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## Linear-like discrete-time fuzzy control in the regulation of irrigation canals

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**Abstract:** A linear-like discrete-time fuzzy controller was designed to control and stabilize a single-pool irrigation canal. Saint Venant equations for open-channel flow were linearized using the Taylor series and a finite-difference approximation of the original nonlinear partial differential equations. Using the linear optimal control theory, a traditional linear quadratic regulator (LQR) was first developed for an irrigation canal with a single-pool, and the results were observed. Then a linear-like global system representation of a discrete-time fuzzy system was proposed by viewing a discrete-time fuzzy system in a global concept and unifying the individual matrices into synthetic matrices. This linear-like representation aided development of a design scheme for a global optimal fuzzy controller in the way of the general linear quadratic approach. Based on this kind of system representation, a discrete-time optimal fuzzy control law that can achieve global minimum effect was developed. An example problem with a single-pool was considered for evaluating the performance of the discrete-time optimal fuzzy controller in the control of irrigation canals. The results obtained with the optimal fuzzy controller were compared to the results obtained with a traditional linear quadratic regulator. The discrete-time fuzzy controller was the best for the operation of the canal system, reaching the optimal performance index under unknown demands.

**Key words:** Optimal fuzzy controller, linear quadratic regulator, canal automation

### Doğrusal ayrık zamanlı bulanık kontrol metodu ile sulama kanallarının regülasyonu

**Özet:** Bu çalışmada tek havuzlu bir sulama kanalında akışın kontrolü ve regülasyonu için doğrusal ayrık zamanlı bulanık kontrol yöntemine göre bir kontrol algoritması geliştirilmiştir. Sulama kanallarında akışı ifade eden Saint Venant eşitlikleri, Taylor serileri ve doğrusal olmayan kısmi diferansiyel eşitliklerin sonlu-farklar yaklaşımı kullanılarak doğrusal hale getirilmiştir. Optimal kontrol teorisi kullanılarak tek havuzlu sulama kanalı için geleneksel doğrusal karesel regülatör (LQR) geliştirilmiş ve simülasyon sonuçları gözlemlenmiştir. Sonraki aşamada, sistem matrislerinin sentetik matris şeklinde bütünleştirilmesi ile mevcut problemin ayrık zamanlı bulanık model hali tanımlanmıştır. Bu model ile global optimal bulanık kontrol tekniğinin doğrusal karesel regülatör yaklaşımı şeklinde ifade edilmesi sağlanmıştır. Bu yaklaşım modelinden hareket ile, dışarıdan gelen etkilerden minimum düzeyde etkilenen ayrık zamanlı optimal bulanık kontrol modeli tasarlanmıştır. Geliştirilen bu modelin sulama kanallarının kontrolünde göstereceği performansı değerlendirmek için tek havuzlu bir sulama kanalı örnek problem olarak seçilmiştir. Simülasyon sonrası hem optimal bulanık kontrolcu hemde geleneksel doğrusal karesel regülatörden elde edilen sonuçlar karşılaştırılmıştır. Sulama kanalları işletiminde kanaldan bilinmeyen taleplerin olması durumunda ayrık zamanlı bulanık kontrolcunun, doğrusal karesel regülatöre göre daha iyi performans gösterdiği ve akış düzeni sağladığı tespit edilmiştir.

**Anahtar sözcükler:** Optimal bulanık kontrolcu, doğrusal karesel regülatör, kanal otomasyonu

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## Introduction

Uncertainty is always bothersome in controlling a real system, as a physical system is usually only partly known and difficult to describe, and has few measurements available. Irrigation canals are operated using a variety of delivery schedules. Providing the right quantity of water at the right time increases agricultural production. Supply-oriented systems have not been able to provide the needed flexibility, in terms of the quantity of water and timing, to improve crop yields and water-use efficiency. This calls for a more flexible delivery schedule, called demand delivery.

The demand delivery schedule provides more flexibility to water users than other delivery schedules that are in use today. With demand delivery operation of irrigation canals, variations in water withdrawal rates into lateral canals (disturbances) are not known in advance; hence, these variations in flow rates are classified as random disturbance actions on the supply canal. In other words, the level of uncertainty in the demand delivery schedule is high. In the absence of information on the disturbances being obtained in advance, meeting the random demands when operating canals becomes a difficult task.

In the past, the concepts of optimal control theory have been applied to derive closed-loop control algorithms for real-time control of irrigation canals (Balogun 1985; Reddy et al. 1992; Malaterre 1994; Begovich et al. 2007a, 2007b). Balogun et al. (1988), Hubbard et al. (1987), Reddy (1999), and Durdu (2003) applied the linear quadratic regulator (LQR) technique to open-channel flow control using a linearized, spatially discretized version of the Saint Venant equations; however, most of these studies dealt with the traditional linear quadratic regulator technique. None of the above studies used an optimal discrete-time fuzzy controller to control irrigation canals. Begovich et al. (2003) developed a controller for the real-time implementation and evaluation of a fuzzy gain scheduling control regulating the downstream levels at the end of the pools of a 4-pool open irrigation canal prototype. Durdu (2005) designed an optimal fuzzy filter that employed the Lyapunov function to formulate the fuzzy interference rules to solve the state estimation problem of controlled irrigation canals. Most fuzzy control

studies are based on the Takagi-Sugeno (T-S)-type fuzzy model, combined with the parallel distribution compensation (PDC) concept and application of Lyapunov's method for stability analysis (Wu and Lin 2002).

Wu and Lin (2002) developed a fuzzy system representation proposed to mature the formulation and simplification of the quadratic optimal fuzzy control problem. This linear-like representation motivates one to develop the design scheme for a fuzzy controller using the general linear quadratic (LQ) approach. Fuzzy modeling can mimic a real system well and fuzzy control can support more robust control than linear control is capable of (Wu and Lin 2002). Irrigation canals are regulated using spatially distributed control structures (gates). In a global fuzzy control algorithm, the variation in the opening of the gate in the system is computed based upon the information on water levels in the pools. The fuzzy controller of an irrigation canal needs to know the current water level and must be able to set the gate. The controller's input will be the water level error (desired water level minus actual water level) and its output will be the rate at which the gate is opened or closed. The goal of the present study was to determine the effectiveness of a global optimal fuzzy controller for the operation of irrigation canals in the presence of arbitrary external disturbances (unknown demands), and to evaluate the performance of the controller algorithm, as compared to a traditional linear quadratic regulator.

## Materials and methods

### Mathematical modeling of open-channel flow

In the operation of irrigation canals, decisions regarding changes in gate opening in response to arbitrary (random) changes in water withdrawal rates into lateral or branch canals are required to maintain the flow rate into the lateral canals close to the desired value. This is accomplished by maintaining the depth of flow or the volume of water in a given pool at a target value. This problem is similar to the process control problem in which the state of the system is maintained close to the desired value using real-time feedback control. Linear control theory is well developed and is easier to apply than nonlinear

control theory (Reddy 1991). In the present study, therefore, a linear-like system representation of a fuzzy system was employed.

### Water conveyance equations

The Saint Venant equations, presented below, were used to model flow in a canal

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \quad (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + \left( gA \frac{\partial y}{\partial x} - S_0 + S_f \right) = 0 \quad (2)$$

in which  $Q$  = flow in the canal ( $\text{m}^3 \text{s}^{-1}$ ),  $A$  = wetted area ( $\text{m}^2$ ),  $q_l$  = lateral flow ( $\text{m}^2 \text{s}^{-1}$ ),  $y$  = water depth (m),  $t$  = time (s),  $x$  = longitudinal direction of the channel (m),  $g$  = gravitational acceleration ( $\text{m}^2 \text{s}^{-1}$ ),  $S_0$  = canal bottom slope (m/m),  $R$  = hydraulic radius,  $A/P$  (m),  $P$  = wetted perimeter (m),  $n$  = roughness coefficient ( $\text{s}/\text{m}^{1/3}$ ), and  $S_f$  = the friction slope (m/m), which is defined as:

$$S_f = Q|Q|/K^2 \quad (3)$$

in which  $K$  = hydraulic conveyance of the canal ( $AR^{2/3}/n$ ) and  $R$  = hydraulic radius (m). In deriving Eq. (2), the effect of the net acceleration terms stemming from removal of a fraction of the surface stream was assumed to be negligible.

### Lateral flow rates

Lateral canals in the main canal are usually scattered throughout the length of the supply canal. Manually controlled discharge regulators are used at the head of lateral canals. The mathematical representation of flow through these structures is given as follows:

$$q_l = C_d b_l w_l (2g(Z - Z_l))^{1/2} \text{ for submerged flow} \quad (4)$$

$$q_l = C_d b_l w_l (2g(Z - E_s))^{1/2} \text{ for free flow} \quad (5)$$

in which  $q_l$  = lateral discharge rate ( $\text{m}^3 \text{s}^{-1}$ ),  $C_d$  = outlet discharge coefficient,  $b_l$  = width of the outlet structure (m),  $w_l$  = height of gate opening of the outlet structure (m),  $Z$  = water surface elevation in the supply canal (m),  $Z_l$  = water surface elevation in the lateral canal (m), and  $E_s$  = sill elevation of the head regulator (m). Obviously, the flow rate through a head regulator depends upon the water surface elevation in the supply canal. The water surface elevation in the lateral canal is a function of the discharge rate through the head regulator. As such, this equation is an implicit equation. In the case of free flow, the discharge rate through the head regulator is independent of the water surface elevation in the lateral canal; therefore, once the required discharge into a lateral is specified, the gate opening is adjusted to obtain the required flow rate through the head regulator, assuming that the water surface elevation in the supply canal is maintained constant at the target level. When a manually controlled head regulator is used, for simulation purposes the gate opening or the variation in the gate opening is specified as a function of time. Conversely, when an automated discharge rate regulator is used, for simulation purposes the lateral discharge rate, as a function of time, is specified as a known input, i.e.  $q_l = f_q(t)$ .

### Control structures (gates)

In the regulation of the main canal, decisions regarding the opening of gates in response to random changes in water withdrawal rates into lateral canals are required in order to maintain the flow rate into laterals close to the desired value. This is accomplished by either maintaining the depth of flow in the immediate vicinity of the turnout structures in the supply canal constant or by maintaining the volume of water in the canal pools at the target value. When the latter option is used, the outlets are often fitted with discharge rate regulators. The water levels or the volume of water stored in the canal pools are regulated using a series of spatially distributed gates (control elements); hence, irrigation canals are modeled as distributed control systems. As such, in the solutions of Eqs. (1) and (2), additional boundary conditions are specified at the control structures, in terms of the flow continuity and the gate discharge equations, which are given by:

$$Q_{i-1,N} = Q_{gi} = Q_{i,1} \text{ (continuity)} \quad (6)$$

$$Q_{gi} = C_{di} b_i u_i (2g(Z_{i-1,N} - Z_{i,1}))^{1/2} \text{ (gate discharge)} \quad (7)$$

in which  $Q_{i-1,N}$  = flow rate through the downstream gate (or node N) of pool  $i-1$  ( $\text{m}^3 \text{s}^{-1}$ ),  $Q_{gi}$  = flow rate through the upstream gate of pool  $i$  ( $\text{m}^3 \text{s}^{-1}$ ),  $Q_{i,1}$  = flow rate through the upstream gate (or node 1) of pool  $i$  ( $\text{m}^3 \text{s}^{-1}$ ),  $C_{di}$  = discharge coefficient of gate  $i$ ,  $b_i$  = width of gate  $i$  (m),  $u_i$  = opening of gate  $i$  (m),  $Z_{i-1,N}$  = water surface elevation at node N of pool  $i-1$  (m),  $Z_{i,1}$  = water surface elevation at node 1 of pool  $i$  (m), and  $i$  = pool index ( $i = 0$  refers to the upstream constant level of the reservoir).

### Linearization and discretization of system equations

The Saint Venant open-channel equations are linearized about equations and the average operating condition of the canal is used to apply the linear control theory concepts to the problem (Malaterre 1997). After applying a finite-difference approximation and the Taylor series expansions to Eqs. (1) and (2), a set of linear ordinary differential equations is obtained for the canal with control gates and turnouts (Durdu 2003):

$$A_{11} \delta Q_j^+ + A_{12} \delta z_j^+ + A_{13} \delta Q_{j+1}^+ + A_{14} \delta z_{j+1}^+ = A'_{11} \delta Q_j + A'_{12} \delta z_j + A'_{13} \delta Q_{j+1} + A'_{14} \delta z_{j+1} + C_1 \quad (8)$$

$$A_{21} \delta Q_j^+ + A_{22} \delta z_j^+ + A_{23} \delta Q_{j+1}^+ + A_{24} \delta z_{j+1}^+ = A'_{21} \delta Q_j + A'_{22} \delta z_j + A'_{23} \delta Q_{j+1} + A'_{24} \delta z_{j+1} + C_2 \quad (9)$$

where  $\delta Q_j^+$  and  $\delta z_j^+$  = discharge and water-level increments from time level  $t + 1$  at node  $j$ ,  $\delta Q_j$  and  $\delta z_j$  = discharge and water-level increments from time level  $t$  at node  $j$ , and  $A_{11}$ ,  $A'_{21}$ , ...,  $A_{12}$ ,  $A_{22}$  are the coefficients of the continuity and momentum equations, respectively, computed with known values at time level ( $t$ ). Similar equations are derived for channel segments that contain a gate structure, a weir, or some other type of hydraulic structure. The matrix form of the about equations for the canal can be defined as follows (Malaterre 1994):

$$\underbrace{\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ -\left(\frac{\partial f}{\partial z_j}\right)_e & 1 - \left(\frac{\partial f}{\partial z_{j+2}}\right)_e \end{bmatrix}}_{A_L} \begin{bmatrix} \partial Q_j^+ \\ \partial z_j^+ \\ \partial Q_{j+1}^+ \\ \partial z_{j+1}^+ \\ \partial Q_{j+2}^+ \\ \partial z_{j+2}^+ \end{bmatrix} = \underbrace{\begin{bmatrix} A'_{11} & A'_{12} & A'_{13} & A'_{14} \\ A'_{21} & A'_{22} & A'_{23} & A'_{24} \\ -\left(\frac{\partial f}{\partial z_j}\right)_e & 1 - \left(\frac{\partial f}{\partial z_{j+2}}\right)_e \end{bmatrix}}_{A_R} \begin{bmatrix} \partial Q_j \\ \partial z_j \\ \partial Q_{j+1} \\ \partial z_{j+1} \\ \partial Q_{j+2} \\ \partial z_{j+2} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \left(\frac{\partial f}{\partial u_j}\right)_e \end{bmatrix}}_B \Delta \delta + \underbrace{\begin{bmatrix} A_{11} - A'_{11} \\ A_{21} - A'_{21} \end{bmatrix}}_C \begin{bmatrix} \delta Q_p \\ \delta Q_p^+ \end{bmatrix} \quad (10)$$

where  $f$  is any dependent variable on  $Q$  or  $z$ . From the matrix form of the above equations, the state of the system equation at any sampling interval ( $k$ ) can be written, in a compact form as follows:

$$A_L \delta x(k+1) = A_R \delta x(k) + B \delta u(k) + C \Delta \delta q(k) \quad (11)$$

where  $A_L = n \times n$  system feedback matrix [left-hand side coefficients of Eqs. (8) and (9)],  $A_R = n \times n$  system feedback matrix [right-hand side coefficients of the Eqs. (8) and (9)],  $B = n \times m$  control distribution matrix,  $C = p \times n$  disturbance matrix,  $\delta x(k) = n \times 1$  state vector,  $\delta u(k) = m \times 1$  control vector,  $\Delta \delta q =$  variation in demands (or disturbances) at the turnouts ( $\text{m}^2 \text{s}^{-1}$ ),  $n =$  number of dependent (state) variables in the system,  $m =$  number of controls (gates) in the canal,  $p =$  number of outlets in the canal, and  $k =$  time increment (s). The elements of matrices  $A$ ,  $B$ , and  $C$  depend upon the initial condition. The dimensions of control distribution matrix  $B$  depend on the number

of state variables and the number of gates in the canal. The dimensions of disturbance matrix  $C$  depend on the number of disturbances acting on the canal system and the number of dependent state variables. Eq. (11) can be written in a state-variable form, along with the output equations as follows (Reddy 1991):

$$\delta x(k+1) = \Phi \delta x(k) + \Gamma \delta u(k) + \Psi \delta q(k) \quad (12)$$

$$\delta y(k) = H \delta x(k) \quad (13)$$

where  $\Phi = (A_L)^{-1} * A_R$ ,  $\Gamma = (A_L)^{-1} * B$ ,  $\Psi = (A_L)^{-1} * C$ ,  $\delta y(k) = r \times 1$  vector of output (measured variables),  $H = r \times n$  output matrix, and  $r =$  number of outputs. The elements of matrices  $\Phi$ ,  $\Gamma$ , and  $\Psi$  depend upon the canal parameters, the sampling interval, and the assumed average operating condition of the canal. In Eq. (12), the vector of state variables is defined as follows:

$$\delta x = (\delta Q_{i,1}, \delta Z_{i,2}, \delta Q_{i,2}, \dots, \delta Z_{i,N-1}, \delta Q_{i,N-1}, \delta Q_{i,N}) \quad (14)$$

### Linear optimal quadratic control

In the irrigation control literature much attention has been devoted to linear quadratic regulator design problems, largely as a result of their elegant problem formulation, solution tractability, and robust properties with respect to fairly large variations in system parameters. The problem of designing a linear feedback control system to minimize the quadratic performance index can be reduced to the problem of obtaining a positive definite solution of a matrix Riccati equation. An important characteristic of transient performance of an open canal is its stability. Once a canal is disturbed from its original equilibrium condition responses to the disturbance will result in a stable, neutral, or unstable condition.

The stability requirement of any system is defined in terms of eigenvalues, which are the roots of the characteristic equation of matrix  $\Phi$  and must have values less than unity. The oscillatory behavior of a canal's water surface is associated with the presence of complex roots in the solution of the characteristic

equation of the system. The response amplitude grows continuously if the absolute value of the complex roots is greater than unity, decays to zero if the absolute value is less than unity, and oscillates at a constant amplitude if the real part of the roots is zero. Additionally, because of inertia, it is almost impossible to derive the deviation in water surface elevation (error) instantaneously to zero. Thus, the output of the system lags the desired input and results in overshoot or oscillation of the water level about its equilibrium position.

The objective of control theory is to find a control law that will bring an initially disturbed water surface to the desired target water level in the presence of external disturbances acting on the canal. This can be accomplished by applying a large proportional control in which change in gate opening is proportional to changes in flow depths and flow rates, as follows (Reddy 1991):

$$\delta u(k) = -K(k) \delta x(k) \quad (15)$$

where  $K(k) =$  controller gain matrix. Controllability ensures the stability of the system and maintains the water level at any desired value by suppressing the influence of external disturbances. A canal is said to be controllable if it is possible to derive it from any initial water level to any specified water level (state) within a finite number of steps. Eq. (15), which was used throughout the study, is called the discrete state equation and control law. This equation describes the condition or evolution of the basic internal variables of the system. The variables in the equation (i.e.  $\delta x$ ) are called the state variables. In optimal control theory the elements of gain matrix  $K$  can be obtained by formulating the control problem as an optimization problem in which the cost function to be minimized is given as follows (Reddy 1999):

$$J = \sum_{i=1}^{K_{\infty}} [\delta x^T(k) Q_{x_{\text{mxn}}} \delta x(k) + \delta u^T(k) R_{\text{mxm}} \delta u(k)] \quad (16)$$

subject to the constraint that:

$$\begin{aligned} -\delta x(k+1) + \Phi \delta x(k) + \Gamma \delta u(k) &= 0 \\ k &= 0, \dots, K_{\infty} \end{aligned} \quad (17)$$

where  $K_\infty$  = number of sampling intervals considered to derive the steady state controller,  $Qx_{n \times n}$  = state cost weighting matrix, and  $R_{m \times m}$  = control cost weighting matrix. The matrices  $Qx$  and  $R$  are symmetric, and to satisfy the non-negative definite condition they are usually selected to be diagonal, with all diagonal elements positive or zero.

The first term in Eq. (16) represents the penalty on the deviation of the state variables from the average operating (or target) condition, whereas the second term represents the cost of control. This term is included in an attempt to limit the magnitude of the control signal  $\delta u(k)$ . Unless a cost is imposed for the use of control, the design that emerges is liable to generate control signals that cannot be achieved by the actuator. In this case saturation of the control signal will occur, resulting in system behavior that differs from the closed loop system behavior that was predicted assuming that saturation will not occur. Therefore, the control signal weighting matrix elements are selected to be large enough to avoid saturation of the control signal under normal operating conditions. Eqs. (16) and (17) constitute a constrained-minimization problem that can be solved using the method of Lagrange multipliers. This produces a set of coupled difference equations that must be solved recursively backwards in time; however, because irrigation canals run for a long time and the dynamics of the canals are usually very slow, a steady state controller is more desirable. For the steady state case, the solution for  $\delta u(k)$  is the same form as Eq. (15), except that  $K$  is given by:

$$K = [R + \Gamma^T P \Gamma]^{-1} \Gamma^T P \Phi \quad (18)$$

$P$  is a solution of the discrete algebraic Riccati equation (DARE):

$$\Phi^T P \Phi - \Phi^T P \Gamma S^{-1} \Gamma^T P \Phi + Qx = P \quad (19)$$

where  $S = R + \Gamma^T S \Gamma$ ,  $R = R^T > 0$ , and  $Qx = Qx^T = H^T H \geq 0$ . The solution of the discrete algebraic Riccati equation is fundamental to the implementation of optimal control. The control law defined by Eq. (15) brings an initially disturbed system to an equilibrium

condition in the absence of any external disturbances acting on the system (Reddy 1991). In the presence of these external disturbances, the system cannot be returned to the equilibrium condition using Eq. (15). An integral control, in which the cumulative (or integrated) deviation of a selected output variable is used in the feedback control loop, is required to return the system to the equilibrium condition in the presence of external disturbances (Kwakernaak and Sivan 1972; Kailath 1980). Integral control is achieved by appending additional variables of the following form to the system dynamic equation (Reddy et al. 1992):

$$-\delta x_i(k+1) = D\delta x(k) + \Gamma \delta x_i(k) \quad (20)$$

in which  $\delta x_i$  = integral state variables and  $D$  = the integral feedback matrix. This produces a new control law to the form:

$$\delta u(k) = -K\delta x(k) - K_i \delta x_i(k) \quad (21)$$

The first term in Eq. (21) accounts for initial disturbances, whereas the second term accounts for external disturbances. Eq. (21) predicts the desired gate openings as a function of the measured deviations in the values of the state variables (Reddy et al. 1992). In hydraulic engineering problems, the depth of flow, flow rate, and velocity as a function of distance can be considered as the state or internal variables. Sometimes, the volume of water in a given reach of a canal can also be considered as a state variable. In the present study the water surface elevation and flow rate were considered the state variables. Given initial conditions  $[\delta x(0)]$ ,  $\delta u$ , and  $\delta q$ , Eq. (16) can be solved for variations in flow depth and flow rate as a function of time (Reddy 1990). If the system is really at equilibrium [i.e.  $\delta x(0) = 0$  at time  $t = 0$ ] and there is no change in the lateral withdrawal rates (disturbances), the system would continue to be at equilibrium forever; then there is no need for any control action (Reddy 1990). Conversely, in the presence of disturbances (known or random) the system would deviate from the equilibrium condition. The actual condition of the system may be either

above or below the equilibrium condition, depending upon the sign and magnitude of the disturbances. If the irrigation canal system departs from the stability condition, the discharge rates into the turnouts might be different (either more or less) than the targeted rates. However, in canal conveyance systems, the purpose is to keep these departures to a minimum so that a nearly constant rate of discharge is maintained through the turnouts.

### Optimal quadratic fuzzy controller

In the present study a linear-like quadratic fuzzy control problem, developed by Wu and Lin (2002), was used to formulate the single-pool irrigation canal system representation (Figure 1). This system representation matures the formulation of the quadratic optimal fuzzy control problem and a sound unification of the individual matrices into synthetic matrices to generate a linear-like global system representation of a fuzzy system, which aided in the derivation of a theoretical design scheme for the quadratic optimal fuzzy controller (Wu and Lin 2002). The considered T-S type fuzzy model for the single-pool irrigation canal is as follows:

$$\begin{aligned} R^i : \text{If } x_n \text{ is } T_{1i}, \dots, x_n \text{ is } T_{ni}, \text{ then} \\ \delta x(k+1) = \Phi_i(k)\delta x(k) + \Gamma_i(k)\delta u(k) \\ \delta y(k) = H(k)\delta x(k), \quad i = 1, \dots, r \end{aligned} \quad (22)$$

where  $R^i = i^{th}$  rule of the fuzzy model,  $x_1, \dots, x_n =$  system states,  $T_{1i}, \dots, T_{ni} =$  input fuzzy terms in the  $i^{th}$  rule,  $\delta x(k) = [x_1, \dots, x_n]^T =$  state vector,  $\delta y(k) =$  the output vector (measured variables), and  $\delta u(k) =$  system input (i.e. control output or changes in gate openings).  $\Phi_i(k)$ ,  $\Gamma_i(k)$ , and  $H(k)$  are, respectively,  $n \times n$ ,  $n \times m$ , and  $n \times n$  matrices, whose elements are real-value functions defined on non-negative real numbers,  $N$ . Throughout in this report, it is assumed that  $\Phi_i(k)$  is nonsingular for all  $k$  to ensure no deadbeat response; in that case,  $\Phi_i(k+1)$  and  $u(k)$  cannot define  $\Phi_i(k)$  uniquely, and the poles of the resultant closed-loop system are all located at zero points (Wu and Lin 2002). If the desired controller is a rule-based non-linear fuzzy controller, then the form of the equation is

$$\begin{aligned} R^i : \text{If } y_1 \text{ is } S_{1i}, \dots, x_n \text{ is } S_{ni}, \text{ then } \delta u(k) = \delta r_i(k), \\ i = 1, \dots, \lambda \end{aligned} \quad (23)$$

where  $y_1, \dots, y_n =$  elements of output vector  $\delta y(k)$ ,  $S_{1i}, \dots, S_{ni} =$  input fuzzy terms in the  $i^{th}$  control rule, and  $\delta u(k)$  or  $\delta r_i(k) =$  the control output (changes in gate opening) vector (Wu and Lin 2000). To describe a quadratic optimal fuzzy control problem for the given T-S type rule-based fuzzy system in Eq. (22) with  $\delta x(k_0) = \delta x_0$  and a rule-based non-linear fuzzy controller in Eq. (23),  $k [k_0, k_1 - 1]$ , find a control input (changes in gate opening)  $\delta u(.)$  that can minimize the quadratic cost function

$$\begin{aligned} J(\delta u(.)) = \sum_{k=k_0}^{k_1-1} [\delta x^T(k) Q_{\text{mxm}} \delta x(k) + \delta u^T(k) \delta u(k)] + \\ \delta x^T(k_1) R_{\text{mxm}} \delta x(k_1) \end{aligned} \quad (24)$$

over all possible  $\delta u(.)$  of class piecewise continuous, where  $Qx$  and  $R$  belong to symmetric positive semi-definite  $n \times n$  matrices. Since each penalty term in the performance index [Eq.( 24)] is in reference to the entire fuzzy system and the controller, it is possible to formulate the distributed fuzzy subsystems and rule-based fuzzy controller into one equation from the global concept. As such, the well-known T-S-type fuzzy model was used to obtain the system state equations as follows (Wu and Lin 2002):

$$\begin{aligned} \delta x(k+1) = \sum_{i=1}^r h_i(\delta x(k)) \Phi_i(k) \delta x(k) + \\ \sum_{i=1}^r \sum_{j=1}^{\lambda} h_i(\delta x(k)) w_j(\delta y(k)) \Gamma_i(k) r_j(k) \\ \delta y(k) = H(k) \delta x(k) \end{aligned} \quad (25)$$

and the control law is

$$\delta u(k) = \sum_{j=1}^{\lambda} w_j(\delta y(k)) r_j(k) \quad (26)$$

with  $\sum_{i=1}^r h_i(\delta x(k)) = 1$  and  $\sum_{j=1}^{\lambda} w_j(\delta y(k)) = 1$ , where  $h_i(\delta x(k))$  and  $w_j(\delta y(k))$  denote, respectively, the normalized firing strength of the  $i^{th}$  rule of the



discrete-time fuzzy model and that of the  $i^{\text{th}}$  fuzzy control rule (Wu and Lin 2002). Given the entire system state equations in Eq. (25), with the fuzzy controller law  $\delta u(k)$  in Eq. (26) and  $\delta x(k_0) = \delta x_0$ ,  $k \in [k_0, k_1 - 1]$ , find a control input law (changes in gate opening),  $\delta r_p$ ,  $i = 1, \dots, \lambda$ , to minimize the quadratic performance index

$$J(\delta u(\cdot)) = \sum_{k=k_0}^{k_1-1} [\delta x^T(k) Q_{\text{X}_{\text{Ist}}} \delta x(k) + \sum_{i=1}^{\lambda} \sum_{j=1}^1 w_j (\delta y(k)) w_j \delta y(k) \delta r_i^T(k) \delta r_j(k)] + \delta x^T(k_1) R_{\text{mxm}} \delta x(k_1) \quad (27)$$

This kind of quadratic optimal control problem is obviously still too tough to deal with; therefore, the following synthetic matrices,  $H(\delta x(k))$ ,  $W(\delta y(k))$ ,  $\Phi(k)$ ,  $\Gamma(k)$ , and  $U(k)$ , can be used (Wu and Lin 2002):

$$H(\delta x(k)) = [h_1(\delta x(k)) I_n \dots h_r(\delta x(k)) I_n] \\ W(\delta y(k)) = [w_1(\delta x(k)) I_m \dots w_l(\delta x(k)) I_m] \quad (28)$$

$$\Phi(k) = \begin{bmatrix} \Phi_1(k) \\ \vdots \\ \Phi_r(k) \end{bmatrix} \Gamma(k) = \begin{bmatrix} \Gamma_1(k) \\ \vdots \\ \Gamma_r(k) \end{bmatrix} \Phi(k) = \begin{bmatrix} \delta_1(k) \\ \vdots \\ \delta_r(k) \end{bmatrix}$$

where  $I_n$  and  $I_m$  denote the identity matrices of dimension  $n$  and  $m$ , respectively. Based on these synthetic matrices, Eqs. (25) and (28) can be rewritten as follows:

$$\delta x(k+1) = H(\delta x(k)) \Phi(k) \delta x(k) + H(\delta x(k)) \Gamma(k) W(\delta y(k)) U(k), \delta y(k) = H(k) \delta x(k) \quad (29)$$

With  $\delta x(k_0) = \delta x_0$ , find the optimal control law (changes in gate opening),  $U(\cdot)$ , to minimize the quadratic performance index

$$J(U(\cdot)) = \sum_{k=k_0}^{k_1-1} [\delta x^T(k) Q_{\text{X}_{\text{mxn}}} \delta x(k) + U^T(k) W^T(\delta y(k)) W(\delta y(k)) U(k)] + \delta x^T(k_1) R_{\text{mxm}} \delta x(k_1) \quad (30)$$

Eqs. (29) and (30) represent the entire T-S-type fuzzy system that materializes the design of the global optimal fuzzy controller in the way of the general linear quadratic (LQ) approach (Figure 2). It is necessary for the process of integrating all distributed fuzzy subsystems into one equation to describe the entire fuzzy system in order to determine the global optimal controller. Eq. (25) provides a practical way to work out the global optimal solution; however, even though each fuzzy subsystem in the T-S model is linear, Eq. (25) is complicated and non-linear. Therefore, the synthetic form of system state equation [Eq. (29)] is lower down the order and adds to the difficulty of the problem (Wu and Lin 2002). Now, a discrete-time optimal fuzzy controller for a single-pool irrigation canal can be described using Eqs. (29) and (30). For each segmental dynamic fuzzy system

$$\delta x(k+1) = H_i \Phi(k) \delta x(k) + H_i W_i \Gamma(k) U(k), \\ \delta y(k) = H \delta x(k) \quad (31)$$

where  $H_i = i^{\text{th}}$  stage of  $H$ ,  $W_i = i^{\text{th}}$  stage of  $W$ , and  $\delta x(k_0^i)$  is known. Then there is a unique  $n \times n$  symmetric positive semi-definite solution ( $P$ ) of the discrete time algebraic Riccati-like equation

$$K = -W_i^T [W_i W_i^T]^{-1} \Gamma^T H_i^T P [I_n + H_i \Gamma^T H_i^T P]^{-1} H_i \Phi \quad (32)$$

then the optimal control law will be  $\delta u(k) = K \delta x(k)$ , more clearly

$$\delta u(k) = -W_i^T [W_i W_i^T]^{-1} \Gamma^T H_i^T P [I_n + H_i \Gamma^T H_i^T P]^{-1} H_i \Phi \delta x(k) \quad k \in [k_0, \infty) \quad (33)$$

Eq. (33) will minimize the quadratic performance index

$$J(U(\cdot)) = \sum_{k=k_0}^{\infty} [\delta x^T(k) Q_{\text{X}_{\text{Ist}}} \delta x(k) + U^T(k) W_i^T W_i U(k)] \quad (34)$$

The first term in Eq. (34) represents the penalty on the deviation of the state variables (flow depth and flow rates) from the average operation condition, whereas the second term represents the cost of control. Eqs. (34) and (33) constitute a discrete-time optimal fuzzy control problem (a constrained-minimization problem) that can be solved using the calculus of variations methods combined with the Lagrange multiplier method to obtain the necessary and sufficient condition for global optimum (Wu and Lin 2002). This produces a set of coupled difference equations that must be solved recursively, backward in time. This procedure yields a time-varying controller gain matrix ( $K$ ), which is defined by Eq. (32). A commonly employed numerical approach to finding the solution to Eq. (32) is by defining a Hamiltonian matrix ( $H$ ), as follows (Tewari 2002):

$$\begin{bmatrix} \delta x(k+1) \\ P(k+1) \end{bmatrix} = X \begin{bmatrix} \delta x(k) \\ P(k) \end{bmatrix} \quad (35)$$

$$X = \begin{bmatrix} H_i \Phi - H_i \Gamma \Gamma^T [\Phi^T H_i^T Q - H_i \Gamma \Gamma^T H_i^T [\Phi^T H_i^T]^{-1}] & -[\Phi^T H_i^T]^{-1} Q \\ -[\Phi^T H_i^T]^{-1} Q & -[\Phi^T H_i^T]^{-1} \end{bmatrix}$$

Of course, the definition of the Hamiltonian matrix by Eq. (35) requires that  $\Phi$  be non-singular. Finding the eigenvectors of the inverse of the Hamiltonian matrix helps obtain the steady state elements of the controller gain matrix ( $K$ ) (Reddy 1999). Once the equations of the discrete-fuzzy

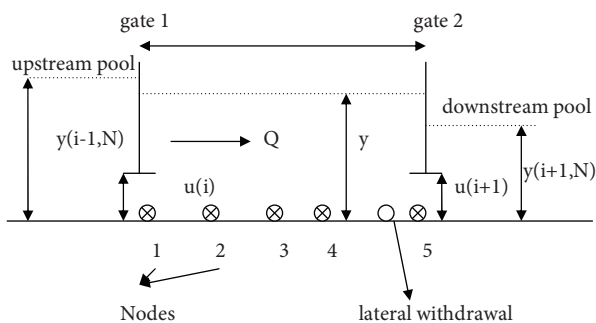


Figure 1. Schematic of an irrigation canal pool.

optimal controller are obtained and measured values for one or more state variables in a given pool are available, the dynamics of the linear system can be simulated for any arbitrarily selected values of external disturbance. In the present study, a single reach of a canal was considered. This model predicts the flow rate,  $Q(x,t)$ , and the depth of flow,  $y(x,t)$ , given the initial boundary conditions. The fuzzy optimal controller equations were added as subroutines to this program. The controller's input will be the water level error (desired water level minus actual water level) and its output will be the rate at which the gate is opened or closed. A first pass at writing a fuzzy controller for this system is as follows:

1. If (water level at downstream is okay) then (upstream gate is unchanged)
2. If (water level at downstream is lower than target depth) then (upstream gate is opened rapidly)
3. If (water level at downstream is higher than target depth) then (upstream gate is closed rapidly)

These 3 rules are not sufficient, as the water level tends to oscillate around the desired level; therefore, another input, the water level's rate of change, should be added to slow down the gate movement when the water level gets close to the right level.

4. If (water level at downstream is higher than target depth) and (variation in water level rate is positive), then (upstream gate is closed slowly)
5. If (water level at downstream is lower than target depth) and (variation in water level rate is negative), then (upstream gate is opened slowly)

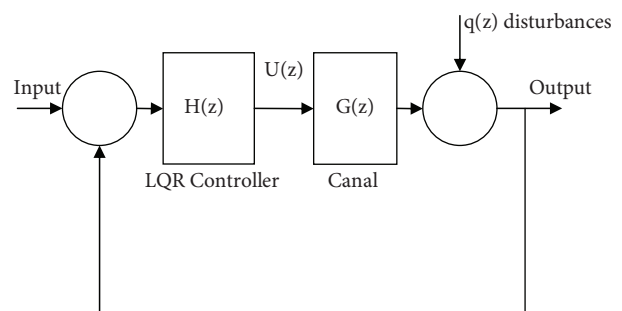


Figure 2. A feedback control system scheme.

These if-then rule statements are used to formulate the conditional statements that comprise fuzzy logic (Wu and Lin 2002). In other words, to minimize the quadratic performance index [Eq. (34)] the best optimal control law [Eq. (33)] should be obtained by finding the controller gain matrix [Eq. (32)]. In those if-then rules, “water level is higher/lower than target depth” and “variations in water level rate is positive” are called the antecedent or premise, while “upstream gate is opened slowly” is called the consequence or conclusion. Note that “target depth” and “positive” are represented as a number between 0 and 1, and so the antecedents are interpretations that return a single number between 0 and 1. On the other hand, “opened slowly” is represented as a fuzzy set, and so the consequence is an assignment that assigns the entire fuzzy set “opened slowly” to the output variable gate opening. In general, the input to an if-then rule is the current value for the input variable (in this case, “water level at downstream” and “variation in water level”) and the output is an entire fuzzy set (in this case, “upstream gate opening fast/slow”). Given the initial flow rate and the target depth at the downstream end of the pool, the model computed the backwater surface elevation. Later, the downstream flow requirement and the withdrawal rate into the lateral were provided as a boundary condition. Known state variables (flow depths and flow rates) were used in the controller subroutine to compute the change in the upstream gate opening in order to bring the depth at the downstream end of the pool close to the target depth. Based upon this gate opening, the new flow rate into the pool at the upstream end was calculated and used as the boundary condition at the upstream end of the pool. This process was repeated during the entire simulation period (7000 s).

## Results

To demonstrate and compare the feasibility of the linear-like fuzzy controller, an optimal regulation problem for a discrete-time single pool irrigation canal was simulated. Using 5 nodes in the pool, a state-variable model with 8 variables was formulated. The state variable equation was supplemented with an output equation, in which the output variables were the flow depths at the nodes of the pool. The variation in the depth of flow at the downstream end of the pool

was the controlled variable, and the control objective was to maintain the flow depth at the downstream end of the pool at a constant in the presence of random disturbance actions on the system. An example problem obtained from Reddy (1990) was used, the data of which was as follows: length of canal reach = 5000 m, number of nodes = 5, number of sub-reaches used = 4,  $\Delta x = 1250$  m, channel slope = 0.0003, side slope = 1.0, bottom width = 1.7 m, turnout demand =  $2.5 \text{ m}^3 \text{ s}^{-1}$ , discharge required at the end of the canal =  $0.52 \text{ m}^3 \text{ s}^{-1}$ , upstream reservoir elevation = 103.2 m, downstream reservoir elevation = 101.14 m, target depth at the downstream end = 1.2 m, gate width = 1.7 m, and gate discharge coefficient = 0.75. These data were used to calculate the steady state values, which in turn were used to compute the initial gate openings and the elements of the  $\Phi$ ,  $I$ , and  $H$  matrices [Eq. (12)], using a sampling interval of 30 s.

The initial gate opening was 0.59 m and 0.37 m, respectively, for the gates upstream and downstream of the given pool. Figure 3 illustrates variations in flow depth in the pool for both the linear-like fuzzy quadratic controller and a standard linear quadratic regulator (LQR). The system response was simulated for an unknown disturbance of  $+0.50 \text{ m}^3 \text{ s}^{-1}$ . The positive sign indicates an increase in the withdrawal rate from the turnouts. At the beginning of the simulation, there were oscillations in the depth of flow at node 1. Later, with the introduction of water release, the flow depth gradually increased and the variations in flow depth approached a maximum deviation of 0.0340 m for the standard LQR and 0.0285 m for the linear-like fuzzy controller at 5000 s. After 5000 s, variations in flow depth at node 1 reached a constant level for both controllers.

It is obvious that at node 1 the variations in flow depth for the fuzzy optimal controller were less than those for the standard LQR controller. At node 2 the depth of flow had some oscillatory behavior at first, because of downstream demand. After the introduction of water release the flow depth increased gradually, and the deviations in flow depth reached a constant value of 0.0150 m for the linear-like fuzzy controller and 0.0175 m for the standard LQR at the end of the simulation. At node 3 the variations in flow depth had some oscillations at the beginning of the simulation, and reached a value of  $-0.02$  m for the

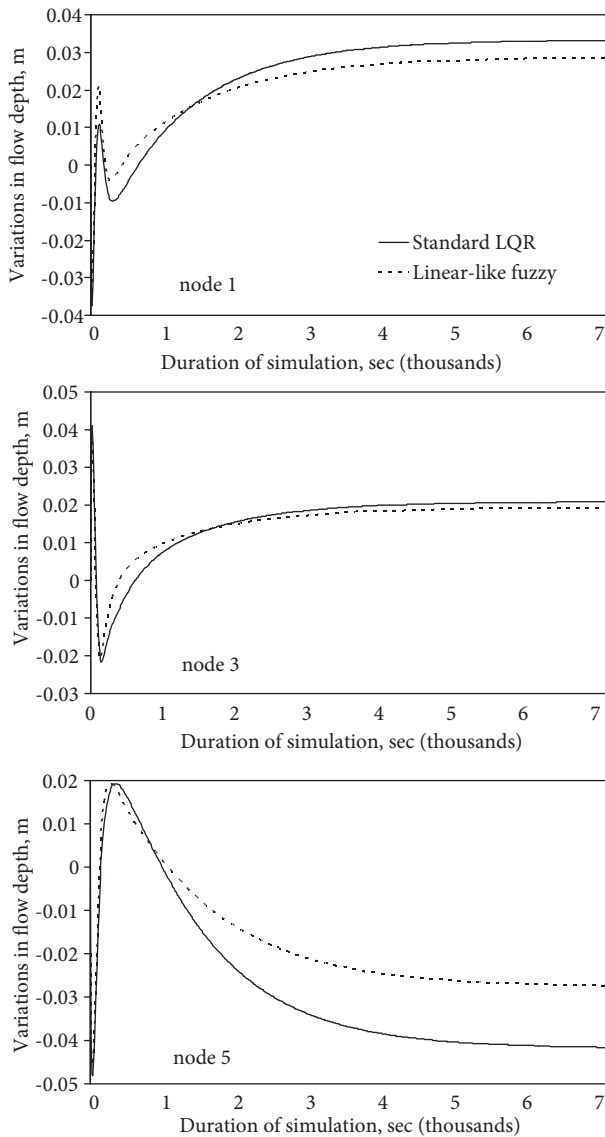


Figure 3. Flow depth variations for standard LQR and optimal fuzzy controllers.

linear-like fuzzy controller and  $-0.021$  m for the standard LQR. With the release of water from upstream of the pool, the depth of flow gradually increased, and the deviations approached a constant value of  $0.019$  m for the fuzzy controller and  $0.021$  m for the standard LQR controller. At node 4, following initial oscillatory behavior the variations in flow depth gradually decreased, and reached  $0.015$  m for the linear-like fuzzy controller and  $0.018$  m for the standard LQR controller at the end of the simulation. The variations in depth of flow at node 5, in other

words at the downstream end of the pool, gradually increased at the beginning of the disturbance period, and finally approached a negative deviation of  $-0.041$  m for the standard LQR and  $-0.027$  m for the linear-like fuzzy controller at the end of the simulation. The negative sign indicates that there was a decrease in flow depth at node 5 because of the demand at the downstream end.

Figure 4 shows that the incremental gate opening was lower for the fuzzy optimal controller than for the standard LQR controller; in other words, gate movement with the fuzzy optimal controller was more stable. At the beginning of the simulation the deviations in gate opening reached a small negative value of  $-9.5 \times 10^{-3}$  m for the standard LQR, versus  $-3 \times 10^{-3}$  for the linear-like fuzzy controller. After 4500 s the incremental gate opening reached an equilibrium position for both controllers. Figure 5 indicates that to bring the initially disturbed system

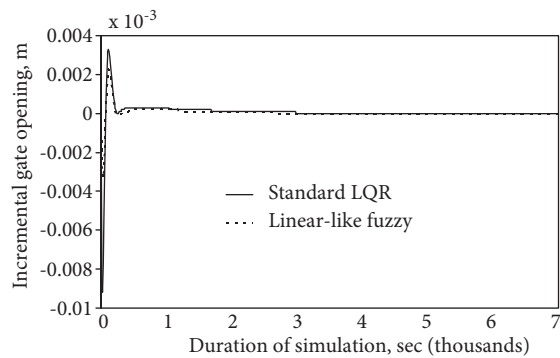


Figure 4. Incremental gate opening for standard LQR and fuzzy optimal controllers.

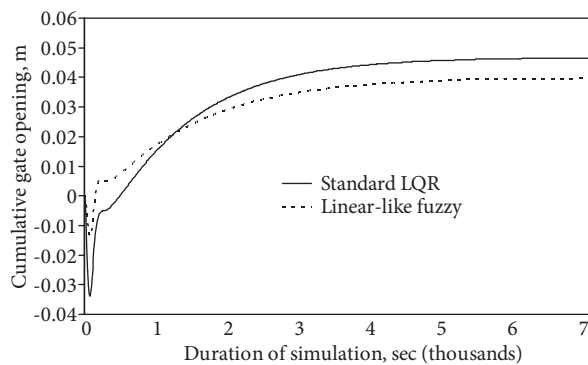


Figure 5. Cumulative gate opening for standard LQR and fuzzy optimal controllers.

to an equilibrium position, the cumulative gate opening reached 0.048 m for the standard LQR and 0.040 m for the fuzzy optimal controller at the end of the simulation. Lastly, the final gate openings for both the linear-like fuzzy controller and the standard LQR controller reached a final value of 0.63 m and 0.638 m, respectively (Figure 6).

**Discussion**

The present study used 1 control variable (the upstream gate) per pool and only 1 variable (either the volume of water in the pool or the depth of flow at any one point in the pool) was maintained at the target value (Kwakernaak and Sivan 1972). Therefore, the upstream end gate alone was used for constant level control by freezing the opening of the downstream end gate at its initial position. After computing steady state values, the control algorithm, written with MATLAB (1992), formulated a standard LQR controller and the results were obtained. Later, the algorithm designed a linear-like fuzzy controller and we compared its results to those obtained with the LQR controller.

The analysis began by evaluating system stability. All the eigenvalues of the feedback matrix were positive and had values less than one. The system was also both controllable and observable. In the derivation of the control matrix elements ( $T$ ) it was assumed that both the upstream and downstream gates of each reach could be manipulated to control

the system dynamics. The downstream end gate position was frozen at the original steady state value, and only the upstream end gate of the given reach was controlled to maintain the system at the equilibrium condition. The effect of variations in the opening of the downstream gate must be taken into account through real-time feedback of the actual depths immediately upstream and downstream of the downstream gate (node  $N$ ). In the derivation of the feedback gain matrix ( $K$ ),  $R$  was set equal to 1000, whereas  $Qx$  was set equal to an identity matrix of dimension 8 (the dimensions of the system). In the absence of a well-defined procedure for selecting the elements of these matrices, these values were selected based upon trial and error.

The downstream end of the pool (node 5) illustrated best that the linear-like fuzzy controller resulted in less deviation in flow depth than did the standard LQR controller. As additional water that was released into the pool reached the downstream end, the depth of flow gradually returned to the steady level, i.e. the depth at the downstream end of the pool was maintained constant. In all the nodes considered maximum deviation in depth of flow occurred at the first and last nodes of the reach. Because the turnouts were located at the downstream end of the reach, as the flow rate into the lateral increased, the depth of flow at the downstream end decreased rapidly. Conversely, maximum increase in depth of flow occurred at the upstream end of the reach. This was due to the increased opening of the upstream gate and compensated for the disturbances at the downstream end or turnout. At all the nodes variations in flow depth obtained with the optimal fuzzy controller were less than those obtained with the standard LQR controller. Thus, the fuzzy optimal controller provided better stability for the controlled single-pool irrigation canal.

Considering the position of the upstream gate, which was close to the equilibrium value at the end of the simulation period, it is evident that the system would eventually return to the equilibrium condition. During the simulation use of the fuzzy optimal controller resulted in less up and down movement of

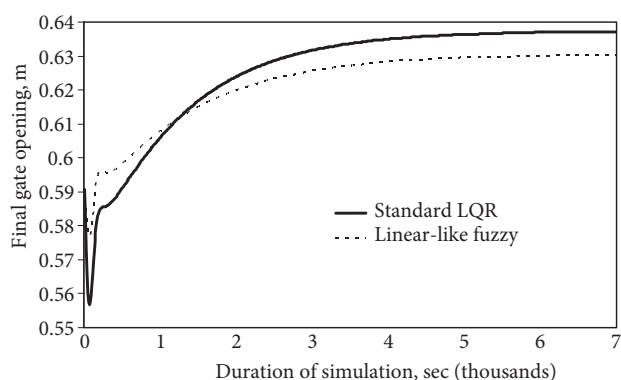


Figure 6. Final gate opening for standard LQR and fuzzy optimal controllers

the gate, and provided enough water for the downstream end of the canal reach. In the absence of any other disturbances (changes in withdrawal rates) the gate will return to its equilibrium position at steady state. Once again, the performance of both controllers was evaluated, assuming that values for all the state variables (8 in this case) were available. Variation in upstream gate opening and deviation in the depth of flow at the downstream end of the pool were satisfactory (Figures 3 and 5). Obviously, there were some differences between the linear-like fuzzy and standard LQR controllers in terms of gate opening and flow depth. Both models maintained the downstream depth at the target value; however, application of the linear-like fuzzy model algorithm maintained the downstream depth at the target value with less deviation in gate opening and flow depth. The overall results of this study show that the proposed linear-like fuzzy controller provides better stability and offers an efficient alternative to a traditional LQR controller when dealing with uncertainty (increase in withdrawal rate from the turnout). Based on the results of the simulation model, it is clear that the optimal fuzzy controller algorithm is more suitable than the traditional LQR method for the regulation of irrigation canals.

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## Conclusions

A quadratic optimal fuzzy control problem was formulated for constant-level control of irrigation canals. A T-S-type fuzzy system that materialized the design of a fuzzy controller based on the way of general linear quadratic (LQ) approach was considered. Individual matrices were unified into synthetic matrices to generate a linear-like global discrete-time fuzzy system. For minimizing the quadratic performance index, the discrete-time fuzzy control law was shown to be the best for the single-pool system. The performance of the linear-like fuzzy controller was compared with that of a standard LQR controller, in terms of variations in the depth of flow and the upstream gate opening. The linear-like fuzzy controller provided both good stability and performance under unknown withdrawals from the irrigation canal. Overall, the performance of the linear-like fuzzy control technique for constant-level control was better than that of the full-state feedback LQR controller (assuming all the state variables in the system were measured). In the present study it was assumed that all the state variables were measured in the canal. As it is very expensive to measure all flow depths and flow rates for each node, an optimal estimator should be used in the design of a regulator and fuzzy controller in any subsequent research.

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