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## Solving a combined economic emission dispatch problem using adaptive wind driven optimization

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**Abstract:** In this paper, the adaptive wind driven optimization (AWDO) algorithm is applied for solving the combined economic emission dispatch (CEED) problem. AWDO is one of the newest hybrid algorithms, which optimizes the selection of coefficients at each iteration, eliminating the need for tuning the coefficients. The evaluation of AWDO performances is carried out on the standard IEEE 30-bus test system with 6 generating units and with various cost curve natures. The results of AWDO use with the test system are compared against the results of use of 3 algorithms: the moth swarm algorithm, firefly algorithm, and hybrid particle swarm optimization and gravitational search algorithm, which were proposed in recent literature for solving this problem. The present paper shows that AWDO gives an accurate and effective solution of the CEED problem and outperforms the other tested algorithms.

**Key words:** Computational intelligence, heuristic algorithms, power generation dispatch, power system analysis computing, power engineering computing

### 1. Introduction

One of the key tasks in the planning and operation of a power system is to minimize fuel cost and emission of pollutants in thermal power plants. This problem is solved as an optimization problem in which the minimization of functions of fuel cost and emission of pollutants is carried out and it is called the combined economic emission dispatch (CEED) problem. Minimization of these functions is performed by adjusting the output powers of generators in the thermal power plants to meet the system load, subject to transmission and operational constraints. Due to the complexity of the objective functions that take the form of a sum of quadratic, sinusoidal, and exponential functions, a large number of stochastic nature-inspired metaheuristic algorithms (MAs) to solve the CEED problem were presented in the literature. In [1] an overview of over 30 MAs proposed in various published papers for solving the CEED problem was given.

A number of improved MAs that enhance performance in terms of convergence speed, global optimality, solution accuracy, and algorithm reliability for solving the CEED problem have been proposed [2–4].

In this paper we apply for the first time the adaptive wind driven optimization (AWDO) algorithm [5] for solving the CEED problem. This MA optimizes its coefficients in each iteration. We carry out the validation of the AWDO algorithm for solving the CEED problem on a standard IEEE test system with 6 generators and by comparing the obtained results with the results of the use of other algorithms.

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## 2. CEED model

The fuel cost function,  $F_g(P_g)$ , of generation unit  $g$  in a thermal power plant can be:

- (i) a quadratic smooth function, when the valve point loading effect (VPLE) in the thermal power plant is not taken into account:
- $$F_g(P_g) = a_g + b_g P_g + c_g P_g^2, g = 1, 2, \dots, G; \quad (1)$$

- (ii) a more complex, nonsmooth, and nonconvex function when considering VPLE [6]:

$$F_g(P_g) = a_g + b_g P_g + c_g P_g^2 + |d_g \sin(e_g (P_g^{\min} - P_g))|, \quad (2)$$

where  $F_g$  is expressed in \$/h;  $P_g$  is the real power of generation unit  $g$  in MW;  $G$  is the total number of generation units;  $P_g^{\min}$  is a lower loading limit of the generation unit  $g$ ; coefficients  $a_g$ ,  $b_g$ , and  $c_g$  are the cost coefficients; and  $d_g$  and  $e_g$  are the coefficients for VPLE.

The emission function,  $E_g(P_g)$ , of a generation unit  $g$  is defined as a sum of quadratic and exponential functions [7,8]:

$$E_g(P_g) = \alpha_g + \beta_g P_g + \eta_g P_g^2 + \xi_g \exp(\lambda_g P_g), \quad (3)$$

where  $E_g$  is expressed in t/h and  $\alpha_g$ ,  $\beta_g$ ,  $\eta_g$ ,  $\xi_g$ , and  $\lambda_g$  are emission coefficients of the generation unit  $g$ .

To solve the CEED problem, Eq. (1) or (2) is combined with Eq. (3) using the weighted sum method [9], i.e.

$$FE = w \sum_{g \in G} F_g(P_g) + (1 - w) \gamma \sum_{g \in G} E_g(P_g), \quad (4)$$

and then the combined function of Eq. (4) is minimized under system constraints. In Eq. (4),  $\gamma$  is the scaling factor and  $w$  is the weight factor, the value of which is within the limits  $0 \leq w \leq 1$ . The limit  $w = 1$  corresponds to the minimization of fuel cost only, while the limit  $w = 0$  corresponds to the minimization of pollutant emission only. By using the scaling factor  $\gamma$ , the biobjective CEED problem is solved as a single-objective problem.

In this minimization process, 2 constraints are satisfied:

- (i) the power equality constraint in the transmission system:

$$\sum_{g \in G} P_g - P_D - P_{loss} = 0, \quad (5)$$

where  $P_{loss}$  and  $P_D$  are the power loss and total load demand, respectively;

- (ii) the generator unit capacity constraint:

$$P_g^{\min} \leq P_g \leq P_g^{\max}, \quad (6)$$

where  $P_g^{\max}$  and  $P_g^{\min}$  are maximum and minimum power values of the generator unit  $g$ .

The power loss of the transmission system is expressed using  $B$ -loss matrices, as follows [9]:

$$P_{loss} = \sum_{g \in G} \sum_{j \in G} P_g B_{gj} P_j + \sum_{g \in G} B_{0g} P_g + B_{00}, \quad (7)$$

where  $B_{00}$ ,  $B_{0g}$ , and  $B_{gj}$  are the coefficients of the  $B$ -loss matrices.

In order to satisfy the constraint of Eq. (5), one of the generators (e.g., generator  $G$ ) is selected to be a dependent generator (the slack generator). Then, from Eq. (5), the value of  $P_G$  is obtained as follows:

$$P_G = P_D + P_{loss} - \sum_{g=1}^{G-1} P_g. \quad (8)$$

By using Eqs. (7) and (8), the values of  $P_G$  and  $P_{loss}$  are obtained as follows: (i) set the initial value of  $P_{loss}$ :  $P_{loss} = P_{loss}^{(0)} = 0$  in Eq. (8); (ii) calculate the initial value  $P_G^{(0)}$  from Eq. (8) for the initial value  $P_{loss}^{(0)} = 0$ ; (iii) calculate the new value  $P_{loss}^{(1)}$  from Eq. (7); (iv) check whether the error value  $\varepsilon$  is below the specified error tolerance value  $\delta$ , i.e.

$$\varepsilon = \left| P_{loss}^{(1)} - P_{loss}^{(0)} \right|, \quad \varepsilon \leq \delta; \quad (9)$$

and (v) obtain the value  $P_G^{(1)}$  from (8) for  $P_{loss} = P_{loss}^{(1)}$ . The power equality constraint of Eq. (5) is met if the condition of Eq. (9) is satisfied. Otherwise, the procedure is repeated. After checking whether the calculated  $P_G$  value satisfies the constraint of Eq. (6), the variable  $P_G^{lim}$  is defined as follows:

$$P_G^{lim} = \begin{cases} P_G^{max} & \text{if } P_G > P_G^{max} \\ P_G^{min} & \text{if } P_G < P_G^{min} \\ P_G & \text{if } P_G^{min} \leq P_G \leq P_G^{max} \end{cases}, \quad (10)$$

where variable  $P_G$  is a dependent variable. Thereafter, the new expanded objective function to be minimized is formed:

$$FE_p = FE + \lambda_p (P_G - P_G^{lim})^2, \quad (11)$$

where the quadratic penalty term with the penalty factor  $\lambda_p$  is added to the objective function  $FE$  of Eq. (4).

### 3. AWDO

Wind driven optimization (WDO) is a nature-inspired population-based algorithm whose use is widespread because of its good exploration and diversity properties [10]. For searching the space of possible solutions, WDO uses the law under which the wind blows, i.e. under which each infinitesimal air parcel (as a member of a population) moves toward an optimum air pressure location to balance its horizontal pressure. The velocity and position of each air parcel are updated on each iteration. This iterative process continues until the air parcels achieve the optimum pressure location to provide the optimum solution. Newton's second law of motion is used to express the motion of the air parcel. There are 4 major forces that can either cause the wind to move in a certain direction or deflect it from its existing path. These forces are the pressure gradient force,  $\vec{F}_{PG} = -\nabla P \delta V$ ; the friction force,  $\vec{F}_F = -\rho \alpha \vec{u}$ ; the gravitational force,  $\vec{F}_G = \rho \delta V \vec{g}$ ; and the Coriolis force,  $\vec{F}_C = -2\Omega \times \vec{u}$ , where  $\nabla P$  is pressure gradient;  $\delta V$  is the finite volume of the air parcel;  $\rho$  is air density for the air parcel;  $\alpha$  is the friction coefficient;  $\vec{u}$  is the velocity vector of the wind;  $g$  is gravitational acceleration; and  $\Omega$  is the rotation of the earth. The sum of these forces is inserted into Newton's second law of motion, which is expressed as follows:

$$\rho \vec{a} = \sum \vec{F}_i, \quad (12)$$

where  $\vec{a}$  is acceleration, and then the velocity and position displacement of each air parcel are computed. The procedure for this computation begins as follows: (i) make a substitution,  $\vec{a} = \Delta \vec{u} / \Delta t$  and, for simplicity, set

$\Delta t = 1$  and  $\delta V = 1$ ; (ii) write  $\rho$  in terms of the pressure  $P$  from the ideal gas law:  $P = \rho RT$ , where  $R$  is the universal gas constant and  $T$  is the temperature; (iii) insert the forces in Eq. (12) and, after editing Eq. (12), write the following equation:

$$\Delta \vec{u} = \vec{g} + \left( -\nabla P \frac{RT}{P_{cur}} \right) - \alpha \vec{u} + \left( \frac{-2\Omega \times \vec{u}RT}{P_{cur}} \right), \quad (13)$$

where  $P_{cur}$  is the pressure of the current location. In WDO the velocity difference  $\Delta \vec{u}$  can be written as  $\Delta \vec{u} = \vec{u}_{new} - \vec{u}_{cur}$ , where  $\vec{u}_{cur}$  and  $\vec{u}_{new}$  are velocities at the current iteration and the next iteration, respectively. The values of  $\vec{g}$  and  $\nabla P$  are written in terms of position  $\vec{x}$  of the parcel:  $\vec{g} = |g| (\vec{0} - \vec{x}_{cur})$  and  $\nabla P = |P_{opt} - P_{cur}| (\vec{x}_{opt} - \vec{x}_{cur})$  [10], where *opt* and *cur* are optimal and current values. After updating Eq. (13), the following is obtained:

$$\vec{u}_{new} = (1 - \alpha) \vec{u}_{cur} + \vec{g} + \left( -\nabla P \frac{RT}{P_{cur}} \right) + \left( \frac{-2\Omega \times \vec{u}RT}{P_{cur}} \right). \quad (14)$$

The following additional substitutions are performed in Eq. (14): (i) the expression  $-2\Omega \times \vec{u}RT$ , which represents the influence of the Coriolis force, is replaced by  $c \vec{u}_{cur}^{other \ dim}$ , where  $c$  is a new constant expressed as  $c = -2\Omega RT$  and  $\vec{u}_{cur}^{other \ dim}$  is velocity of another randomly chosen dimension of the same air parcel; (ii) actual pressure  $P_{cur}$  is replaced by rank  $i$  [10]. Finally, after these substitutions, the velocity update equation becomes:

$$\vec{u}_{new} = (1 - \alpha) \vec{u}_{cur} - g \vec{x}_{cur} + \left( RT \left| \frac{1}{i} - 1 \right| (\vec{x}_{opt} - \vec{x}_{cur}) \right) + \left( \frac{c \vec{u}_{cur}^{other \ dim}}{i} \right). \quad (15)$$

When the velocity  $\vec{u}_{new}$  is calculated, the equation of updating the position will be:

$$\vec{x}_{new} = \vec{x}_{cur} + (\vec{u}_{new} \Delta t), \quad (16)$$

where  $\vec{x}_{cur}$  is the current position of the air parcel,  $\vec{x}_{new}$  is the new position in the next iteration, and  $\Delta t$  is the time step, which is set to  $t = 1$ .

When applying WDO, the user adjusts the coefficients  $\alpha, g, RT$ , and  $c$  in order to adapt the algorithm to the given problem. In some cases, the user can set the coefficients insufficiently precisely, which reduces the efficiency of the algorithm. AWDO removes this possible drawback of WDO. AWDO is a hybrid algorithm consisting of WDO and an evolutionary algorithm, the covariance matrix adaptation evolution strategy (CMAES) [11], which optimizes the above 4 coefficients in each iteration and selects a set of new values. CMAES is suitable for use because it does not require the setting of its coefficients by the user and the size of the population of CMAES is the same as that of WDO. Also, CMAES has a high speed [11]. Fast CMAES is incorporated in AWDO because the additional process of optimizing coefficients can slow down the algorithm. CMAES was described in detail in [11].

WDO starts with initialization and with randomly generating the velocity and position of the air parcel. The next step is to evaluate the pressure for each air parcel, update velocity and position, and check the velocity limits. Then the values of pressure and coefficients  $\alpha, g, RT$ , and  $c$  are entered into CMAES. CMAES optimizes the coefficients and selects a new set of coefficient values and returns them to WDO. In this way, the iterative process takes place in such a way that WDO performs the updating of the position and velocity, and CMAES optimizes the coefficients. Figure 1 shows the flowcharts of AWDO and CMAES.

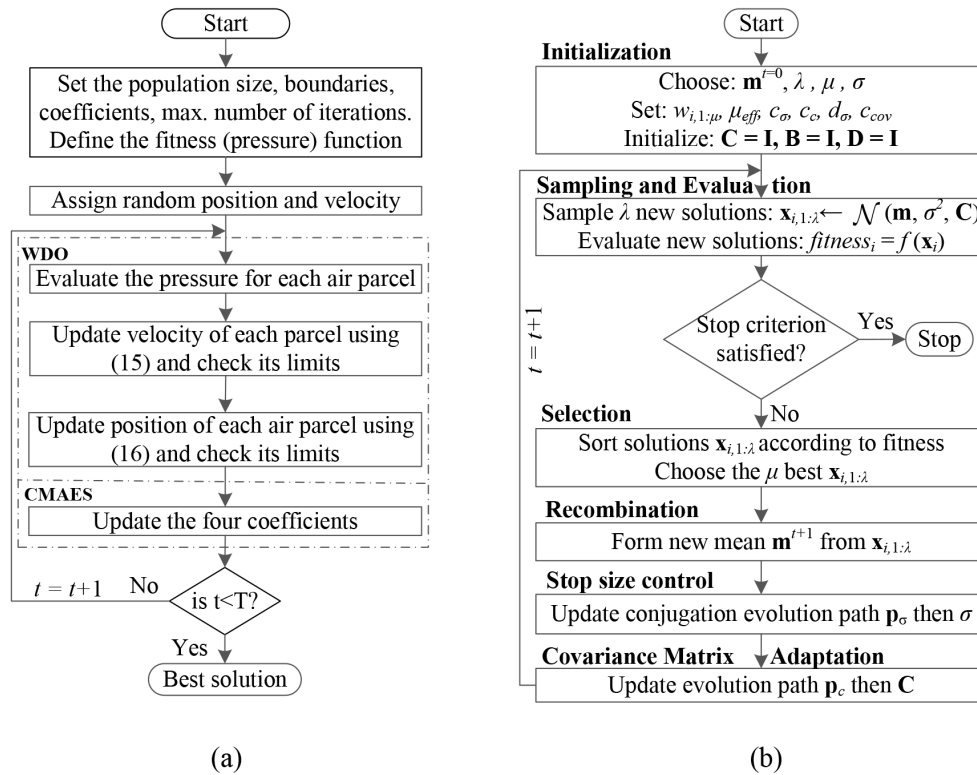


Figure 1. Flowcharts of AWDO (a) and CMAES (b).

CMAES belong to the class of evolutionary algorithms. The population of individuals is a set of solutions to a given optimization problem. The iteration starts with sampling  $\lambda$  candidate solutions,  $\mathbf{x}_i \in \mathbb{R}^N$ , from a multivariate normal distribution:

$$\mathbf{x}_i = \mathcal{N}(\mathbf{m}, \sigma^2 \mathbf{C}) \tag{17}$$

where  $\mathbf{m}$  is distribution mean,  $\sigma$  is the step-size,  $N$  is the problem dimension, and  $\mathbf{C} \in \mathbb{R}^{N \times N}$  is the covariance matrix. The step-size control prevents premature convergence and allows fast convergence. Matrix  $\mathbf{C}$  can be broken apart into its eigenvectors,  $\mathbf{B}$ , and eigenvalues,  $\mathbf{D}$ , as in:

$$\mathbf{C} = \mathbf{B} \mathbf{D}^2 \mathbf{B}^T. \tag{18}$$

After sampling, the candidate solutions  $\mathbf{x}_i$  are evaluated on the objective (fitness) function  $f$  to be minimized. All  $\mathbf{x}_{i,1:\lambda}$  are sorted according to fitness and their best number  $\mu$  (typically  $\mu \approx \lambda/2$ ) is used to compute the mean  $\mathbf{m}$  of the next iteration ( $t+1$ ) as follows:

$$\mathbf{m}^{t+1} = \sum_{i=1}^{\mu} w_i \mathbf{x}_i^t, \tag{19}$$

where  $w_i$  is the recombination weight of the  $i$ th best candidate solution:

$$w_i = \frac{\log_2(\mu + 0.5) - \log_2(i)}{\sum_{j=1}^{\mu} (\log_2(\mu + 0.5) - \log_2(j))} \quad \text{for } i = 1, 2, \dots, \mu. \tag{20}$$

After calculating  $w_i$ , the effective number of children  $\mu_{eff}$  is calculated as

$$\mu_{eff} = \left( \sum_{i=1}^{\mu} w_i^2 \right)^{-1}. \quad (21)$$

Next, the conjugate evolution path  $p_{\sigma}$  and the step-size are updated:

$$p_{\sigma}^{t+1} = (1 - c_{\sigma}) \cdot p_{\sigma}^t + \sqrt{c_{\sigma}(2 - c_{\sigma})} \cdot \frac{\sqrt{\mu_{eff}}}{\sigma^t} (C^t)^{-1/2} (m^{t+1} - m^t), \quad (22)$$

$$\sigma^{t+1} = \sigma^t \exp \left( \frac{c_{\sigma}}{d_{\sigma}} \left( \frac{\|p_{\sigma}^{t+1}\|}{E \|\mathcal{N}(0, I)\|} - 1 \right) \right), \quad (23)$$

where  $c_{\sigma}$  is the learning rate for the step-size control:

$$c_{\sigma} = \frac{\mu_{eff} + 2}{N + \mu_{eff} + 3}, \quad (24)$$

$d_{\sigma}$  is damping factor for step-size control:

$$d_{\sigma} = 1 + 2 \max \left( 0, \sqrt{\frac{\mu_{eff} - 1}{N + 1}} - 1 \right) + c_{\sigma}, \quad (25)$$

and  $E \|\mathcal{N}(0, I)\|$  is expected value of  $\|p_{\sigma}^{t+1}\|$ :

$$E \|\mathcal{N}(0, I)\| \approx \sqrt{N} \left( 1 - \frac{1}{4N} + \frac{1}{21N^2} \right). \quad (26)$$

Finally, the covariance matrix is updated, where the evolution path  $\mathbf{p}_c$  is updated first:

$$p_c^{t+1} = (1 - c_c) \cdot p_c^t + \sqrt{c_c(2 - c_c)} \cdot \frac{\sqrt{\mu_{eff}}}{\sigma^t} (m^{t+1} - m^t), \quad (27)$$

$$C^{t+1} = (1 - c_{cov}) \cdot C^t + \frac{c_{cov}}{\mu_{eff}} p_c^{t+1} (p_c^{t+1})^T + \left( 1 - \frac{1}{\mu_{eff}} \right) \frac{c_{cov}}{(\sigma^t)^2} \sum_{i=1}^{\mu} w_i (x_i^{t+1} - m^t) (x_i^{t+1} - m^t)^T, \quad (28)$$

where  $c_c$  is the learning rate for the rank-one update of the covariance matrix:

$$c_c = \frac{4}{N + 4}, \quad (29)$$

and  $c_{cov}$  is the learning rate for the update of the covariance matrix [11]:

$$c_{cov} = \frac{1}{\mu_{eff}} \frac{2}{(N + \sqrt{2})^2} + \left( 1 - \frac{1}{\mu_{eff}} \right) \min \left( 1, \frac{2\mu_{eff} - 1}{(N + 2)^2 + \mu_{eff}} \right). \quad (30)$$

#### 4. Simulation results

The proposed AWDO algorithm is tested on the standard IEEE 30-bus 6-generator system with total load demand of 283.4 MW, and with  $\text{NO}_x$  emission. The results obtained using the proposed AWDO algorithm are compared with the results of the implementation of 3 other well-established algorithms: the moth swarm algorithm (MSA) [12], firefly algorithm (FA) [13], and hybrid particle swarm optimization and gravitation search algorithm (PSOGSA) [14], which have better performance than other algorithms presented in the existing literature for solving the CEED problem [1,15].

The AWDO, MSA, FA, and PSOGSA are implemented in the MATLAB 2011b computational environment and run on a platform of 2.20 GHz with 3.0 GB RAM. The coefficients used for the simulations are presented in Table 1. The best results of the simulations are obtained after 30 runs. The testing was carried out in 2 cases of the CEED problem: (i) without VPLE (Case I) and (ii) with VPLE (Case II). The power loss in the system is considered in both cases. The specified error tolerance value is  $\delta = 10^{-6}$  MW. The  $B$ -loss matrices, the emission coefficients, and fuel cost coefficients are taken from [9] (see Appendix). In this paper, a scaling factor  $\gamma_{\text{NO}_x}$  of 1000 (\$/t) is used. Minimization is carried out with 3 values of weight factor:  $w = 1$  (fuel cost minimization),  $w = 0$  (emission minimization), and  $w = 0.5$  (minimization of fuel cost and emission, simultaneously). The maximum, minimum, and standard deviation values for cases of application of AWDO, MSA, FA, and PSOGSA to the test system (Case I) are shown in Table 2. From the results seen in Table 2, it is evident that the minimum values of the fuel cost and emission are the same for all 4 algorithms. However, the standard deviations of the results obtained by using AWDO are by far the smallest compared to the standard deviations obtained by using the other 3 algorithms (between 1.2372e-11 for minimization of the fuel cost and 3.6654e-13 for minimization of the emission).

**Table 1.** The coefficients of algorithms that apply to the test system.

AWDO			MSA			FA					PSOGSA					
$N$	$T$	$\alpha, g, RT, c$	$N$	$T$	$N_c$	$N$	$T$	$\alpha$	$\beta_{min}$	$\gamma$	$N$	$T$	$G_0$	$\alpha$	$C_1$	$C_2$
50	200	optimized	50	200	6	50	200	0.25	0.2	1	50	200	1	10	2	2

**Table 2.** Min, max, and SD values of the results and iteration times obtained by using AWDO, MSA, PSOGSA, and FA for the test system (Case I).

Algorithm		AWDO	MSA	PSOGSA	FA
Minimization for $w = 1$	Min	605.99837	605.99837	605.99837	605.99837
	Max	605.99840	605.99841	643.19300	606.40143
	SD	1.2372e-11	9.2426e-06	13.9840	0.0736
	Iteration time (s)	4.64440	6.37231	3.09460	7.17510
Minimization for $w = 0$	Min	0.194179	0.194179	0.222654	0.194179
	Max	0.194179	0.194179	0.194179	0.194179
	SD	3.6654e-13	2.2373e-06	6.3959	1.1483e-05
	Iteration time (s)	6.84278	8.47069	2.86218	8.79489
Minimization for $w = 0.5$	SD	2.74563-11	2.4235e-06	8.6575	6.9793e-08
	Iteration time (s)	5.35274	7.07522	3.12065	7.70115

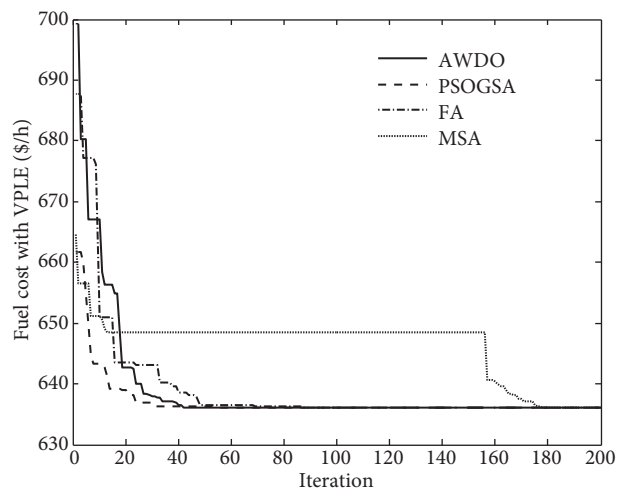


The best solutions for the power outputs, emission, and fuel cost obtained by using AWDO for  $w = 1$ ,  $w = 0$ , and  $w = 0.5$  are given in Table 3.

**Table 3.** The best solutions obtained by using AWDO.

Generation	Case I			Case II		
	$w = 1$	$w = 0$	$w = 0.5$	$w = 1$	$w = 0$	$w = 0.5$
$P_1$ (MW)	12.09691	41.09250	22.55426	5.01518	41.09250	5.00000
$P_2$ (MW)	28.63120	46.36678	35.45564	11.62726	46.36678	18.61854
$P_3$ (MW)	58.35573	54.44194	57.00525	83.44629	54.44194	80.05706
$P_4$ (MW)	99.28542	39.03737	74.53983	74.03373	39.03737	74.81317
$P_5$ (MW)	52.39702	54.44590	54.82118	79.19164	54.44590	77.91437
$P_6$ (MW)	35.18992	51.54851	41.55653	30.08590	51.54851	28.95097
Fuel cost (\$/h)	605.99837	646.20700	612.25279	635.82242	728.66748	638.92308
$\text{NO}_x$ (ton/h)	0.220729	0.194179	0.203570	0.226734	0.194179	0.222780
$P_{loss}$ (MW)	2.55619	3.53300	2.53270	1.87934	3.53300	1.95412
Iter.time (s)	4.6444	6.8428	5.3527	7.3134	6.9214	5.8684

The convergence characteristics of the proposed AWDO and those of MSA, FA, and PSO-GSA for minimization of fuel cost considering VPLE (Case II) are illustrated in Figure 2. From Figure 2, it can be seen that ascending speeds are high at the beginning for all 4 algorithms, which shows the high convergence. However, AWDO can achieve an optimal solution after a smaller number of iterations than the other 3 algorithms. Thus, AWDO is demonstrated to have a better convergence property in comparison with the MSA, PSO-GSA, and FA.



**Figure 2.** Comparative convergence characteristics of AWDO, MSA, FA, and PSO-GSA in the case of minimization of fuel cost considering the VPLE.

Table 4 shows a comparison of the best solutions for the fuel cost and  $\text{NO}_x$  emission obtained by using AWDO, MSA, PSO-GSA, and FA considering VPLE (Case II). It can be seen that AWDO gives the best results for minimal cost and an equal value of minimal emission compared to the other 3 algorithms. The minimal power losses, in the case of application of AWDO, are better or very close to those of the other 3 algorithms.

From the results seen in Table 4 it follows that the value of fuel cost obtained by using AWDO is 0.0529 \$/h less than the value obtained by using the MSA, 0.0163 \$/h less than the value obtained by the PSO-GSA, and 0.0370 \$/h less than the value obtained by the FA. Therefore, AWDO can result in better economic effects than the MSA, PSO-GSA, and FA. Lower fuel cost (Table 4) and smaller standard deviation (Table 2) of evaluation values result in a higher quality solution of AWDO than the other 3 algorithms.

**Table 4.** Comparison of the best solutions for the test system (Case II) using AWDO (1), MSA (2), PSO-GSA (3), and FA (4).

Algorithm	Optimization for $w=1$			Optimization for $w=0$			Optimization for $w=0.5$		
	Fuel cost (\$/h)	Emission (t/h)	$P_{loss}$ (MW)	Fuel cost (\$/h)	Emission (t/h)	$P_{loss}$ (MW)	Fuel cost (\$/h)	Emission (t/h)	$P_{loss}$ (MW)
1	635.82242	0.226734	1.879	728.66748	0.194179	3.533	638.92308	0.222780	1.954
2	635.87530	0.226533	1.882	728.66962	0.194179	3.533	639.11337	0.222607	1.962
3	635.83871	0.226902	1.880	728.66748	0.194179	3.533	639.11517	0.222590	1.961
4	635.85945	0.227061	1.878	728.66693	0.194179	3.533	638.79420	0.222911	1.950

The results obtained by AWDO for the test system along with corresponding data from the literature (Case I) are summarized in Table 5. As can be seen in Table 5, AWDO provided better values for the minimum fuel cost in regard to the values obtained by the algorithms proposed in [16–20] as well as ones that are the same as the results obtained by the algorithms from [1] and [15]. The minimum values of  $\text{NO}_x$  emission calculated by AWDO are the same as the associated results reported in [1] and [15] or better than the results in [16–20].

**Table 5.** A comparison of the best solutions for the fuel cost and  $\text{NO}_x$  emission (Case I).

Algorithm	Optimization for $w=1$		Optimization for $w=0$		Optimization for $w=0.5$	
	Fuel cost (\$/h)	Emission (t/h)	Fuel cost (\$/h)	Emission (t/h)	Fuel cost (\$/h)	Emission (t/h)
<b>AWDO</b>	605.99837	0.220729	646.20700	0.194179	612.25279	0.203570
MSA [1]	605.99837	0.220728	646.20486	0.194179	612.25190	0.203571
PSO-GSA [15]	605.99837	0.220728	646.20838	0.194179	612.25222	0.203571
FA [1]	605.99837	0.220728	646.20731	0.194179	612.25302	0.203570
MBFA [16]	607.6700	0.2198	644.4300	0.1942	616.496	0.2002
MOPSO [17]	607.7900	0.2193	644.7400	0.1942	615.000	0.2021
PSO [18]	607.8400	0.2192	642.9000	0.1942	-	-
DE [19]	608.0658	0.2193	645.0850	0.1942	-	-
MODE/PSO [20]	606.0073	0.2209	646.0243	0.1942	-	-

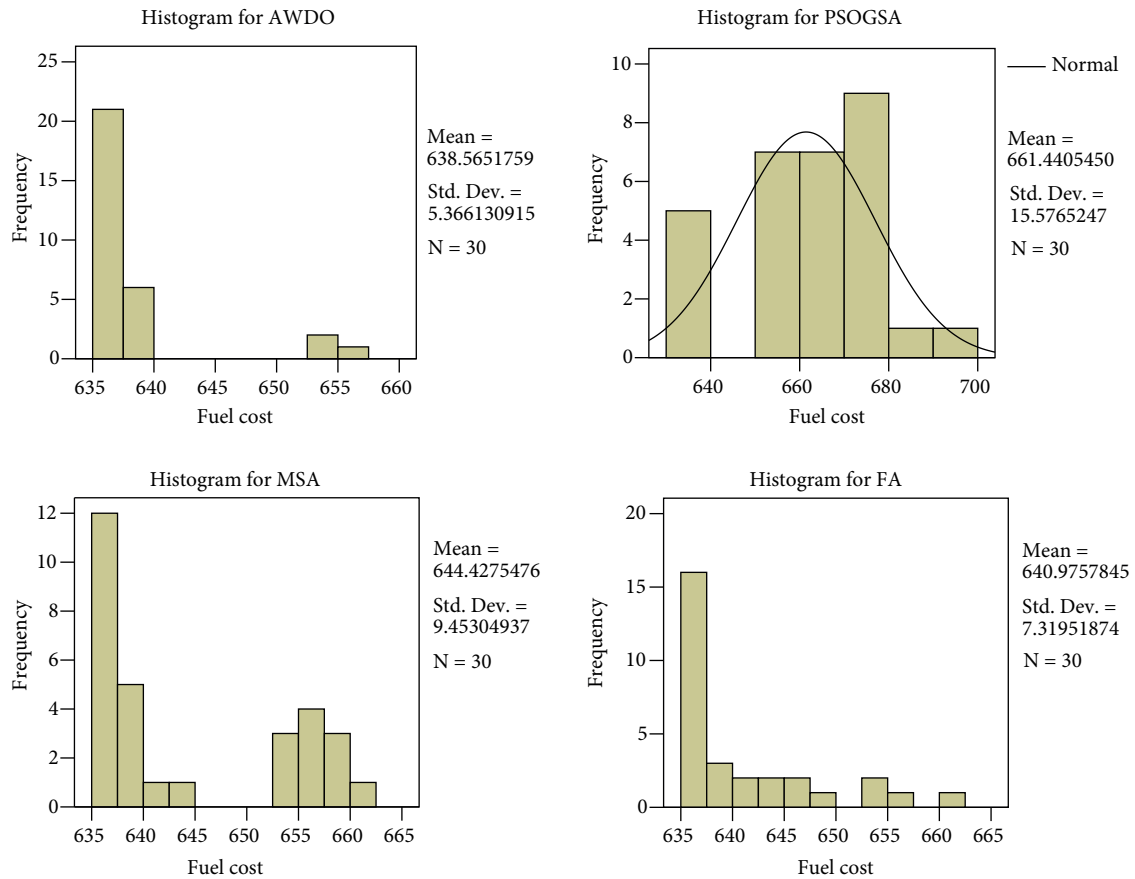
Randomness is one of properties of stochastic metaheuristic algorithms because the initialization of the population is carried out with random numbers. Table 6 shows the frequency of attaining the fuel cost within different iteration ranges out of 30 runs for Case II. From Table 6 it can be seen that AWDO is robust and most consistent in comparison with other algorithms.

Figure 3 shows the histograms that represent a frequency of results obtained after 30 runs of algorithms over fuel cost function with VPLE (case II). From Figure 3 it can be seen that AWDO has the smallest dispersion

**Table 6.** Frequency of convergence of attaining fuel cost for Case II out of 30 runs.

Iteration	AWDO	MSA	PSOGSA	FA
0–10	0	7	12	2
10–20	1	3	9	2
20–30	2	1	2	0
30–50	27	0	1	9
5–100	0	0	3	17
100–200	0	19	3	0

and largest frequency of results and the PSOGSA follows the normal distribution. The Q-Q plots (graphical presentations of the differences between quartiles from results observed and those from the normal distributions) for AWDO and PSOGSA are presented in Figure 4 and they confirm the property of normality for PSOGSA. Detailed statistical analysis of the algorithms' behavior can be performed using parametric and nonparametric tests [21] and that will be the subject of our future work. Figure 3 and Figure 4 are obtained by using the statistical software package SPSS.



**Figure 3.** Histograms of frequency of results obtained using algorithms AWDO, PSOGSA, MSA, and FA over fuel cost function considering VPLE (Case II).

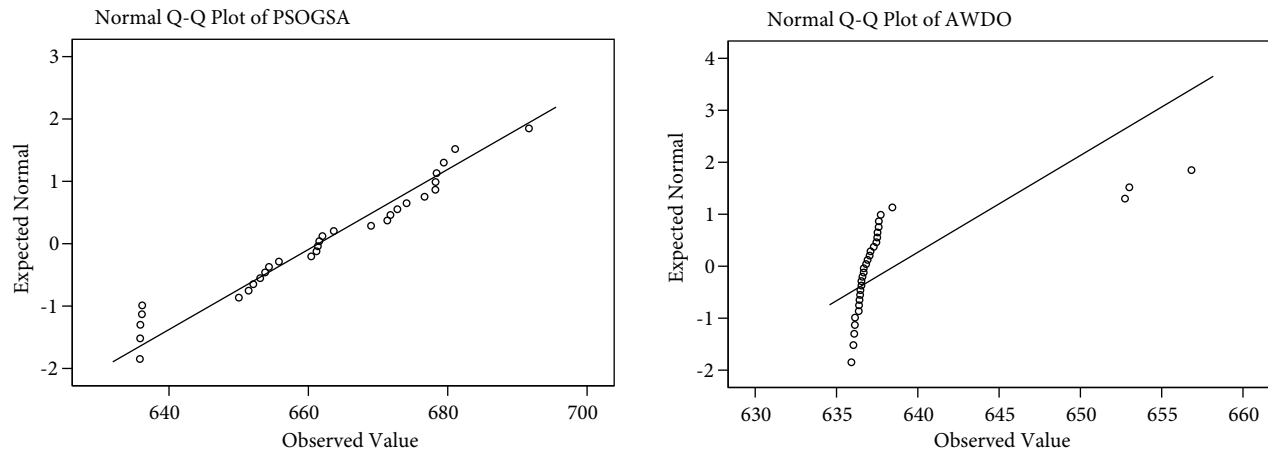


Figure 4. Q-Q plots for PSO GSA and AWDO on fuel cost function considering VP LE.

## 5. Conclusion

In this paper the AWDO algorithm is proposed to solve the multiobjective CEED problem with nonconvex and nonsmooth functions and with the nonlinearities of VP LE and  $P_{loss}$ . The multiobjective function is converted into a single-objective function by means of the weighted sum method. The proposed algorithm for solving the CEED problem was tested on the IEEE 30-bus 6-generator test system. The testing was carried out in all cases of the CEED problem: (i) optimization of fuel cost only; (ii) optimization of pollutant emission only; (iii) optimization of cost and emission, simultaneously; (iv) without VP LE; (v) with VP LE. The power loss in the system is considered in all cases. The AWDO is an adaptive algorithm and it optimizes the coefficients in each iteration, which improves the process of the global optimal solution search. The results obtained from the proposed AWDO algorithm are compared with the results of implementation of the 3 other well-established algorithms, MSA, PSO GSA, and FA, which have better performance than other algorithms presented in the existing literature for solving the CEED problem. The simulation results show that the proposed AWDO gives best results for fuel cost and equal value of emission as the other 3 implemented algorithms. Therefore, AWDO can result in better economic effects. The standard deviation of results is much smaller in the case of AWDO than in the case of other algorithms. AWDO can achieve an optimal solution after a smaller number of iterations than the other 3 algorithms, which results in its better convergence property. Finally, the comparison of the numerical results, the convergence profiles, and the statistical analysis of results confirm the robustness, effectiveness, and superiority of the proposed AWDO algorithm for solving the CEED problem with different functions.

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## Appendix

**Table A1.** The *b-loss* matrices for the test system [9].

Matrices	Matrix elements
$B$	$\begin{bmatrix} 0.1382 & -0.0299 & 0.0044 & -0.0022 & -0.0010 & -0.0008 \\ -0.0299 & 0.0487 & -0.0025 & 0.0004 & 0.0016 & 0.0041 \\ 0.0044 & -0.0025 & 0.0182 & -0.0070 & -0.0066 & -0.0066 \\ -0.0022 & 0.0004 & -0.0070 & 0.0137 & 0.0050 & 0.0033 \\ -0.0010 & 0.0016 & -0.0066 & 0.0050 & 0.0109 & 0.0005 \\ -0.0008 & 0.0041 & -0.0066 & 0.0033 & 0.0005 & 0.0244 \end{bmatrix}$
$B_0$	$[-0.0107 \quad 0.0060 \quad -0.0017 \quad 0.0009 \quad 0.0002 \quad 0.0030]$
$B_{00}$	[0.00098573]

**Table A2.** Fuel cost coefficients, NO<sub>x</sub> emission coefficients, and generation limits for the test system [9].

$g$	$a_g$	$b_g$	$c_g$	$\alpha_g$	$\beta_g$	$\eta_g$	$\xi_g$	$\lambda_g$	$P_g^{\min}$	$P_g^{\max}$
1	10	200	100	4.091e-2	-5.554e-2	6.490e-2	2.0e-4	2.857	5	150
2	10	150	120	2.543e-2	-6.047e-2	5.638e-2	5.0e-4	3.333	5	150
3	20	180	40	4.258e-2	-5.094e-2	4.586e-2	1.0e-6	8.0	5	150
4	10	100	60	5.326e-2	-3.550e-2	3.380e-2	2.0e-3	2.0	5	150
5	20	180	40	4.258e-2	-5.094e-2	4.586e-2	1.0e-6	8.0	5	150
6	10	150	100	6.131e-2	-5.555e-2	5.151e-2	1.0e-5	6.667	5	150