

1-1-2018

The application of analytical mechanics in a multimachine power system

ZHIJIE WANG

ZHIYUAN LIU

JIE WANG

XIUCHEN JIANG

SANMING LIU

See next page for additional authors

Follow this and additional works at: <https://journals.tubitak.gov.tr/elektrik>



Part of the [Computer Engineering Commons](#), [Computer Sciences Commons](#), and the [Electrical and Computer Engineering Commons](#)

Recommended Citation

WANG, ZHIJIE; LIU, ZHIYUAN; WANG, JIE; JIANG, XIUCHEN; LIU, SANMING; LIU, YIFANG; SHENG, GEHAO; and LIU, TIANYU (2018) "The application of analytical mechanics in a multimachine power system,"

Turkish Journal of Electrical Engineering and Computer Sciences: Vol. 26: No. 3, Article 33.

<https://doi.org/10.3906/elk-1707-239>

Available at: <https://journals.tubitak.gov.tr/elektrik/vol26/iss3/33>

This Article is brought to you for free and open access by TÜBİTAK Academic Journals. It has been accepted for inclusion in Turkish Journal of Electrical Engineering and Computer Sciences by an authorized editor of TÜBİTAK Academic Journals. For more information, please contact academic.publications@tubitak.gov.tr.

The application of analytical mechanics in a multimachine power system

Authors

ZHIJIE WANG, ZHIYUAN LIU, JIE WANG, XIUCHEN JIANG, SANMING LIU, YIFANG LIU, GEHAO SHENG, and
TIANYU LIU

The application of analytical mechanics in a multimachine power system

Zhijie WANG^{1,2}, Zhiyuan LIU^{2,*}, Jie WANG², Xiuchen JIANG², Sanming LIU¹, Yifang LIU^{1,3},
Gehao SHENG², Tianyu LIU¹

¹College of Electrical Engineering, Shanghai Dianji University, Shanghai, P.R. China

²Department of Electrical Engineering, Shanghai Jiaotong University, Shanghai, P.R. China

³School of Energy and Power Engineering, Huazhong University of Science and Technology, Wuhan, P.R. China

Received: 19.07.2017

Accepted/Published Online: 03.04.2018

Final Version: 30.05.2018

Abstract: Analytical mechanics methods play an important role in determining the energy function and Hamilton realization of a system. In this paper, the Hamilton function of the system is obtained directly from the system differential equation by means of analytical mechanics. In addition, the system Hamiltonization process requires that the system must be even order, and the commonly used generator model is a single-axis third-order model. For this reason, the odd-order power system model is extended to an even-order system. The Hamilton function of the system is deduced with coordinate transformation and the generalized force. Then the standard form of Hamilton realization for the multimachine power system is given. Based on the basic principle of nonconservative analytical mechanics and the state feedback control law of the power system, the stability control law of the multimachine power system is designed, which makes the system become asymptotically stable in the neighborhood of the equilibrium point. The control of the IEEE 14 is simulated with MATLAB utilizing the proposed controller. The control effect of the controller under the three-phase short-circuit fault is subsequently studied in the simulation, wherein the effectiveness of the proposed control strategy is verified.

Key words: Hamilton realization, analytical mechanics, energy function, asymptotical stability

1. Introduction

With the increasing scale of power systems and the increasingly complex structure of power grids, the problem of power system stability analysis and control becomes extremely prominent [1–3]. In recent years, the generalized Hamiltonian system theory has been studied in depth and this theory has extended applications in stability analysis and control design [4,5].

For a general nonlinear controlled system, the Hamiltonian system theory was applied to the general nonlinear control system for Hamiltonian realization [6]. The literature [7] proposed the concept of a pseudogeneralized Hamilton system with a wide range of application value. Moreover, the authors designed the excitation control strategy of the power system using energy balance and damping injection methods. On the basis of the transient energy function of a power system, the new Lyapunov function was proposed for the first time by Hamiltonian system theory, and the transient stability of a simple power system with cutting load regulation was analyzed [8]. The realization of Hamilton is the precondition and foundation of controller design, but the general sufficient conditions of traditional Hamiltonian realization are complex and difficult to be satisfied. In the literature [9], aiming at a single-machine infinite-bus system with steam-valve control, constant value

*Correspondence: onionlv@163.com

realization, which is a pseudo-Hamilton realization with structure matrix as constant value, is used to express the pseudo-Hamilton system, and the electromechanical disturbance controller of excitation and steam-value coordinate controlled are designed for the first time. In the literature [10], the Hamilton energy function with the concept of system oscillation energy is constructed, including the description of a multiple equivalent power system of STATCOM in the generalized Hamilton system form. In order to provide suitable inertia and damping ratio to suppress the oscillation of the power system, a virtual synchronous generator control algorithm based on dissipative Hamilton control was designed in the literature [11]. The effectiveness of the control strategy is verified by simulation and experiment. In [12], the stabilization control of time-varying controlled Hamiltonian systems is studied by means of an energy forming method, and the problem of power system analysis with periodic interference is discussed. In [13], the passive control method was used to realize the control of the port-controlled Hamiltonian system, and the system's stabilization control strategy was proposed to improve the system performance.

The application of the energy function method in the stability and control of power systems has gained considerable results. In [14], the influence of unit inertia variation on power system transient stability under various operating modes was studied by deriving and analyzing the 2-machine system based on the transient energy function method. The current research focus is how to establish the transient energy function, and then use a variety of control algorithms for analysis and design [15]. In [16], a dynamic and practical model with good accuracy for transient stability analysis is proposed. Based on this, the equivalent transient energy function is constructed and the stability of the system is checked. Despite the Hamiltonian system theory based on the energy function method being widely used in the power system, the specific construction process of the Hamilton function from the dynamic differential equation of the power system was not given.

Utilizing the theory of analytical mechanics, we deduce the energy function of the mechanical system and provide a method for the extension and further application of the energy function [17]. The Hamiltonian system theory, which is based on analytical mechanics, has been studied in many fields, but it is rarely used in the power system to get the corresponding Hamilton function.

In previous studies, analytical mechanics methods were commonly used in the field of machinery and were less used in the field of electricity. In [18], from the view of energy, the author used the classical method of analytical mechanics to analyze wind turbines. In [19], on the basis of the Lagrange equation in analytical mechanics, the Lagrange equation is used to solve the RLC oscillation problem in the circuit. Both of these papers get the Lagrange equation through the energy of system, and this method is difficult to apply to multimachine systems.

In this paper, we study the Hamiltonian system in analytical mechanics and apply it to the dynamic model of the odd-order power system. The Hamilton function of the power system is deduced and then the corresponding control law is given. This method is not only used in single-machine systems; it also can be applied to multimachine systems. Finally, the validity of the proposed control strategy is verified.

2. Basic theory

2.1. Basic theory

The Hamiltonian implementation of a nonlinear system can be described as follows:

$$\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x} + g(x) u, \quad (1)$$

where $x \in R^n$ and u are the state and the control input, respectively; H represents the Hamiltonian function; $J(x)$ and $g(x)$ are the structure matrix of the system, respectively; $R(x)$ is the additional dissipative structure matrix. Moreover, $J(x)$ and $R(x)$ are the antisymmetric matrix and semisymmetric matrix, respectively.

From system (1), we can get

$$\dot{H} = - \left(\frac{\partial H}{\partial x} \right)^T R(x) \left(\frac{\partial H}{\partial x} \right) + u^T g^T(x) \frac{\partial H}{\partial x} \tag{2}$$

Obviously, when $u = 0, \dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}$. Besides, if $R > 0$, then $\dot{H} = - \left(\frac{\partial H}{\partial x} \right)^T R(x) \left(\frac{\partial H}{\partial x} \right) < 0$.

That is to say, the total energy of the system, $H(x)$, is quasi-Lyapunov function when $u = 0$. However, we usually study the stable point of the system when $u \neq 0$. Thus, we expand the system into

$$\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x} + g(x) u'' + g(x) u''', \tag{3}$$

where

$$u' = u'' + u''' \tag{4}$$

We supposed that \hat{x} satisfies

$$[J(\bar{x}) - R(\bar{x})] \frac{\partial H}{\partial \bar{x}} + g(x) u''' = 0 \tag{5}$$

It is obvious that $\bar{H}(\bar{x}) = 0$ when $\bar{H}(x) = H(x) - H(\bar{x})$. Therefore, there is a control strategy $u = -Kg^T \frac{\partial H}{\partial x}$ that can make the nonlinear dynamic system (3) asymptotically stable in the neighborhood of stable point \bar{x} .

2.2. The method of analytical mechanics

For 2N first-order differential equations, they can be expressed as

$$\dot{x}_\mu = f_\mu(t, x_\nu), \mu, \nu = 1, \dots, 2n \tag{6}$$

Multiply the two ends of Eq. (6) by

$$(\omega_{\mu\nu}) = \begin{bmatrix} 0_{n \times n} & -1_{n \times n} \\ 1_{n \times n} & 0_{n \times n} \end{bmatrix} \tag{7}$$

and get the sum of ν as follows:

$$\omega_{\mu\nu} \dot{x}_\nu = F_\mu(t, x_\nu), \tag{8}$$

where

$$F_\mu = \omega_{\mu\nu} f_\nu \tag{9}$$

If the function F_μ can satisfy

$$\frac{\partial F_\mu}{\partial x_\nu} = \frac{\partial F_\nu}{\partial x_\mu} \tag{10}$$

Eq. (6) is self-adjoint [17]. Thus, Eq. (6) can be written as a Hamilton function:

$$\omega_{\mu\nu} \dot{x}_\nu = \frac{\partial H}{\partial x_\mu}, \tag{11}$$

where H is the Hamilton function [20]

$$H = x_\mu \int_0^1 F_\mu(t, \tau x_\nu) d\tau \quad (12)$$

Therefore, these first-order differential equations include $2N$ variable, \mathbf{p}_i , and \mathbf{q}_i . Moreover, they are a Hamilton canonical equation:

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}, \quad \dot{q}_i = \frac{\partial H}{\partial p_i} \quad (13)$$

Hence, the system is energy-conservation and

$$\dot{H} = \left(\frac{\partial H}{\partial q_i}\right) \dot{q}_i + \left(\frac{\partial H}{\partial p_i}\right) \dot{p}_i = 0. \quad (14)$$

If Eqs, (6) are not self-adjoin, we can introduce the generalized force Q ,

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i} + Q_i \quad (15)$$

3. Hamilton realization and controller design of a power system

3.1. Multimachine power system model

Consider the following n -generator power system

$$\dot{\delta}_i = \omega_i - \omega_0, \quad (16)$$

$$\dot{\omega}_i = \frac{\omega_0}{M_i} \left(P_{mi} - D \left(\frac{\omega_i}{\omega_0} - 1 \right) - \sum_{j=1}^n E'_{qi} E'_{qj} B_{ij} \sin(\delta_i - \delta_j) - E'^2_{qi} G_{ii} \right) \quad (17)$$

$$\dot{E}'_{qi} = -\frac{1}{T'_{d0i}} \left(E'_{qi} - (x_{di} - x'_{di}) \sum_{j=1}^n E'_{qj} B_{ij} \cos(\delta_i - \delta_j) - u_{fi} - u_{fi0} \right), \quad (18)$$

where the subscription i stands for the i -th generator, δ_i is the generator power angle, ω_i is the angular velocity of the generator, and ω_0 is the synchronous speed. x_{di} is the d-axis synchronous reactance of the generator, x'_{di} is the d-axis transient reactance of the generator, and $x'_{di} < x_{di}$. T'_{d0i} is the d-axis transient open-time constant. G_{ii} and B_{ii} are self-conductance and mutual susceptance, respectively. E'_{qi} is the q-axis transient potential of the generator. P_{mi} is the mechanical input power and it is a constant. $u_{fi} + u_{fi0}$ is the generator excitation input and u_{fi0} is a constant [20].

3.2. The process of Hamiltonian realization

For the system (16), (17), (18), we supposed that $\delta_i = M_i^\alpha \delta'_i$, where α is undetermined coefficient. Then we can get

$$\dot{\delta}'_i = \frac{1}{M_i^\alpha} (\omega_i - \omega_0) \quad (19)$$

$$\dot{E}'_{qi} = \bar{E}'_{qi} \tag{20}$$

$$\frac{1}{M_i^\alpha} \dot{\omega}_i = \frac{\omega_0}{M_i^{\alpha+1}} P_{mi} - \frac{D}{M_i^\alpha} \dot{\delta}'_i - \frac{\omega_0}{M_i^{\alpha+1}} \left(\sum_{j=1}^n E'_{qi} E'_{qj} B_{ij} \sin(M_i^\alpha \delta'_i - M_j^\alpha \delta'_j) + E'^2_{qi} G_{ii} \right) \tag{21}$$

$$\begin{aligned} \dot{E}'_{qi} = & -\frac{1}{T'_{d0i}} \left((x_{di} - x'_{di}) \sum_{j=1}^n \left(-\dot{E}'_{qj} B_{ij} \cos(M_i^\alpha \delta'_i - M_j^\alpha \delta'_j) + E'_{qj} B_{ij} (M_i^\alpha \dot{\delta}'_i - M_j^\alpha \dot{\delta}'_j) \sin(M_i^\alpha \delta'_i - M_j^\alpha \delta'_j) \right) \right. \\ & \left. + \dot{E}'_{qi} - \dot{u}_{fi} \right) \end{aligned} \tag{22}$$

We multiply the two ends of Eqs. (19), (20), (21), (22) by

$$(\omega_{\mu\nu}) = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{23}$$

Then we can get

$$-\frac{1}{M_i^\alpha} \dot{\omega}_i = -\frac{\omega_0}{M_i^{\alpha+1}} P_{mi} + \frac{D}{M_i^\alpha} \dot{\delta}'_i + \frac{\omega_0}{M_i^{\alpha+1}} \left(\sum_{j=1}^n E'_{qi} E'_{qj} B_{ij} \sin(M_i^\alpha \delta'_i - M_j^\alpha \delta'_j) + E'^2_{qi} G_{ii} \right) \tag{24}$$

$$\begin{aligned} -\dot{E}'_{qi} = & \frac{1}{T'_{d0i}} \left((x_{di} - x'_{di}) \sum_{j=1}^n \left(-\dot{E}'_{qj} B_{ij} \cos(M_i^\alpha \delta'_i - M_j^\alpha \delta'_j) + E'_{qj} B_{ij} (M_i^\alpha \dot{\delta}'_i - M_j^\alpha \dot{\delta}'_j) \sin(M_i^\alpha \delta'_i - M_j^\alpha \delta'_j) \right) \right. \\ & \left. + \dot{E}'_{qi} - \dot{u}_{fi} \right) \end{aligned} \tag{25}$$

$$\dot{\delta}'_i = \frac{1}{M_i^\alpha} (\omega_i - \omega_0) \tag{26}$$

$$\dot{E}'_{qi} = \bar{E}'_{qi} \tag{27}$$

For simplicity of calculation, we introduce the generalized force Q_i and Q'_i :

$$Q_i = -\frac{D_i}{M_i} \dot{\delta}'_i \tag{28}$$

$$\begin{aligned} Q'_i = & -\frac{1}{T'_{d0i}} \left(\dot{E}'_{qi} - \dot{u}_{fi} \right) + \frac{\omega_0}{\sqrt{M_i}} 2G_{ii} E'_{qi} \delta'_i - \sum_{j=1}^n \omega_0 E'_{qj} B_{ji} \cos(M^{-1/2} \delta'_j - M^{-1/2} \delta'_i) \\ & + \frac{1}{T'_{d0i}} (x_{di} - \bar{x}_{di}) \cdot \sum_{j=1}^n (B_{ji} \left(\dot{E}'_{qj} \cos(M^{-1/2} \delta'_j - M^{-1/2} \delta'_i) \right. \\ & \left. - E'_{qj} (M^{-1/2} \dot{\delta}'_j - M^{-1/2} \dot{\delta}'_i) \sin(M^{-1/2} \delta'_j - M^{-1/2} \delta'_i) \right)) \end{aligned} \tag{29}$$

To meet the self-adjoint conditions (10), we can get $\alpha = -1/2$.

Hence, we can get the Hamilton function of system (16), (17), (18) through Eq. (12):

$$\begin{aligned}
 H = & \sum_{i=1}^n \left(\frac{1}{2} \left(\dot{\delta}'^2 + \dot{E}'_{qi}{}^2 \right) - \frac{\omega_0}{\sqrt{M_i}} P_{mi} \delta' \right) - \sum_{1 \leq i < j \leq n} \omega_0 E'_{qi} E'_{qj} B_{ij} \cos \left(M_i^{-1/2} \delta'_i - M_j^{-1/2} \delta'_j \right) \\
 & + \sum_{i=1}^n \left(\frac{\omega_0}{\sqrt{M_i}} G_{ii} E'_{qi}{}^2 \delta'_i - \frac{1}{2} \omega_0 B_{ii} E'_{qi}{}^2 \right)
 \end{aligned} \tag{30}$$

Among them, we can make the generalized variable:

$q_i = \delta'_i = \sqrt{M_i} \delta_i, \bar{q}_i = E'_{qi}, p_i = \dot{\delta}'_i = \sqrt{M_i} (\omega_i - \omega_0), \bar{p}_i = \dot{E}'_{qi}$. Then we get the Hamilton function as follows:

$$\begin{aligned}
 H = & \sum_{i=1}^n \left(\frac{1}{2} (p_i^2 + \bar{p}_i^2) - \frac{\omega_0}{\sqrt{M_i}} P_{mi} q_i \right) - \sum_{1 \leq i < j \leq n} \omega_0 \bar{q}_i \bar{q}_j B_{ij} \cos \left(M_i^{-1/2} q_i - M_j^{-1/2} q_j \right) \\
 & + \sum_{i=1}^n \left(\frac{\omega_0}{\sqrt{M_i}} G_{ii} \bar{q}_i^2 q_i - \frac{1}{2} \omega_0 B_{ii} \bar{q}_i^2 \right)
 \end{aligned} \tag{31}$$

and Eq. (31) can satisfy Eq. (13).

In order to facilitate the calculation, we introduced ξ_i :

$$\begin{aligned}
 \xi_i = & \frac{\omega_0}{\sqrt{M_i}} 2G_{ii} E'_{qi} \delta'_i - \sum_{j=1}^n \left(\omega_0 E'_{qj} B_{ji} \cos \left(M_i^{-1/2} \delta'_i - M_j^{-1/2} \delta'_j \right) \right) \\
 & + \frac{1}{T'_{d0i}} (x_{di} - x'_{di}) \sum_{j=1}^n \left(\dot{E}'_{qj} B_{ij} \cos \left(M_i^{-1/2} \delta'_i - M_j^{-1/2} \delta'_j \right) \right) \\
 & + E'_{qj} B_{ij} \left(M_i^{-1/2} \dot{\delta}'_i - M_j^{-1/2} \dot{\delta}'_j \right) \sin \left(M_i^{-1/2} \delta'_i - M_j^{-1/2} \delta'_j \right)
 \end{aligned} \tag{32}$$

In order to realize the Hamiltonization of the system, we construct the new control variable

$$\dot{u}''_{fi} = T'_{d0i} \xi_i + \dot{u}_{fi}. \tag{33}$$

Thus, the standard Hamiltonian realization equation is

$$\begin{pmatrix} \dot{q}_i \\ \dot{\bar{q}}_i \\ \dot{p}_i \\ \dot{\bar{p}}_i \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -\frac{D}{M} & 0 \\ 0 & -1 & 0 & -\frac{1}{T'_{d0i}} \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial q_i} \\ \frac{\partial H}{\partial \bar{q}_i} \\ \frac{\partial H}{\partial p_i} \\ \frac{\partial H}{\partial \bar{p}_i} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1/T'_{d0i} \end{pmatrix} \dot{u}''_{dc} \tag{34}$$

In order to make the expression more concise, we introduce the vector: $x_i^T = (\delta_i \ \omega_i \ E'_{qi})$, $\zeta_i^T = (\dot{q}_i \ \dot{\bar{q}}_i \ \dot{p}_i \ \dot{\bar{p}}_i)$, and $M(\zeta) = J(\zeta) - R(\zeta)$. The system can be written as

$$\dot{\zeta} = M(\zeta) \frac{\partial H}{\partial \zeta} + g(\zeta) u \tag{35}$$

Among them, $g = \left(0 \ 0 \ 0 \ 1/T'_{d0i} \right)^T$, $J(\zeta) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$, $R(\zeta) = \text{diag} \left(0 \ 0 \ D/M \ 1/T'_{d0i} \right)$.

For the system (34), $q_i = \delta'_i = \sqrt{M_i}\delta_i$, $\bar{q}_i = E'_{qi}$, $p_i = \dot{\delta}'_i = \sqrt{M_i}(\omega_i - \omega_0)$, $\bar{p}_i = \dot{E}'_{qi}$. The resulting Hamiltonian realization can show the function (16), (17), (18). It can be noted that seeking the second derivative will cause the original system to have a larger solution. In addition, the stability point of the system is not unique after the transformation, but contains the equilibrium point of the original system.

$$\begin{aligned} \frac{\partial H}{\partial q_i} &= -\frac{\omega_0}{\sqrt{M_i}}P_{mi} + \frac{\omega_0}{\sqrt{M_i}}G_{ii}E'^2_{qi} - \frac{\omega_0}{\sqrt{M_i}} \sum_{i>j} \left(E'_{qi}E'_{qj}B_{ji} \sin \left(M_j^{-1/2}\delta'_j - M_i^{-1/2}\delta'_i \right) \right. \\ &\quad \left. - E'_{qi}E'_{qj}B_{ji} \sin \left(M_i^{-1/2}\delta'_i - M_j^{-1/2}\delta'_j \right) \right) = -p_i = -\dot{\delta}'_i \end{aligned} \tag{36}$$

$$\frac{\partial H}{\partial \bar{q}_i} = -\omega_0 B_{ii}E'_{qi} + \frac{2\omega_0}{\sqrt{M_i}}\delta' E'^2_{qi} G_{ii} - \omega_0 \sum_{i \neq j} E'_{qj} B_{ij} \cos(\delta_i - \delta_j) = -\bar{p}_i = \ddot{E}'_{qi} \tag{37}$$

$$\frac{\partial H}{\partial p_i} = p_i = \dot{q}_i = \dot{\delta}' \tag{38}$$

$$\frac{\partial H}{\partial \bar{p}_i} = \bar{p}_i = \dot{\bar{q}}_i = \dot{E}'_{qi} \tag{39}$$

3.3. The design of the controller

Hamilton's system (34) can be analyzed in consideration of the design of the controller when the Hamiltonian realization process needs to satisfy the control condition (i.e. Eq. (33)).

The controller u''_{fi} satisfies $\dot{u}'_{fi} = -Kg^T \frac{\partial H}{\partial \zeta_i}$, that is to say,

$$u''_{fi} = \int_{t_0}^t \left(-Kg^T \frac{\partial H}{\partial \zeta_i} \right) dt + u''_{fi0} \tag{40}$$

We take $K = K^T > 0$, $\dot{\zeta} = [M(\zeta) - gKg^T] \cdot \frac{\partial H}{\partial \zeta_i}$. Then it can be obtained that $\dot{H} < 0$, and H is not always constant in the neighborhood of the equilibrium point. Thus the dynamic system is asymptotically stable in the neighborhood of the stability point. Hence, the controller design idea is dividing the controller of system (16), (17), (18) into two parts, $u_{fi} = u''_{fi} + u'_{fi}$. u''_{fi} is part of the controller designed to meet the stability of the system at the stability point and u'_{fi} is part of the controller plan to satisfy the Lagrange function. Therefore,

we can obtain the part of the original system controller as follows:

$$\begin{aligned}
 u'_{fi} = & -Kg^T \frac{\partial H}{\partial x} - \frac{2\omega_0}{\sqrt{M_i}} T'_{d0i} G_{ii} E'_{qi} \delta'_i + T'_{d0i} \sum_{j=1}^n \omega_0 E'_{qj} B_{ji} \cos \left(M^{-1/2} \delta'_j - M^{-1/2} \delta'_i \right) \\
 & - (x_{di} - \bar{x}_{di}) \cdot \sum_{j=1}^n (B_{ji} \left(\dot{E}'_{qj} \cos \left(M^{-1/2} \delta'_j - M^{-1/2} \delta'_i \right) \right. \\
 & \left. - E'_{qj} \left(M^{-1/2} \delta'_j - M^{-1/2} \delta'_i \right) \sin \left(M^{-1/2} \delta'_j - M^{-1/2} \delta'_i \right) \right))
 \end{aligned} \tag{41}$$

Then we can get the controller u_{fi} of the original system. Under the above control law, the system is asymptotically stable in the neighborhood of the stability point. The controller (41) is a full state feedback controller. In the specific project, the unmeasurable input signal of the controller can be switched to other measurable signals by using the equality relation in the power system, such as:

$$\delta = \int_{t_0}^t (\omega - \omega_0) dt + \delta_0 \tag{42}$$

$$E'_{qi} = U_{ti} + \frac{x_{di} Q_{ei}}{U_{ti}} - (x_{di} - x'_{di}) \frac{Q_{ei} U_{ti}}{(U_{ti}^2 + x_{di} Q_{ei})} \tag{43}$$

Among them, U_{ti} is the generator terminal voltage and Q_{ei} is the reactive power of the generator. In practice, these parameters are relatively easy to measure. Through the function (16), (17), (18), it is possible to obtain a control law that can make the system asymptotically stable and achievable in the neighborhood of the stability point.

4. Simulation

The control of the IEEE 14 is simulated and the system simulation model shown in Figure 1. $\omega_0 = 2\pi f = 314\text{rad/s}$. The parameters of five generators are shown in the Table.

Table. The parameters of five generators.

Generators	$\delta/^\circ$	E'_q/pu	M/s	x_d/pu	x'_d/pu	T'_{d0}/s
1	0.2219	1.0311	40	0.16	0.045	6.00
2	0.2493	1.0019	39.2	0.306	0.048	8.2
3	2.2716	1.0566	47.28	0.146	0.0608	8.96
4	19.7316	1.0502	12.80	0.8958	0.1198	6.00
5	13.1664	1.0170	6.02	1.3125	0.1813	5.89

Interference is a 3-phase ground short circuit at lines 6–13 near bus 13 at 1 s. The simulation time is 30 s and the numerical integration step is determined to be 0.05 s. The simulation results are shown in Figures 2–5. In the figures, "Uncontrolled" indicates that the system controls the response curve as a constant; "Controlled" indicates the response curve after adding the control variable u_{fi} at 1.5 s, which is satisfied by Eq. (30).

It can be seen that the δ , ω , and E_q of the generator fluctuate greatly in the fault state when the system is uncontrolled, and the stable state can be restored after a certain time. After adding the control, the fluctuation of the δ , ω , and E_q of the generator in the fault state is greatly reduced, and the time of stabilizing

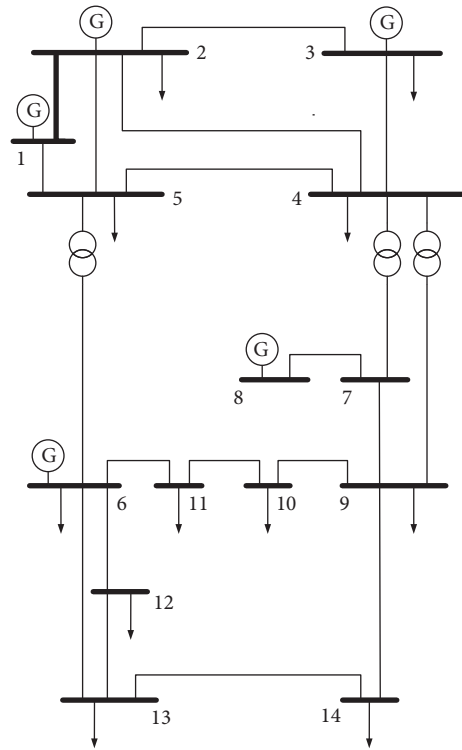


Figure 1. Diagram of the IEEE14 bus system.

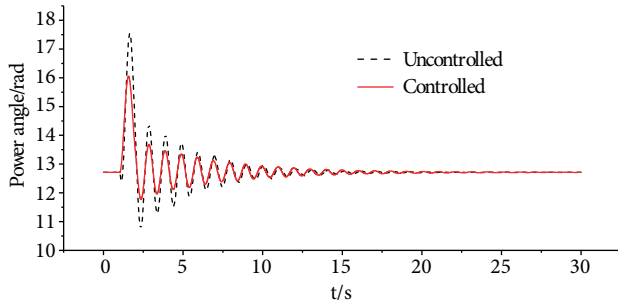


Figure 2. The δ of generator 1.

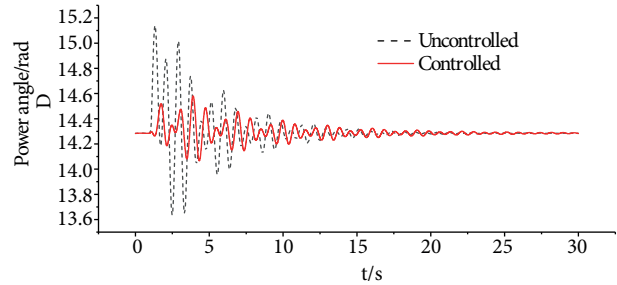


Figure 3. The δ of generator 6.

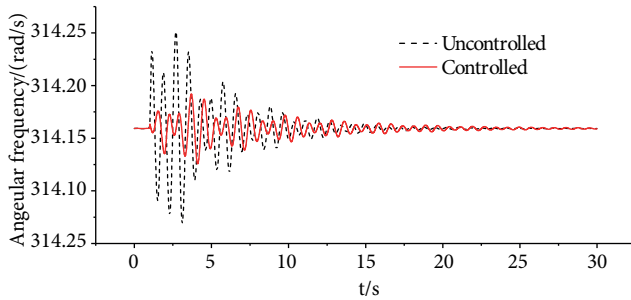


Figure 4. The ω of generator 6.

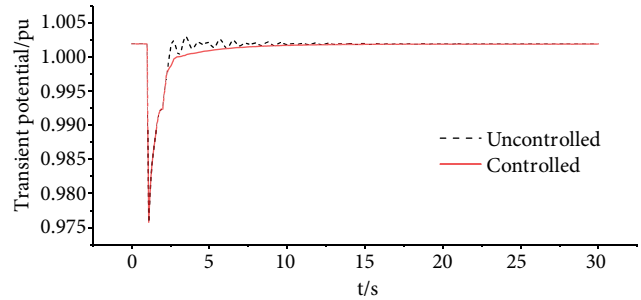


Figure 5. The E_q of generator 6.

the state is also reduced. Through the comparison of the response curve, it can be seen that using the control idea presented in this article can effectively enhance the speed of transient response and the stability of the system more significantly.

5. Conclusion

The Hamiltonian system is an important research object of nonlinear scientific research. The theory and method of analytical dynamics have been widely used in various fields, especially the Hamiltonian process, which can reflect the physical meaning of the Hamilton function.

This paper has done the following work:

1. The Hamilton function of the power system is obtained by the system extension and the coordinate transformation and the generalized force method. On this basis, a controller is designed that can stabilize asymptotically in the neighborhood of the equilibrium point.
2. The construction of a Hamiltonian function by analytic mechanics and the design process of the system stability controller are extended to the application of a multimachine system. The IEEE 14 is simulated and the control effect of the system under the three-phase short-circuit fault is compared.
3. The combination of nonlinear system and analytical mechanics plays an important role in the Hamiltonian realization and control of power systems. Future work is to apply this method of Hamiltonian realization and controller design to wind power generation system analysis and control and optimize the controller design.

Acknowledgments

The authors would like to thank the anonymous reviewers for helpful suggestions to improve this manuscript. This work was supported by the National Natural Science Foundation of China (No.51477099), Natural Science Foundation of Shanghai (No.15ZR1417300, No.14ZR1417200), and Innovation Program of Shanghai Municipal Education Commission (No.14YZ157, No.15ZZ106).

References

- [1] Yao L, Wu J, Wang Z, Lu Z. Pattern analysis of future HVDC grid development. Proceedings of the CSEE 2014; 34, 6007-6020 (article in Chinese with an abstract in English).
- [2] Lu S. Sliding mode adaptive excitation system control based on back-stepping approach. Power Sys CLE 2010; 26: 77-79+87 (article in Chinese with an abstract in English).
- [3] Wang H, Tang Y, Hou J, Liu N, Li B, Zhang H. Equivalent method of integrated power generation system of wind, photovoltaic and energy storage in power flow calculation and transient simulation. Proceedings of the CSEE 2012; 32: 1-8 (article in Chinese with an abstract in English).
- [4] Ortega R, Galaz M, Astolfi A, Sun Y, Shen T. Transient stabilization of multimachine power systems with nontrivial transfer conductances. IEEE T Autom Control 2005; 50: 60-75.
- [5] Wang K, Lan Z, Gan D. A survey on adaptive control of nonlinear systems and its application in power systems. Power Sys Techno 2007; 31: 11-16 (article in Chinese with an abstract in English).
- [6] Liu S, Zhang M, Zhong Y, Zhu C. Nonlinear control for power system with SVC based on Hamiltonian energy function. Proceedings of the CSU-EPSCA 2006; 18: 24-28 (article in Chinese with an abstract in English).
- [7] Shi F, Wang J. Stabilising control of multi-machine power systems with transmission losses based on pseudo-generalised Hamiltonian theory. IET Control Theory A 2011; 6: 173-181.
- [8] Sun Y, Peng J. A direct method of transient stability analysis for controlled power systems based on Hamiltonian theory. Power Sys Techno 2002; 26: 1-6 (article in Chinese with an abstract in English).

- [9] Ding L, Wang DL, Ma NN, Liu Y. The research of electromechanical disturbance suppression based on constant value of Hamilton energy function. *Electron Eng* 2017; 4: 33-37 (article in Chinese).
- [10] Zhang LN, Liu Q. STATCOM control strategy study on restraining inter-area low frequency oscillation based on Hamilton theory. *Power Capacitor & React Power Compen* 2017; 4: 19-23 (article in Chinese with an abstract in English).
- [11] Ma YF, Yu RN, Liu HQ, Zhao SQ. The analysis of VSG control algorithm based on Hamiltonian system. *Power Sys Techno* 2017; 8: 2543-2552 (article in Chinese with an abstract in English).
- [12] Gu Y, Cheng D. Stabilization of time-varying Hamiltonian systems. *IEEE T Contr Syst T* 2006; 14, 871-880.
- [13] Ortega R, Schaft A, Castanos F, Astolfi A. Control by interconnection and standard passivity-based control of port-Hamilton systems. *IEEE T Automat Contr* 2008; 53: 2527-2542.
- [14] Zhao Z, Chen W, Dai C, Zhang X. Influence of system inertia time constants on transient stability level of interconnected AC power grid. *Power Sys Techno* 2012; 36: 102-107 (article in Chinese with an abstract in English).
- [15] Liu Y, Chen T, Li C, Wang Y, Chu B. Energy-based L2 disturbance attenuation excitation control of differential algebraic power systems. *IEEE T Circuits Syst* 2008; 55: 1081-1085.
- [16] Wang X, Liu D, Wu J, Wang R. Energy function-based power system transient stability analysis. *Power Sys Techno* 2011; 35: 114-118 (article in Chinese with an abstract in English).
- [17] Santilli RM. *Foundations of Theoretical Mechanics: the Inverse Problem in Newtonian Mechanics*. New York, NY, USA: Springer-Verlag Press, 1978.
- [18] Liu X, Wang X. Dynamics modeling and analysis of wind turbine. *Automat Electron Power Sys* 2015; 39: 15-19 (article in Chinese with an abstract in English).
- [19] Jin X, Lu W. Application of Lagrange equation to motion analysis of electrical system and electromechanical system. *J Shanghai Univ Electr Power* 2003; 19: 1-5 (article in Chinese with an abstract in English).
- [20] Shi F, Wang J. Pseudo-generalized Hamiltonian theory and its application to multi-machine power system nonlinear excitation control. *Proceedings of the CSEE* 2011; 31: 67-74 (article in Chinese with an abstract in English).