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Abstract

The Author summarizes the evidence that his quantum theory of the electric charge depends in a nontrivial way on the numerical value of the fine structure constant.

1. Introduction

The quantum theory of the electric charge formulated by the Author in Ref.[1] does explain the universality of the electric charge i.e. its quantization in terms of a single universal constant. The natural question arises if this theory says something about the numerical value of this constant. I present below the evidence that this is indeed the case.

2. The inequality of Berestetsky, Lifshitz, and Pitaevsky

Berestetsky, Lifshitz, and Pitaevsky [2] say that the electromagnetic field $F_{\mu\nu}$ is approximately classical if ($\hbar = 1 = c$)

$$\sqrt{F_{01}^2 + F_{02}^2 + F_{03}^2} \;\; (\Delta x^0)^2 \gg 1, \tag{1}$$

where Δx^0 is the observation time over which the field can be averaged without being significantly changed. For a static field this time is obviously infinite and therefore, conclude B.L.P., a static field is always classical. This conclusion, if applicable to the Coulomb field, would make the phenomenon of charge quantization even more mysterious. Fortunately, the B.L.P. inequality is not inconsistent with the phenomenon of charge quantization. The total electric charge is determined from the Gauss law as an integral over a sphere $r =$ const at the spatial infinity, $r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2} \to \infty$. At

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the spatial infinity the total available time is limited by the opening of the light cone, $| \Delta x^0 | \leq 2r$. Therefore the B.L.P. inequality takes on the form

$$\frac{| Q |}{r^2} (2r)^2 \gg 1 \quad i.e. \quad | Q | \gg \frac{1}{4}, \tag{2}$$

where Q is the total charge. In the natural units $\hbar = c = 1$ $e = \frac{1}{\sqrt{137}}$ and therefore

$$| Q | \gg \frac{1}{4}\sqrt{137}e = 2.93 \ e. \tag{3}$$

The total electric charge is approximately classical if it is substantially larger than three elementary charges. This eminently sensible result follows from the observed value of the fine structure constant and from the limitation of available time implicit in the notion of space-like infinity. The limitation is relevant since the total charge "lives" at the spatial infinity.

3. Critical values of the fine structure constant

The theory described in Ref.[1] is a closed mathematical scheme akin to the theory of angular momentum, but with infinite number of degrees of freedom. It predicts a number of critical values for the parameter $\frac{e^2}{\pi \hbar c}$, where $\frac{e^2}{\hbar c}$ is the fine structure constant in the unrationalized Gaussian units. By a critical value I mean a value which separates two qualitatively distinct regimes of the theory. In the following $\hbar = c = 1$.

(a) $\frac{e^2}{\pi} = 1$. This value appears in the theorem proved in [3]. The quantum Coulomb field, when decomposed into unitary irreducible representations of the proper, orthochronous Lorentz group, contains only the main series for $\frac{e^2}{\pi} > 1$; for $0 < \frac{e^2}{\pi} < 1$ it contains the main series and a single representation from the supplementary series corresponding to the special value of the Casimir operator

$$C_1 = -\frac{1}{2} M_{\mu\nu} M^{\mu\nu} = \frac{e^2}{\pi}(2 - \frac{e^2}{\pi}) < 1. \tag{4}$$

This theorem seems to be of fundamental importance since it establishes a functional relation between the fine structure constant and the parameter z, $0 < z < 1$, which selects a single representation from the supplementary series.

(b) $\frac{e^2}{\pi} = \frac{1}{2}$. This value appears in a theorem to be published soon. The probability distribution for the observable $M_{01} + M_{12}$ in the quantum Coulomb field is regular at the origin for $\frac{1}{2} < \frac{e^2}{\pi} < 1$ but singular for $0 < \frac{e^2}{\pi} < \frac{1}{2}$. The operator $M_{01} + M_{12}$ generates parabolic Lorentz transformations which can be geometrically characterized as those which preserve a null plane in space-time.

(c) $\frac{e^2}{\pi} = \frac{1}{4}$. The two previous values are critical in the sense stated above. They separate two regimes in which certain observables associated with the quantum Coulomb field behave in a qualitatively different way. The case of $\frac{e^2}{\pi} = \frac{1}{4}$ is different and not yet fully understood: certain matrix elements which involve the quantum Coulomb field are given by integrals convergent for $\frac{1}{4} < \frac{e^2}{\pi} < 1$ and divergent for $0 < \frac{e^2}{\pi} < \frac{1}{4}$. It is not clear at present if this analytical fact has some observable consequences because the divergent integrals can be evaluated by analytical continuation from the segment $\frac{1}{4} < \frac{e^2}{\pi} < 1$, apparently without violating anything of importance.

(d) Any of the above values divided by n^2, $n = 2, 3, 4, \ldots$. The quantum theory of the electric charge described in [1] allows to construct the quantum Coulomb field with the charge $Q = ne$, $n = \pm 1, \pm 2, \ldots$. This clearly amounts to multiplying the fine structure constant by n^2 or to dividing each critical value by n^2.

References

[1] A. Staruszkiewicz, *Ann. Phys. (N.Y.)* **190** (1989) 354.

[2] W.B. Berestetsky, E.M. Lifshitz, and L.P. Pitaevsky, *Relativistic Quantum Theory*, (Moscow, 1968), in Russian.

[3] A. Staruszkiewicz, *Acta Phys. Pol.* **B23** (1992) 591. This paper is marred by several misprints, which are there because the journal did not take into account my proofs. For a corrected version see ERRATUM, *Acta Phys. Pol.* **B23** (1992) 959.