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Simulation of a Feedback Control Technique Through Irrigation Canal Junctions

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Abstract: A linear quadratic controller based algorithm was developed for simulating the dynamics of a single-reach irrigation canal with a channel junction. Using the concepts of feedback control theory, an expression for an upstream gate opening of an irrigation canal reach with a channel junction operated based upon a constant-level control was obtained. In the derivation, the canal reach between 2 gates was divided into 5 nodes, and the finite difference forms of the continuity and momentum equations were written for each node. The Taylor series was applied to linearize the equations around equilibrium conditions. At the third node of the canal reach, a channel junction occured and the equations were derived based on the channel junction parameters. The hydraulic description of flow at channel junctions is difficult because of flow approaching angles, energy losses and turbulence. An example problem with a single pool was considered for evaluating the technique used to design a linear quadratic controller (LQR) for irrigation canals with a channel junction. Considering the computational complexity and the accuracy of the results obtained, the LQR feedback control theory was found to be adequate for irrigation canals with channel junctions.

Key Words: channel junctions, linear quadratic controller, mathematical modeling, open-channel flow

Sulama Kanallarının Birleştiği Noktalarda Geri-dönüşümlü Kontrol Tekniğinin Simülasyonu

Özet: Sulama kanallarının birleştiği noktalarda suyun kontrolu için doğrusal karesel control tekniğine dayalı bir algoritma geliştirilerek simülasyon yapılmıştır. Geribeslemeli control tekniği kullanılarak kanalların birleştiği noktanın memba kısmındaki kapağının açılıp kapanması için sabit-seviyeli control methoduna göre bir ifade geliştirilmiştir. Bu ifadenin çıkartılmasında göz önüne alınan havuz (iki kapak arasında kalan kısım) beş boğuma bölünmüş ve her bir boğum için sonlu farklar ile ifade edilen süreklilik ve momentum eşitlikleri çıkartılmıştır. Taylor serileri de bir denge konumu göz önüne alınarak bu eşitliklere uygulanmıştır. Havuzun üçüncü boğumunda kanal bir başka kanal ile birleştiğinden süreklilik ve momentum eşitlikleri de buna göre çıkartılmıştır. Birleşen kanaldan gelen akışın yaklaşım açısı, enerji kayıpları ve türbülansı birleşim yerindeki akışın hidrolik tanımını zorlaştırmaktadır. Geliştirilen doğrusal karesel kontrol algoritmasını değerlendirmek için bir örnek problem çözülmüştür. Hesaplamalardaki zorluklar ve simülasyondan elde edilen sonuçlar göz önüne alınarak, doğrusal karesel kontrol tekniğinin birleşen kanallar için yeterli ve uygun bir teknik olduğu sonucuna varılmıştır.

Anahtar Sözcükler: kanallarda birleşmeler, doğrusal karesel control tekniği, matematiksel modelleme, açık kanallarda akım.

Introduction

In recent years, the better operation of canals has become an increasingly important and attainable goal. Improved operation of irrigation conveyance systems will improve service to water users, conserve water through increased efficiency, reduce operation and maintenance costs, increase delivery flexibility, and provide more responsive reactions to emergencies. Timely delivery of the required quantity of water is necessary for improved agricultural production. The supply-oriented operational concept has not been able to provide the needed flexibility in terms of water quantity and timing to achieve improved crop yields and water-use efficiency. The demand-oriented operational concept bases operations on downstream conditions (Bureau of Reclamation, 1995). Most irrigation systems should use this downstream concept. The canal system should be operated to satisfy downstream needs, responding to what is taken out of the system rather than to what is put in. A significant portion of wasted irrigation water is attributed to inefficient canal system operation. Feedback control systems can improve the efficiency of the operation of

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irrigation canal networks and increase the dependability of water supply at farm level. With automatic feedback controllers, irrigation distribution networks can have better robustness, reliability and safety, as well as minimize water waste and reduce the cost of operation and maintenance when compared with manual operations (Bureau of Reclamation, 1995).

Feedback control systems for gate structures and turnouts have been applied to irrigation canal systems in the past. Over the years, the concepts of feedback control theory have been applied for deriving control algorithms for the real-time control of irrigation canals (Balogun et al., 1988; Begovitch and Ortega, 1989; Reddy et al., 1991; Goussard, 1993; Rodellar et al., 1993; Malaterre, 1994; Malaterre, 1998). Balogun et al. (1988), Garcia et al. (1992), and Durdu (2003, 2004a, 2004b) simulated feedback control concepts for deriving global control algorithms for irrigation canals. However, those algorithms dealt with an irrigation canal without channel junctions and they were designed for an independent irrigation canal. Generally, the canal network is a relatively large system. Accordingly, even using numerical method, simultaneous prediction of flow variables is a challenging problem. Because of this difficulty, the above control algorithms consider a single canal in an irrigation network. This assumption may be used in certain situations. However, the flow phenomenon in an irrigation network is much more complicated than that in a single independent canal. The hydraulic description of the flow at the channel junction is complicated and difficult because of the high degree of flow mixing, separation, turbulence and energy losses (Choi, 1991). Furthermore, the approaching angles of flow into the main canal are important for the numerical solution of flow equations. Therefore, the flow equations at a channel junction node will be different than those at other regular nodes. The objective of this paper is to present a discrete feedback control scheme for the operation of irrigation canals with a converging channel junction.

Model Development

The governing equations of continuity and momentum (or energy) express the phenomena of the water flow in the irrigation canal network. However, they furnish no direct answers as to the values of discharge and water depths, which are solutions of the basic flow equations, which are functions of time and space. The theoretical analysis is so complicated that solutions can only be obtained by a numerical method. In the operation of irrigation canals, in order to maintain the flow rate into the laterals close to the desired value, decisions regarding the opening of gates in response to random changes in the water withdrawal rates into lateral canals are required. This is accomplished by maintaining the depth of flow or the volume of water in a given pool at a target value. This problem is similar to the process-control problem, in which the state of the system is maintained close to the desired value by using real-time feedback control. In this study, only a single reach of a canal with a channel junction was considered. The reach was divided into N-1 sub-reaches of length Δx. There are 5 nodes (N = 5) in the canal reach (Figure 1). For convenience, the spacing between the nodes (Δx) was assumed constant. Using a finite-difference approximation, the continuity and momentum equations are written for each node. At the third node, there is a converging channel junction into the main irrigation canal. Because of the approaching angles of flow, energy loss, turbulence, and flow mixing, the continuity and momentum equations will be different at this particular node than those at other regular nodes. In this study, it was assumed that the flow was subcritical throughout the canal and the canal was horizontal and trapezoidal.

Gradually varied unsteady channel flow is described by 2 basic partial differential equations (Saint-Venant equations): the continuity and momentum equations. The continuity equation considers the conservation of mass and the momentum equation expresses the conservation of momentum. The Saint-Venant equations, presented below, are used to model flow in irrigation canals:

$$\frac{\partial A}{\partial t} - \frac{\partial Q}{\partial x} - q_1 = 0 \qquad (1)$$

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial Q}{\partial x}\left(\frac{Q^2}{A}\right) = gS_0 - gS_f - g\,\frac{\partial x}{\partial z} \qquad (2)$$

in which Q = discharge, ($m^3\ s^{-1}$); A = cross-sectional area of flow, (m^2); q_l = lateral flow, ($m^2\ s^{-1}$); z = water surface elevation, (m); t = time, (s); x = longitudinal direction of channel, (m); g = gravitational acceleration, ($m^2\ s^{-1}$); S_o = canal bottom slope (mm^{-1}); and S_f = the friction slope, (mm^{-1}). In deriving Eq. (2), the effect of the net

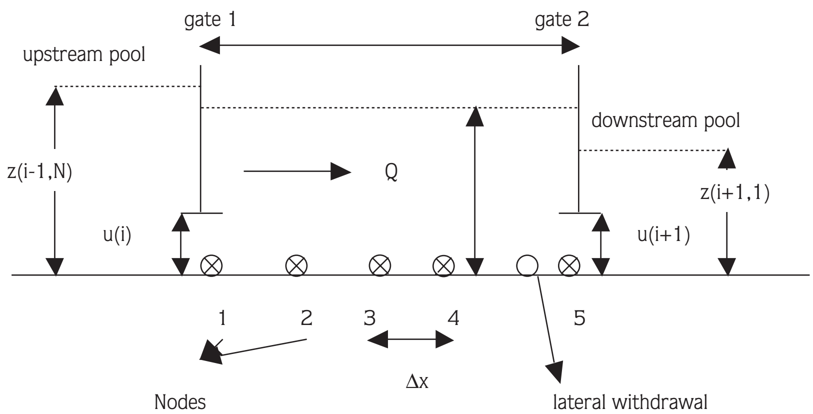

Figure 1. Schematic of a single canal reach.

acceleration terms stemming from removal of a fraction of the surface stream was assumed negligible. In Eqs. (1) and (2), the spatial derivatives were replaced by finite difference approximations by dividing the pool into a few segments (N number of nodes). A central difference scheme was used for the interior nodes ($1<j<N$), and a forward difference and a backward difference were applied to the first and last nodes, respectively. To solve those equations, the boundary conditions were expressed in terms of the continuity equation (Reddy, 1996):

$$Q_{i-l,N} = Q_{i,l} = Q_{gi} \tag{3}$$

and the gate discharge equation

$$Q_{gi} = C_{di} b_i u_i \sqrt{2g(z_{i-l,N} - z_{i,l})} \tag{4}$$

in which $Q_{i-1,N}$ = flow rate through the downstream gate (or node N) of pool i-1, (m^3 s^{-1}); Q_{gi} = flow rate through upstream gate of pool i, (m^3 s^{-1}); $Q_{i,1}$ = flow rate through the upstream gate (or node 1) of pool i, (m^3 s^{-1}); C_{di} = discharge coefficient of gate i; b_i = width of gate i, (m); u_i = opening of gate i, (m); $z_{i-1,N}$ = water surface elevation at node N of pool i-1, (m); $z_{i,1}$ = water surface elevation at node 1 of pool i, (m); and i = pool index (i = 0 refers to the upstream constant level reservoir). In Eq. (4), the change in the bottom elevation of the canal across the gate was assumed negligible. At the upstream and downstream nodes of the canal reach, the momentum equation was replaced by the gate discharge equation, Eq. (4), which is basically expressed in terms of depth of flow at these nodes. At each intermediate node, there are 2 unknowns (Q and z). At the upstream and downstream ends, there are 2 unknowns per node (z and u). With N as the number of nodes in the reach, the total number of equations for the reach is 2(N-2) + 2. Therefore, to solve the set of equations, the 2 boundary conditions must be specified.

The precise hydraulic description of the flow at a channel junction is complicated and difficult because of the high degree of flow mixing, separation, turbulence and energy losses (Choi, 1991). The influence of the approaching angles (γ) is different, depending upon the discharge ratio between the upstream and downstream (Choi, 1991). The maximum depth difference between the upstream and downstream occurred near the depth ratios of 0.4 to 0.6 in the 30 and 60 degree junctions. On the other hand, the maximum depth difference occurred near the depth ratio of 1.0 in the 90 degree junction. It is noted that the relative upstream flow depth is a function of discharge distribution for various angles of junction (Choi, 1991). In a canal junction, the process of flow mixing and turbulence causes a loss of energy in addition to the friction loss that occurs in the canal itself. The other major process that effects a loss of energy at a junction is flow separation. The separation zone is developed downstream from the entrance of a lateral channel. The momentum of the lateral canal ensures that the flow detaches from the sidewall as it enters the main canal and a separation zone of lower speed with circulating flow is introduced (Choi, 1991). Best and Reid (1984) conducted experiments to determine the maximum width and length of flow separation in the main canal at the section downstream of the junction. Their experiment was based on 15, 45, 70 and 90 degree canal junctions, with Froude numbers between 0.1 and 0.3.

The experiment showed that the separation width increases with an increase in discharge ratios between the lateral canal and the downstream canal (Choi, 1991).

The accurate description of the junction hydraulics is important for realistic and reliable simulation of a feedback controller in an irrigation canals network. In addition to the continuity relationship, the dynamic relationship at the channel junction can be represented by either the energy or momentum equations. In the past, the momentum equation was rarely used because the terms of this equation are vector quantities, and the momentum contribution from upstream channels, especially from the laterally connected upstream channels, is usually difficult to define (Choi, 1991). Even though difficulties exist in applying the vector quantities in the momentum equation for channel junctions, the conservation of momentum at channel junctions may be necessary to consider the effects of channel junction angles and to account for other upstream channel discharge effects. Using lateral inflow angles to the main canal instead of channel junction angles, based on the momentum flux contribution, a momentum equation at channel junctions can be derived.

The hydraulic description of flow at channel junctions can be represented by the conservation of momentum between upstream and downstream canal segments, and the junction continuity equation. The momentum equation can be derived by considering the contribution of momentum flux by the upstream and downstream channels. The momentum flux in a downstream channel is equal to the sum of momentum flux from the upstream channels. As illustrated in Figure 2, γ is the lateral inflow angle into the downstream canal as opposed to the channel junction angle α. The relationship between γ and α in channel junctions has been studied experimentally (Best and Reid, 1984). The lateral inflow angle (γ) to the downstream channel was shown by Best and Reid (1984) to be dependent upon discharge ratios between lateral inflow and downstream outflow and channel junctions angles (α). The momentum and continuity equations at channel junctions can be presented as follows (Choi, 1991):

$$\frac{\partial A_x}{\partial t} + \frac{\partial Q_x}{\partial x} = q_1 \qquad (5)$$

$$\frac{\partial Q_x}{\partial t} + \frac{\partial (V_x Q)}{\partial x} + gA_x\left(\frac{\partial z}{\partial x} - S_0 + S_f\right) = 0 \qquad (6)$$

where A_x is the projection of the cross-sectional area in the longitudinal x-direction in the one-dimensional equation (A_x = A*cos γ), V_x is the velocity component in the x direction, which is longitudinal direction in a dimensional equation (V_x = V*cos γ), and Q_x is the discharge in the x direction. The turnouts were assumed to be located immediately upstream of the last node in the canal reach (Figure 1). Although the lateral withdrawals were concentrated at one point, for modeling purposes, it was assumed to be uniformly distributed between the adjacent nodes, and was related to q_l of Eq. (1) as follows. The mathematical representation of flow through turnout structures is given as follows (Reddy, 1996):

$$q_1 = \frac{\sum_{n=1}^{O_{i,j}} q_{i,n}}{s} \qquad (7)$$

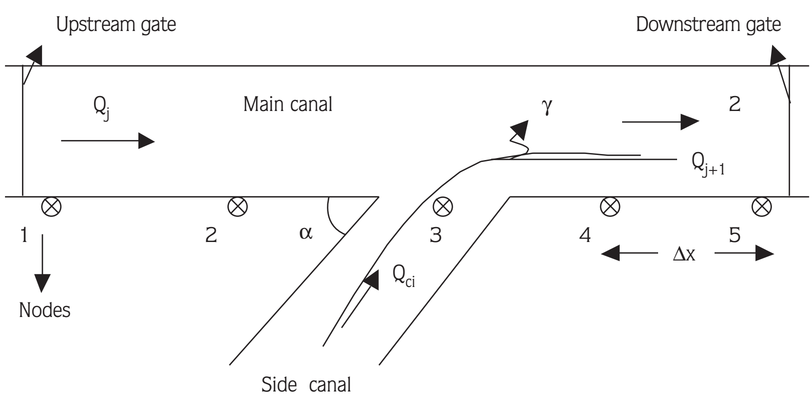

Figure 2. Channel junction profile.

where $s = \Delta x$ in the case of a backward/forward difference scheme; and $s = 2\Delta x$ in a central difference scheme; Δx = distance between 2 nodes, (m); $q_{i,n}$ = withdrawal rate from outlet n of pool i, ($m^3\ s^{-1}$); and $O_{i,j}$ = number of outlets represented around node j of pool i.

The Saint-Venant open-channel equations were linearized about an average operating condition of the canal to apply the linear control theory concepts to the problem (Balogun, 1985). Linear control theory is well developed and is easier to apply than nonlinear control theory. To apply linear control theory, Eqs. (1, 2, 5, 6) were linearized about an average operating point. In this study, the derivation of equations for regular nodes was not emphasized. For details about the derivation of the equations for regular nodes, refer to Balogun (1985), Reddy (1990), or Durdu (2003). Using the Taylor series around the average operating point, and truncating terms higher than the first-order, the deviation variables were obtained as follows:

$$\delta Q_{i,j} = Q_{i,j} - Q^0_{i,j} \quad (8)$$

$$\delta z_{i,j} = z_{i,j} - z^0_{i,j} \quad (9)$$

$$\delta u_i = u_i - u^0_i \quad (10)$$

$$\delta u_{i+1} = u_{i+1} - u^0_{i+1} \quad (11)$$

in which Q^0, z^0, and u^0 represent the flow rate, water surface elevation, and gate opening, respectively, at the equilibrium condition; and δQ, δy, and δu represent the deviations in flow rate, water surface elevation, and gate opening, respectively, from the equilibrium condition. In Eqs. (8-11), i is the index of the canal reach, and j is the index of the node in the canal reach. Application of the finite-difference technique, and substitution of Eqs. (8-11) into Eqs. (1) and (2) results in a set of linear equations (Reddy, 1990) for the regular nodes. For the channel junction node, Eqs. (8-11) were substituted into Eqs. (5) and (6) as follows (Choi, 1991):

Continuity equation at channel junction:

$$\theta[Q^+_{j+1} - (\delta Q^+_j + \delta Q^+_{cj}*\cos\gamma)/\Delta x] + (1 - \theta)[\delta Q_{j+1} - (\delta Q_j + \delta Q_{cj}*\cos\gamma)/\Delta x] + (1/2\ \Delta t)b_{j+1/2} (\delta z^+_j + \delta z^+_{j+1} - \delta z_j - \delta z_{j+1}) = \theta q_l{}^+_{j+1/2} + (1-\theta)\, q_{l\ j+1/2} \quad (12)$$

Momentum equation at channel junction:

$$(1/2\ \Delta t)[(\delta Q^+_j + \delta Q^+_{cj}*\cos\gamma) - (\delta Q_j + \delta Q_{cj}*\cos\gamma) + \delta Q^+_{j+1} - \delta Q_{j+1}) + (\theta/\Delta x)[\delta V_{j+1}\delta Q^+_{j+1} - \delta V_{cj}*\cos\gamma\,(\delta Q^+_j + \delta Q^+_{cj}*\cos\gamma)] + (1-\theta)/\Delta x[\delta V_{j+1}\delta Q_{j+1} - \delta V_{cj}*\cos\gamma(\delta Q^+_j + \delta Q^+_{cj}*\cos\gamma)] + ((\theta g A_{x(j+j+1)/2})/\Delta x)(\delta z^+_{j+1} - \delta z^+_j) + ((1-\theta) g A_{x(j+j+1)/2})/\Delta x)\,(\delta z_{j+1} - \delta z_j) = ((\theta g A_{x(j+j+1)/2})/\, 2)(So^+_{(j+j+1)/2}) + ((1-\theta) g A_{x(j+j+1)/2}/2)\ (S_{0(j+j+1)/2}) - ((\theta g A_{x(j+j+1)/2})/2)[Sf_{j+1} + (2Sf_{j+1}/Q_{j+1})\,(\delta Q^+_{j+1} - \delta Q_{j+1}) - (2Sf_{j+1}/K_{j+1})(\partial K_{j+1}/\partial h)(\delta z^+_{j+1} - \delta z_{j+1})] - ((\theta g A_{x(j+j+1)/2})/2)[Sf_j + (2Sf_j/(\delta Q_j + \delta Q_{cj}*\cos\gamma))\, ((\delta Q^+_j + \delta Q^+_{cj}*\cos\gamma) - (\delta Q_j + \delta Q_{cj}*\cos\gamma)) - (2Sf_j/K_j)\, (\partial K_j/\partial z)(\delta z^+_j - \delta z_j)] - ((1-\theta) g A_{x(j+j+1)/2})/2)\ Sf_{j+1} - ((1-\theta) g A_{x(j+j+1)/2})/\, 2)\ Sf_j + (\theta/2) q_l{}^+_{(j+j+1/2)}\ V_l{}^+_{(j+j+1)/2} + ((1-\theta)/2) q_{l\,(j+j+1)/2}\ V_{l(j+j+1)/2} \quad (13)$$

where θ is a weighting coefficient, $0 \le \theta \le 1$, which controls the stability of the numerical results; γ is flow approaching angles at the channel junction (Figure 2); δQ^+_j is discharge from time level n+1 at node j ($m^3\ s^{-1}$); V_{cj} is velocity of flow at the channel junction (ms^{-1}); Q_{cj} is discharge at the secondary canal at time level n ($m^3\ s^{-1}$); Sf is friction slope; So is channel bed slope, and q_l is lateral flow, ($m^2\ s^{-1}$); j is node index; and i is canal reach index. After the substitution of Eqs. (8-11) into momentum and continuity equations for both regular nodes and channel junction node, a set of linear equations was obtained as follows (Malaterre, 1994):

$$A_{11}\delta Q^+_j + A_{12}\delta z^+_j + A_{13}\delta Q^+_{j+1} + A_{14}\delta z^+_{j+1} = A'_{11}\delta Q_j + A'_{12}\delta z_j + A'_{13}\delta Q_{j+1} + A'_{14}\delta z_{j+1} + C_1 \quad (14)$$

$$A_{21}\delta Q^+_j + A_{22}\delta z^+_j + A_{23}\delta Q^+_{j+1} + A_{24}\delta z^+_{j+1} = A'_{21}\delta Q_j + A'_{22}\delta z_j + A'_{23}\delta Q_{j+1} + A'_{24}\delta z_{j+1} + C_2 \quad (15)$$

where δQ^+_j and δz^+_j are discharge and water-level increments from time level n+1 at node j; δQ_j and δz_j are discharge and water-level increments from time level n at node j; and A_{11}, A'_{21},…. A_{12}, A_{22} are the coefficients of the continuity and momentum equations, respectively, computed with known values at time level n. For details about coefficients of linear equations for channel junctions, refer to Choi (1991). Similar equations are derived for channel segments that contain a lateral, a gate structure, a weir or some other type of hydraulic structure. From the equations above, the state of system equation at any sampling interval k can be written, in compact form as follows:

$$A_L\ \delta x(k+1) = A_R \delta x(k) + B\delta u(k) + C\delta q(k) \quad (16)$$

where A = l x l system feedback matrix, B = l x m control distribution matrix, k = time increment (s); and Δq = variation in demands (or disturbances) at the turnouts ($m^2 s^{-1}$). The elements of the matrices A, B, and C depend upon the initial condition. Eq. (16) can be written in a state-variable form along with the output equations as follows (Reddy, 1999):

$$\delta x(k+1) = \Phi\delta x(k) + \Gamma\delta u(k) + \Psi\delta q(k) \quad (17)$$

$$\delta y(k) = H\delta x(k) \quad (18)$$

where $\Phi = (A_L)^{-1} * A_R$, $\Gamma = (A_L)^{-1} * B$, and $\Psi = (A_L)^{-1} * C$, $\delta x(k)$ = l x 1 state vector, $\delta u(k)$ = m x 1 control vector, $\delta q(k)$ = p x l matrix representing external disturbances (changes in water withdrawal rates) acting on the system, $\delta y(k)$ = r x 1 vector of output (measured variables), H = r x l output matrix, l = number of dependent (state) variables in the system, m = number of controls (gates) in the canal, p = number of outlets in the canal, and r = number of outputs. The elements of the matrices Φ, Γ, and Ψ depend upon the canal parameters, the sampling interval, and the assumed average operating condition of the canal. In Eq. (17), the vector of state variables is defined as follows (Reddy, 1996):

$$\delta x = (\delta Q_{i,1}, \delta z_{i,2}, \delta Q_{i,2}, \ldots\ldots \delta z_{i,N-1}, \delta Q_{i,n-1}, \delta Q_{i,N}) \quad (19)$$

Feedback Control

Feedback control is another term to describe closed-loop control (Figure 3). A closed-loop control system utilizes an additional measure of the actual output to compare the actual output with a desired output response. The measure of the output is called the feedback signal. In irrigation canals, feedback type control systems are used to minimize the magnitude and durations of the mismatch between the supply and the demand. Eq. (17) is called the discrete state equation in this study. This equation describes the condition or evolution of the basic internal variables. In hydraulic engineering problems, the depth of flow, flow rate, and velocity as a function of distance can be considered the state or internal variables. Sometimes, the volume of water in a given reach of a canal can also be considered a state variable (Reddy, 1990). In this paper, the water surface elevation and flow rate were considered the state variables. Given the initial conditions [$\delta x(0)$], δu, and δq, Eq. (17) can be solved for variations in flow depth and flow rate as a function of time. If the system is really at equilibrium [i.e. $\delta x(0) = 0$ at time t = 0] and there is no change in the lateral withdrawal rates (disturbances), the system would continue to be at equilibrium forever; then there is no need for any control action. Conversely, in the presence of disturbances (known or random), the system would deviate from the equilibrium condition. The actual condition of the system may be either above or below the equilibrium condition, depending upon the sign and magnitude of the disturbances. If the system deviates significantly from the equilibrium condition, the discharge rates into the laterals will be different (either more or less) than the desired values. However, in canal operations, the main objective is to keep these deviations to a minimum so that a nearly constant rate of discharge is maintained through the turnouts. The objective of control theory is to find a control law that will bring an initially disturbed system to the desired target value in the presence of external disturbances acting on the system (Reddy, 1996). During the last 2 decades, the concepts of the optimal control theory have been applied for deriving feedback control algorithms for the real-time control of irrigation canals. The application of optimal control theory for the derivation of canal control algorithms eliminates the need for the trial and error procedure that has been traditionally used. In order to maintain the target water levels in the pools at specified values, in the presence of random disturbances acting on the system,

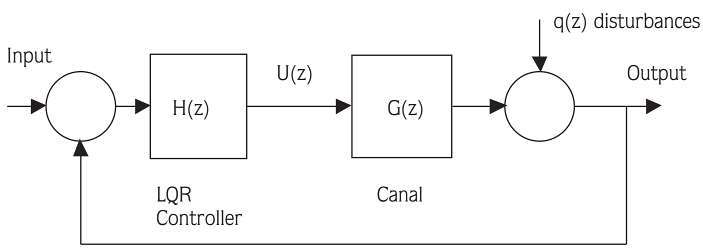

Figure 3. A feedback control system scheme.

the control structures (gates) in the irrigation canal are frequently adjusted. This can be accomplished by applying a large proportional control, in which the change in gate opening is proportional to the changes in flow depths and flow rates, of the following form (Reddy 1996):

$$\delta u(k) = -K\, \delta x(k) \tag{20}$$

where K the m x l controller gain matrix. The selection of values for the elements of the gain matrix is the design problem that is posed here. In the case of a multi-input and multi-output (MIMO) system, since the gains are not uniquely determined, a systematic procedure for the selection of the elements of matrix K is required. The LQR control problem is an optimization problem in which the cost function, J, to be minimized is given as follows:

$$J = \sum_{i=1}^{K_\infty} [\delta x^T(k) Qx_{lxl} \delta x(k) + \delta u^T(k) R_{mxm} \delta u(k)] \tag{21}$$

subject to the constraint that

$$-\delta x(k+1) + \Phi \delta x(k) + \Gamma \delta u(k) = 0 \quad k = 0, ..., K_\infty \tag{22}$$

where K_∞ = number of sampling intervals considered to derive the steady state controller; Qx_{lxl} = state cost weighting matrix; and R_{mxm} = control cost weighting matrix. The matrices Qx and R are symmetric, and to satisfy the non-negative definite condition, they are usually selected to be diagonal with all diagonal elements positive or zero. The first term in Eq. (21) represents the penalty on the deviation of the state variables from the average operating (or target) condition, where the second term represents the cost of control. This term is included in an attempt to limit the magnitude of the control signal δu(k). Unless a cost is imposed for the use of control, the design that emerges is liable to generate control signals that cannot be achieved by the actuator (Reddy, 1996). In this case, the saturation of the control signal will occur, resulting in system behavior that is different from the closed loop system behavior that was predicted assuming that saturation will not occur (Tewari, 2002). Therefore, the control signal weighting matrix elements are selected to be large enough to avoid saturation of the control signal under normal operating conditions. Eqs (21) and (22) constitute a constrained-minimization problem that can be solved using the method of Lagrange multipliers. This produces a set of coupled difference equations that must be solved recursively backwards in time. For the steady-state case, the solution for δu(k) is the same form as Eq. (20), except that K is given by

$$K = [R + \Gamma^T S \Gamma]^{-1} \Gamma^T S \Phi \tag{23}$$

S is a solution of the discrete algebraic Riccati equation (DARE):

$$\Phi^T S \Phi - \Phi^T S \Gamma [R + \Gamma^T S \Gamma]^{-1} \Gamma^T S \Phi + Qx = S \tag{24}$$

where $R = R^T > 0$ and $Qx = Qx^T \geq 0$. The control law defined by Eq. (20) brings an initially disturbed system to an equilibrium condition in the absence of any external disturbances acting on the system. In hydraulic engineering problems, the depth of flow, flow rate, and velocity as a function of distance can be considered the state or internal variables. Sometimes, the volume of water in a given reach of a canal can also be considered a state variable.

Results and Discussion

To demonstrate and compare the feasibility of a linear quadratic regulator (LQR) controller, an optimal regulation problem for a discrete-time single pool irrigation canal with a converging channel junction was simulated (Figure 2). An example problem obtained from Reddy (1990) was used in the study. The data used were as follows: length of canal reach = 5000 m, number of nodes = 5, number of subreaches used = 4, Δx = 1250 m, channel slope = 0.0003, side slope = 1.0, bottom width = 1.7 m, turnout demand = 2.5 $m^3 s^{-1}$, discharge required at the end of the canal = 0.52 $m^3 s^{-1}$, upstream reservoir elevation = 103.2 m, downstream reservoir elevation = 101.14 m, target depth at downstream end = 1.2 m, gate width = 1.7 m, and gate discharge coefficient = 0.75. First, these data were used to calculate the steady-state values, which in turn were used to compute the initial gate openings and the elements of the Φ, Γ, H matrices using a sampling interval of 30 s. The values of the initial gate opening were u_1 = 0.8 m and u_2 = 0.4 m. The analysis was started by evaluating the system stability. All the eigenvalues of the feedback matrix were positive and had values less than one. The system was also found to be both controllable and observable. In the derivation of the control matrix, Γ, elements, it was assumed that both the upstream and downstream gates of each reach could be manipulated to control the system dynamics. The downstream-end gate position was frozen at the original steady-state value, and only the upstream-end gate of the given reach was controlled to maintain the system at the equilibrium condition. The effect of variations in the opening of the downstream gate must be taken into account through real-time feedback of the actual depths immediately upstream and downstream of the downstream gate (node N). In the derivation of the feedback gain matrix, R was set equal to 100,000, whereas Qx was set equal to an identity matrix of dimension 8 (the dimensions of the system). In the absence of a well-defined procedure for selecting the elements of these matrices, these values were selected based upon trial and error.

First a linear quadratic controller problem was simulated for a single reach canal without a channel junction. The system response was simulated for 7500 s. As demonstrated in Figure 4, the deviations in flow depth at nodes 1, 2 and 5 were high, whereas at nodes 4 and 3, the deviations were close to the equilibrium condition. The highest flow deviation occured at node 1. The deviation in flow depth reached almost 16 cm at this particular node. Since the turnout was located at the downstream end of the reach, the maximum decrease in depth of flow occurred at the last node. Due to the increased opening of the upstream gate to compensate for disturbances at the turnouts, the maximum increase in depth of flow occurred at the upstream gate (Figure 5). At the beginning the upstream gate opening was 0.05 m. However, at the end of the simulation, the position of the upstream gate is below the equilibrium value. This is evidence that the system would eventually return to the equilibrium condition. To bring the system to equilibrium, the variation in gate opening reached its highest value of approximately 0.05 m, and gradually decreased to a small negative value of -0.016 m at 7500 s.

After simulating the system for an independent single reach canal, the system was simulated for a single reach canal with a channel junction at node 3. The flow in the side channel converges into the main canal with a 12 degree approaching angle (γ) (Figure 2). The lateral inflow angle (γ) to the downstream canal was shown by Best and Reid (1984) to be dependent upon discharge ratios between lateral inflow and downstream outflow, and the channel junction angle (α). The side channel discharge was 3 $m^3 s^{-1}$. Because of flow mixing and flow approaching angles, there was a change in backwater surface profiles and in water depth in the main canal. As shown in Figure 6, the flow depth at node 3 was increasing at the beginning of the simulation but it was coming close to the equilibrium position at 7500 s. Along the simulation, the variations in flow depth at node 3 have positive values if compared with the first simulation. Again the highest deviations in flow depth were at nodes 1, 2 and 5. The reason for these deviations was the increased opening of the upstream gate and the decrease in depth of flow at the last node. In addition, because of the channel junction at node 3, there were backwater surface effects at all nodes and water surface levels increased along the main canal. The upstream gate opening increased to 0.09 m at 1500 s and later decreased to below the equilibrium condition at the end of the simulation (Figure 7). In other words, the flow

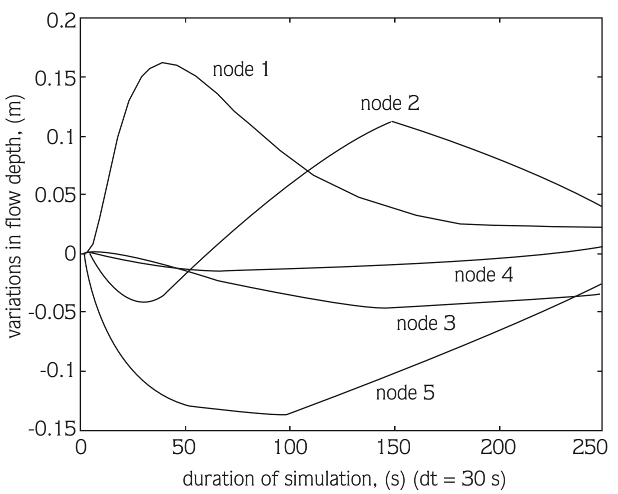

Figure 4. Variations in flow depth for the main canal without channel junction.

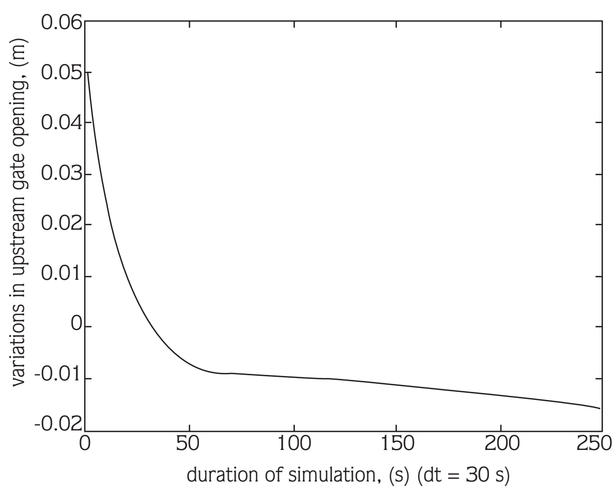

Figure 5. Variations in upstream gate opening for the main canal without channel junction.

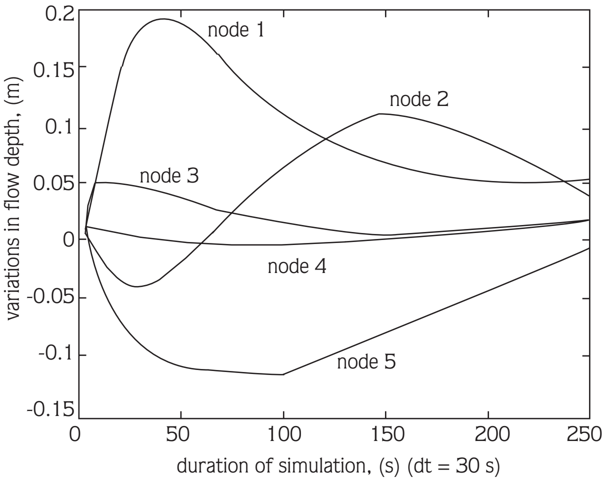

Figure 6. Variations in depth of flow for the main canal with channel junction.

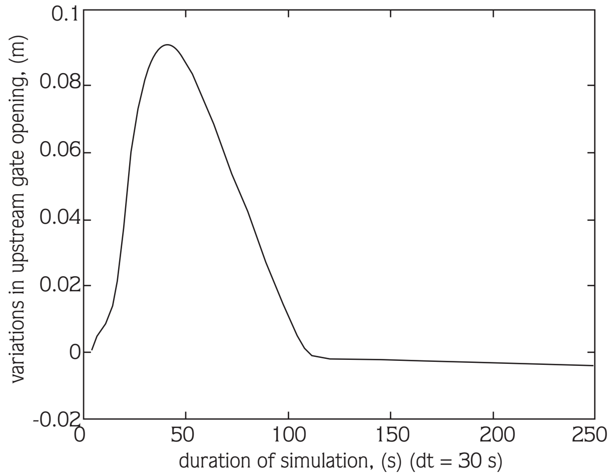

Figure 7. Variations in upstream gate opening for the main canal with channel junction.

from the side channel into the main canal starts to show its response to backwater surface profile at 1500 s. When water surface profile increased in the canal reach, the variation in the upstream gate was decreasing steeply. At 2700 s, variations in gate opening approached a constant value, indicating that a new equilibrium condition was established. At the end of the simulation the deviation in gate opening was decreasing to a small negative value of -0.004 m at 7500 s. In the absence of changes in the withdrawal rates, the gate will return to its equilibrium position at steady state.

Conclusions

The feedback (closed-loop) control theory was applied to derive a global control algorithm for an irrigation canal with a converged channel junction. The control problem was formulated as an optimal control problem directly using the discrete linear quadratic regulator (LQR) theory. The hydraulic formulation of the flow at a channel junction was complicated because of energy losses, lateral flow approaching angles and flow mixing. Using lateral inflow angles to the downstream channel instead of channel junction angles, based on the momentum flux contribution a momentum equation at channel junctions was derived. The performance of the model was evaluated in terms of deviations in the depths of flow and the upstream gate opening. The variation in the upstream gate opening and the deviations in the depth of flow in the canal reach were not erratic during the simulation period. The system either approached a constant value or came close to the equilibrium condition. This indicates that the optimal control theory (feedback loop) is still applicable on irrigation canals with a combining channel junction. Although a single canal reach was considered in this study to demonstrate in detail a procedure to derive a control algorithm in the presence of a converging channel junction, the same procedure can be used to derive a feedback control algorithm to run irrigation canals with diverging channel junctions or channel junctions with multiple reaches.

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