbital reservoir. The final result is shrinkage of the orbit, period decrease and spin-up of the components. The evolution of the orbital period contains, therefore, information on the braking mechanism, that in its turn is related to the star internal structure.

The analysis of period changes on short timescales cannot neglect another mechanism that has been proposed to explain the quasi-periodic changes on very short timescales (\leq decades). These are quite frequent and have been detected in a variety of close binaries with at least one late-type component. Its basic idea is that the rotation of the star external layers can vary without AML: just because of angular momentum transfer from or to the inner layers. The variation of the internal velocity profile changes the star quadrupole moment, that is strongly dependent on its oblateness and therefore on the surface rotation rate. According to Applegate [7], the variation of the quadrupole moment induces an instantaneous change of the orbital period (i.e. on a scale much shorter than the tidal one).

This paper presents a review of the mechanisms possibly at work in binaries with late-type components. Section 1 will be devoted to the models of AML by magnetic braking, while Section 2 will describe the effect of AMT and the problems related to the interpretation of the period changes.

2. Angular momentum loss in close late-type binaries

Several observational evidences support the model of AML by magnetic braking:

- the already mentioned decay of stellar rotation [8] for spectral types later than F5, that suggests a straightforward link with the presence of convective envelopes (and dynamos).
- the correlation of the magnetic activity indicators with the rotational period, see for instance [9], that turns out to be a reversible property: if for any reason a star is spun-up, its magnetic activity will also increase.

The basic idea of Schatzman's model is that in presence of a magnetic field the outgoing ionized wind is forced to corotate up to the Alfvén radius, i.e. the distance at which the wind speed equals the Alfvén velocity $v_A = B_A^2/4\pi\rho$, where B_A and ρ are the local magnetic field and wind density. The lever arm for AML can therefore be much larger than the stellar radius, and even a negligible mass loss by the wind can provide substantial AML.

In single stars and wide binaries (where the tidal forces are negligible) the effect of AML is, of course, the decay of the stellar rotation. As we already mentioned in the introduction, the case of tidally locked binaries is different. Thanks to efficient spin-orbit coupling the angular momentum taken away from the wind is actually subtracted from the orbital reservoir.

If the mass loss is negligible, we can write $\dot{H}_{orb} \propto P_K^{-2/3} \dot{P}_K$, that implies a decrease of the orbital period with decreasing H_{orb} . The net effect of AML is, therefore, the shrinking of the orbit and the spin-up of the components. This mechanism is indeed considered as the main way to form late type contact binaries from close systems, given the difficulties to form contact binaries from protostellar clouds [10]. This hypothesis is supported by the absence of W UMa binaries in young clusters [11].

The secular evolution of a close binary under the effects of AML and tidal forces has been computed by several authors. Detailed models have been published by Maceroni and Van't Veer [12] (hereafter MV) and Stepien [13] (hereafter ST).

In all these studies the observational constraints are derived from the observed period distribution of some sample of binaries (the G-type field systems for MV and the Hyades late-type binaries in ST). The logical scheme of these papers is simple: the initial period distribution (IPD) of the sample of binaries under study is evolved into a present period distribution (PPD). The transformation is steered by the period evolution function (PEF) that depends on the combined action of the braking torque and the tides.

The models presented in MV and ST differ with respect to the definition of the braking law, to the treatment of synchronization and to the choice of the initial period distribution (IPD) of close binaries.

In MV the models are restricted to systems formed by G-type binaries of the same mass. The braking law is derived evolving the IPD of Abt and Levy [14] into the PPD of G-type field binaries by Farinella et al [15], after correction for some selection and detectability effects. The relation giving the best fit of the observed distribution is adopted as braking law. The synchronization mechanism is treated in detail and different hypothesis are considered for the unknown coupling between the star core and envelope (rigid

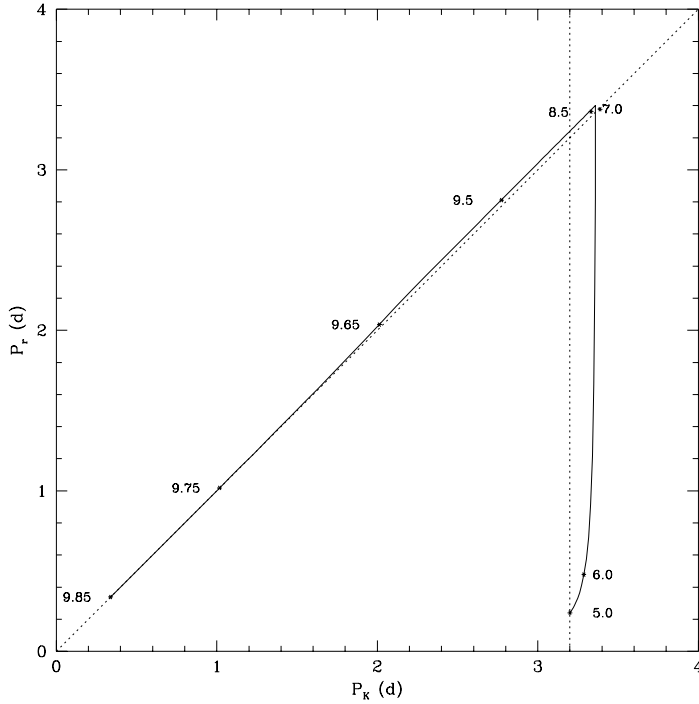


Figure 1. The rotational evolution of G-type binaries showing the period of rotation of the components as function of the orbital period. The track is labeled with $\log t$, being t the time in Gy counted from ZAMS. The dotted diagonal line corresponds to perfect synchronization. The binary gets very quickly that line, then both P_K and P_r decrease with time. The orbital period corresponding to a contact configuration for two MS G5 components ($P_K = 0.33$ d) is reached in 8 Gy.

body rotation or core completely decoupled from the envelope).

As an example, Fig. 1 shows the evolution of the spin and orbital period of a system with initial orbital period of 3.2 days, as computed in [16]. For that particular model rigid body rotation was assumed and spin-orbit synchronization according to Zahn [4].

The main feature of the derived braking law is to become rotation-independent for rotational velocity $\omega > 12\omega_\odot$ [16]. After an initial short phase ($\sim 10^7$ y) during which both AML and tidal forces act to brake the initially fast rotation (note the slight increase of the orbital period due to the fact that the angular momentum transfer by the tides takes place from the spin to the orbit), the system reaches almost perfect synchronization and evolves to shorter period until the contact configuration is reached in 8 Gy.

Similar models by Stepien apply to systems with a few combinations of total mass and mass ratios. Rigid body rotation and instantaneous spin-orbit coupling is assumed. The IPD is taken from the study of Duquennoy and Mayor [1] and the braking law from calibration of the activity versus rotational period, at variance with MV therefore

it is not a parameter to be derived by comparison with the PPD. The results of ST are compared mainly with the shortest orbital period found in the Hyades binaries. The choice of a sample of binaries belonging to one of the best known open clusters has the advantage of an homogeneous and well observed sample, but the obvious weakness of using distributions with very small numbers.

Though the results are somewhat different both papers agree on the fact that the braking law of fast rotators shows a saturation for short period, and that the well known Skumanich relation [17] $\dot{\omega} \propto \omega^3$, actually derived for slow rotators, does not hold for rotational velocities larger than 12-20 ω_{\odot} (the precise value depending on the model). This saturation can be related to that of the activity indicators [9, 18].

The expected relative change of the orbital period, \dot{P}/P , is shown in Fig. 2, where the relations of MV and ST models with two similar components of solar mass are reported, together with a similar relation for the Skumanich braking 'law' and another derived from a combination of Skumanich law and rotation independent braking [16]. Both MV and ST have a smaller slope for short periods and predict, for $P < 1^d$, substantially smaller period changes than the extrapolated Skumanich relation.

A point of interest is if this secular evolution of the period can be detected from the collection of data on period changes at our disposal.

Period changes are usually derived from the study of O-C diagrams. One can estimate the smallest detectable effect by means of the following simple argument that applies to a well observed short period system. Let's take a system of $P \approx 1^d$, that has - say - hundred well distributed measurements of minimum times, *all of them* with a standard deviation $\sigma_i \simeq 10^{-3}$ d. The mean deviation will be $\sigma \simeq 10^{-4}$ d; assuming that the minimum detectable effect on an O-C diagram is of the same order, the minimum continuous period change that can be detected over hundred years is related to the quantity:

$$A_{\min} = \frac{(O - C)_{\min}}{E^2} \approx 10^{-13} \text{d}, \quad (1)$$

(with $E \simeq 3 \cdot 10^4$ being the epoch).

From A_{\min} the relative change with time, $(\dot{P}/P)_{\min}$ can be derived according to:

$$\left(\frac{\dot{P}}{P}\right)_{\min} = \frac{2 A_{\min} \cdot 365.24}{P^2} \simeq 10^{-10} \text{yr}^{-1}, \quad (2)$$

with P in days.

The above estimate might, however, be too optimistic, as we assumed a σ value typical of modern (photometric) observations. The older data of our one-century time-span will in part be photographic or visual observations, whose typical deviations are from several to hundred times the value we assumed, and unfortunately the mean deviation depends strongly on the largest values found in the sample. It might therefore, be more realistic to multiply the previous number by a factor ≤ 10 .

We can conclude, from Fig. 2, that the effect of a variation $\dot{P}/P \approx 10^{-9}$ in systems with periods ~ 1 day would be observable with certainty, over a timespan of a century,

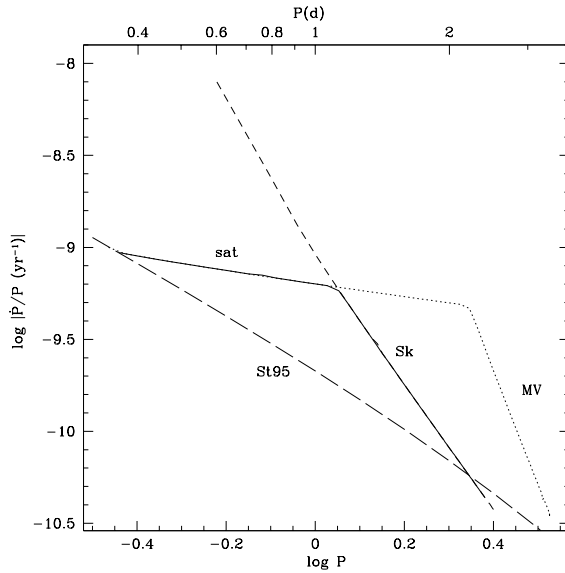


Figure 2. The relation between $\log |\dot{P}/P|$ and $\log P$, labelled according to different braking laws: MV: Maceroni and Van't Veer [11], St95: Stepien [12], Sk: Skumanich [16], sat: Skumanich law + saturation, as in [15]

only if the Skumanich relation held on the whole rotation range. There are however many indications, also independent from the binary configuration, against the latter hypothesis: the already mentioned saturation of the activity indicators that are well correlated with the period of rotation [9, 18], and the results on the rotational braking of single stars [19, 20].

We do not expect therefore to find direct evidences of the secular evolution due to AML from the period variations, and, as far as we know, the few cases reported in the literature (as that of VW Cep described in [21]) use a Skumanich type braking law to compute the expected AML. Anyway, even in presence of more relevant effects the detection would not be a trivial matter: the best candidates to observe the effect of AML by magnetic braking, detached but close systems with late-type components (i.e. RS CVn systems), often show large semi-regular period changes that dominate the O-C diagrams on a timescale ($P/\dot{P} \simeq 10^7$ y) much shorter than the tidal one.

3. Angular momentum transfer and period changes

An extensive study of period changes of some tens of RS CVn binaries was published, several years ago, by Hall and Kreiner [22]. They evidenced that most of the O-C diagrams show similar features: non-strictly periodic, though almost sinusoidal, variations of quite large amplitude. The corresponding modulation "periods" were usually of the order of the decade.

After their work more papers appeared on individual binaries, showing that a similar behaviour of the O-C diagram characterizes a variety of systems, all with at least one late-type component. This sample includes a number of cataclysmic variables, see Richman et al. [23] and references therein, some W UMa binaries [24], and even a binary pulsar, PSR B1957+20 [25].

The irregular but almost sinusoidal shape of the variations rule out the common explanations of period changes: third body and apsidal motion effects are strictly periodic, mass exchange produces monotonic period variations and, moreover, at least the RS CVn's are close but detached systems.

The first interpretation by Hall of the O-C pattern was based on the "rocket effect": he suggested that these very active stars could from time to time produce huge flares, causing anisotropic mass loss. The consequent torque would affect the stellar rotation and, thanks to the tidal locking, the orbital period. This model was however somewhat oversimplified, as it assumed perfect and instantaneous synchronization. Already De Campi and Baliunas [26] had shown that a mass loss of the required amount would quickly desynchronize the system, while most RS CVn's show almost perfect synchronisms.

Anyway the typical periods of the variations and the fact that at least one of the components of those systems is always a late-type and magnetically active star suggests a straightforward connection with the solar magnetic cycles.

Phenomena similar to the solar cycle but of enhanced intensity are known to occur on fast rotating late-type stars. The link of periods changes to those phenomena was however not a trivial one, as it was necessary to find a way to transmit changes occurring on short scale on the stellar surface to the orbital motion. Tidal coupling acts on a too long typical timescale to be efficient [26].

A viable model was presented by Applegate [7]. According to the waxing and waning of the magnetic field, during a stellar activity cycle, could produce angular momentum transfer among internal and external layers of the star. When the field is stronger the layers will rotate closer to rigid body rotation, which at constant angular momentum corresponds to the minimum of rotational energy. The change of the rotational velocity changes the oblateness of the star, inducing in its turn variation of its quadrupole moment.

The basic idea of Applegate's model is that a change of the quadrupole moment is sufficient to change instantaneously the orbital period, without mass or angular momentum loss [7]. The energetic requirements for such a process are acceptable in the hypothesis that the magnetic field is not contributing by itself to change the star shape but is just producing transitions between states of hydrostatic equilibrium.

While this mechanism is very frequently invoked to interpret any type of quasi-periodic variation on a short timescale, its effectiveness is still a matter of debate. In principle there are a few observational tests of the mechanism:

- i) the activity indicators should have the same period as the modulation period, P_{mod} .
- ii) the star luminosity is also expected to change during the cycle, with the same P_{mod} and a phase – that depends on the sign of the radial gradient of the angular velocity. If the inner layers rotate faster than the outer one the luminosity variations are expected to be in phase with the O-C diagram, in the opposite case they would be 180 degrees out

of phase. (We shall add however that as the distribution of angular velocities inside the stars are unknown, the last requirement on phase is not a real constraint; in most cases the sign of the gradient is derived from the phase).

iii) as the stellar radius change is negligible, an increase of luminosity will correspond to higher effective temperature, therefore the system should become bluer at the time of maximum luminosity.

These occurrences have been checked for a few systems with contradictory results. The current status of knowledge is summarized in the Table 1. The quantities appearing in the table are: the orbital period in days, the modulation period and the time-span of observations in years and, when available, the expected luminosity change with respect to the luminosity of the late-type component, L_* .

The most problematic requirement to fulfill is actually that of the luminosity changes. In several cases the expected changes are of non-negligible amount, in others as large as the star luminosity itself, so that the required energetics and time-scale have no theoretical explanation.

For one of the best known systems, RS CVn itself, the result of the check are not very good: the period of activity (as derived from the spot cycles) seems to be twice the modulation period, and the expected luminosity change (larger than the star luminosity) is not observed [27].

It has to be said that the data collected in the Table have very different reliability.

That mainly because the time span of observations is very often comparable to, or even shorter than, the derived value of the modulation period. It has already happened that several short modulation periods proposed for cataclysmic variables or novae, on the base of too short timespan of observations, were not confirmed by more extended records [23]. Observations over a few stellar cycles are therefore absolutely necessary to discriminate recurrent phenomena from accidental events, and to fight the well know tendency of the human eye to pick out from data almost sinusoidal variations on the scale of one or of half period, that are of no real statistical meaning [28].

There is an additional reason for caution in the the interpretation of all the O-C incomplete "cycles" as real period changes. The question has been recently pointed out by Koen and Lombard [35] and Hertz et al. [36] and we briefly summarize it here, following the presentation of Hertz et. al.

The measured times of mid-eclipse of a binary can be written as:

$$t_n = f(n) + e_n \quad (3)$$

being n the cycle number, $f(n)$ the predicted time from the ephemerids and e_n the errors that in normal conditions are independent and identically distributed with variance σ^2 . The residuals:

$$r_n = O - C = t_n - f(n) \quad (4)$$

have the same statistical properties, and their squared sum $\sum r_n^2$ has a χ^2 distribution (so that it has sense to apply least square algorithms to derive the orbital period).

Table 1. Systems interpreted in terms of the Applegate's mechanisms.

Name	type	Sp.	$P_{orb}(d)$	$P_{mod}(y)$	t_{obs}	ref.	comment
Algol	EA	B8V	2.87	32, 180	200	[29]	relatively regular variations 3rd body model possible
CG Cyg	RS	G9+K3V	0.63	50	58	[30]	ΔL and $\Delta(B - V)$ as predicted
RS CVn	RS	F5V+K2IV	4.80	38, 122	100	[27]	$\Delta L > L_*$; P_{mod} differs from the other activity related periods
RT Lac	RS	G9+K1	5.07	12	14	[31]	relatively regular variations 3rd body model possible
V471 Tau	CV	K2+WD	0.52	40, 20, 5	20	[7,32]	3rd body hypothesis for $P_{mod}=20$ y; ΔL in phase with $P_{mod} = 5$ y
SV Cam	RS	G0V	0.59	73	80	[7]	no observational test available
SS Cam	RS	G1 III	4.8	54	75	[7]	very sparse records
V711 Tau	RS	K1IV+G5V	2.84	12	6	[33]	evidence of photometric modulation only; luminosity and activity in phase
SZ Psc	RS	K1IV+F8V	3.96	> 56	56	[34]	$\Delta L > L_*$
AB And	EW		0.33	25, 85	85	[24]	ΔL too large and not observed
PSR 1957+20	PSR		0.38	5	5.5	[25]	Spurious period changes?

At variance with the previous case, if there is in t_n an additional random term of uncertainty, *intrinsic* to the binary and not related to measurement errors, the duration of each cycle will be:

$$d_n = t_n - t_{n-1} = f(n) - f(n-1) + \epsilon_n \quad (5)$$

where ϵ_n is the additional intrinsic uncertainty with a random distribution. One can therefore write:

$$t_n = t_o + \sum_{i=1}^n d_i + e_n = f(n) + \sum_{i=1}^n \epsilon_i + e_n \quad (6)$$

and

$$r_n = \sum_{i=1}^n \epsilon_i + e_n \quad (7)$$

In this case, therefore, the residuals are no longer independent, and, being the sum of random distributed variables, they will execute a random walk with increasing n , even

with the right ephemerids. Random walks are known to be non-stationary processes with no defined mean, and can easily produce what could look as a parabolic O-C plot or as half a sinusoidal cycle.

That is a well known fact to the researchers studying pulsating variables, where it is easier than in binaries to meet intrinsic variability of the time intervals between the light curve extremes. Fig. 6 of the above-mentioned paper of Koen and Lombard presents a simulation of the O-C diagram of a pulsating star (RY Sgr), computed with constant period and the hypothesis of intrinsic variability of the observed system. The result is strikingly similar to the actual O-C plot of Kilkenny [37].

At least in one case [36] a random walk explanation of the apparent period changes has been proposed to explain the O-C plot of a low mass X-ray binary. In that case the orbital period was derived from the pulsar eclipse ingress and egress times, which could be affected by correlated changes (due for instance to atmospheric variability of the non-degenerate component). In most cases however, we will not expect intrinsic changes in the minimum times in eclipsing binaries, being such a system a precise kinematical clock. An exception could however be that of some cataclysmic binaries, where the light curve is dominated by the effect of the hot spot on the accretion disc. This effect, however, should not play an important role in RS CVn's and contact binaries: their surface inhomogeneities, related to the magnetic activity, do produce a shift the minimum times, but its amount is very small [22, 38].

That is one reason more in favour of systematic photometric observations of late-type binaries. This type of programmes can be a promising long term project for small/mean size telescopes. As many of those, once available to the international community, are no longer in operation, one can only rely on the instrumentation belonging to individual institutions. The Turkish astronomical Community, among others, has given in recent years a valuable contribution to this topic.

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