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Abstract

We study the classical dynamics of a bosonic string in the Schwarzschild spacetime using a perturbative scheme which is based on the assumption of a small value of a rescaled string tension parameter. The proposed approximation selfconsistently describes the string dynamics on the scale of large values for the worldsheet time in a fixed gauge.

Key Words: string, perturbation theory, Schwarzschild black hole

1. Introduction

Cosmic strings produced in an early universe phase transition have been proposed as an explanative of density fluctuations and the origin of galaxies [1, 2]. Their large mass per unit length μ makes loops of string very attractive as gravitational sources, yet comparatively little is known about how gravitational fields affect their motion.

Galaxy formation scenarios which involve strings typically require $G\mu$ on the order of 10^{-6} [3, 4]. Their tension, which is also μ in units with $c = 1$, causes the mean curvature of a network of strings to decrease with time. Loops form when a string intersects itself and the ends exchange partners. These oscillating loops interact gravitationally with surrounding material and act as seeds for density enhancements which grow into galaxies, clusters or even, in extreme cases, into black holes [5].

The classical evolution of strings in curved backgrounds is described by a complicated system of second-order non-linear coupled partial differential equations which is integrable only for some special configurations [6-9]. A vast simplification of the equations of motion arises when one neglects string tension and considers the null (tensionless) strings [10]. Their equations of motion are null geodesic equations of General Relativity appended by an additional "stringy" constraint. The exact null string configurations have been studied in Schwarzschild and cosmological spacetimes [11, 12].

However, the important physical information about tensile string dynamics can be obtained from studying the approximate solutions of its equations of motion as it was proposed in [13-15], where a perturbative scheme was considered, based on the assumption of a small value of a rescaled string tension parameter.

The objective of this paper is to apply the expansion scheme as proposed in [13-15] for studing string dynamics in Schwarzschild spacetime.

2. Rescaled Tension as a Perturbation

In this section we shortly discuses the main points of the perturbation scheme of [13-15]. The basic idea is to use for the string action a generalization of the action given for a massless point particle. We assume a perturbative parameter $\varepsilon = G\mu << 1$. It was shown in [13-15] that for the case of small $\varepsilon << 1$ one can introduce a macroscopic "slow" worldsheet time parameter

$$T = \varepsilon\tau, \tag{1}$$

where τ is the proper string time parameter. On the scale of T the string oscillations can be considered as perturbations with respect to the translational motion of the string points and described in the form of an asymptotic expansion

$$x^\mu(T,\sigma) = \varphi^\mu(T) + \varepsilon\psi^\mu(T,\sigma) + \varepsilon^2\chi^\mu(T,\sigma) + \dots, \tag{2}$$

$$\mu = 0, 1, 2, 3$$

with σ being a spacelike worldsheet string coordinate. After introducing the expansion (2) the perturbative equations of motions and constraints in the first approximation have the form [15]

$$\left(D_T^2 - \partial_\sigma^2\right)\psi^\mu + R^\mu_{\nu\rho\kappa}(\varphi)\varphi^\nu_{,T}\varphi^\rho_{,T}\psi^\kappa = 0, \tag{3}$$

$$\varphi_{\mu,T} D_T\psi^\mu = 0, \tag{4}$$

$$\varphi_{\mu,T}\psi^\mu = 0, \tag{5}$$

where $D_T\psi^\mu = \psi^\mu_{,T} + \varphi^\nu_{,T}\Gamma^\mu_{\nu\kappa}(\varphi)\psi^\kappa$, $R^\mu_{\nu\rho\kappa}$ is the Riemann tensor and $(...)_{,T} = \partial/\partial T$. Equations. (3) are of the form of the geodesic deviation equation with an additional term $\partial_\sigma^2\psi^\mu$ describing the elastic string force. The zeroth order equations for $\varphi^\mu(T)$ are geodesic equations for a massless particle in a given curved space, i.e.

$$D_T\varphi^\mu_{,T} = 0, \tag{6}$$

$$\varphi^\mu_{,T}\varphi_{\mu,T} = 0. \tag{7}$$

3. Solution of the Perturbative Equations in Schwarzschild Spacetime

Here we consider the application of the above considered realization of perturbative approach for the solution of the string equations in Schwarzschild spacetime with the element

$$dS^2 = \left(1 - \frac{2GM}{r})dt^2 - \frac{dr^2}{1-\frac{2GM}{r}} - r^2(d\theta^2 + \sin^2\theta \cdot d\varphi^2\right), \tag{8}$$

where G is Newton constant with dimension of L^2 and M is the Schwarzschild mass with the dimension of $L^{-1}(\hbar = c = 1)$.

The general solution of the geodesic equations for a massless particle in Schwarzschild background is well-know. Let's consider a radial motion. In this special case the orbital shape is simply a radial line, moving away from or toward the center:

$$\varphi^0(T) = \varphi^0(0) + ET \pm 2MG\ln\left[1 \pm \frac{ET}{\varphi^1(0) - 2MG}\right], \tag{9}$$

$$\varphi^1(T) = \varphi^1(0) \pm ET, \qquad \varphi^2(T) = \frac{\pi}{2}, \qquad \varphi^3(T) = \varphi^3(T),$$

where $\varphi^i(0) \quad (i = 1, 2, 3)$ and E are the initial data.

Using (8) and the solutions (9) we find that Equations. (3)-(5) are transformed into the following:

$$\psi^1_{,TT} - \psi^1_{,\sigma\sigma} = 0,$$

$$\psi^K_{,TT} - \psi^K_{,\sigma\sigma} \pm \frac{2E}{\varphi^1(0) \pm ET}\psi^K_{,T} = 0, \qquad k = 2, 3, \tag{10}$$

$$\psi^0 = \pm\frac{\psi^1}{1 - \frac{2MG}{\varphi^1(0) \pm ET}}.$$

The solutions for these components are

$$\psi^0(T,\sigma) = \pm\frac{1}{1 - \frac{2MG}{\varphi^1(0) \pm ET}}\left[A^1 + \sum_{n\neq 0}\left(\alpha^1_n e^{in(T-\sigma)} + \beta^1_n e^{-in(T-\sigma)}\right)\right],$$

$$\psi^1(T,\sigma) = A^1 + \sum_{n\neq 0}\left(\alpha^1_n e^{in(T-\sigma)} + \beta^1_n e^{-in(T-\sigma)}\right), \tag{11}$$

$$\begin{aligned}\psi^k(T,\sigma) &= A^k \mp \frac{B^k}{E(\varphi^1(0) \pm ET)} + \\ &+ \frac{1}{\varphi^1(0) \pm ET}\sum_{n\neq 0}\left(\alpha^k_n e^{\frac{in}{E}(\varphi^1(0) \pm ET + E\sigma)} + \beta^k_n e^{-\frac{in}{E}(\varphi^1(0) \pm ET - E\sigma)}\right), \\ k &= 2, 3.\end{aligned}$$

Substituting the solution (9) and (11) into the expansion (2) we find

$$\begin{aligned}t(T,\sigma) &= \varphi^0(0) + ET \pm 2MG\ln\left[1 \pm \frac{ET}{\varphi^1(0) - 2MG}\right] \pm \\ &\pm \frac{\varepsilon A^1}{1 - \frac{2MG}{\varphi^1(0) \pm ET}} \pm \varepsilon\sum_{n\neq 0}\left(\alpha^1_n e^{in(T-\sigma)} + \beta^1_n e^{-in(T-\sigma)}\right) + O(\varepsilon^2),\end{aligned} \tag{12}$$

$$r(T,\sigma) = \left(\varphi^1(0) + \varepsilon A^1\right) \pm ET \pm \varepsilon\sum_{n\neq 0}\left(\alpha^1_n e^{in(T-\sigma)} + \beta^1_n e^{-in(T-\sigma)}\right) + O(\varepsilon^2), \tag{13}$$

$$\begin{aligned}\theta(T,\sigma) &= \left(\frac{\pi}{2} + \varepsilon A^2\right) \mp \frac{\varepsilon B^2}{E(\varphi^1(0) \pm ET)} + \\ &+ \frac{\varepsilon}{\varphi^1(0) \pm ET}\sum_{n\neq 0}\left(\alpha^2_n e^{\frac{in}{E}(\varphi^1(0) \pm ET + E\sigma)} + \beta^2_n e^{-\frac{in}{E}(\varphi^1(0) \pm ET - E\sigma)}\right) + \\ &+ O(\varepsilon^2),\end{aligned} \tag{14}$$

$$\begin{aligned}\varphi(T,\sigma) &= \left(\varphi^3(0) + \varepsilon A^3\right) \mp \frac{\varepsilon B^3}{E(\varphi^1(0) \pm ET)} + \\ &+ \frac{\varepsilon}{\varphi^1(0) \pm ET}\sum_{n\neq 0}\left(\alpha^3_n e^{\frac{in}{E}(\varphi^1(0) \pm ET + E\sigma)} + \beta^3_n e^{-\frac{in}{E}(\varphi^1(0) \pm ET - E\sigma)}\right) + \\ &+ O(\varepsilon^2).\end{aligned} \tag{15}$$

As follows from the representations (11), the perturbative corrections are connected with string oscillations in the directions ortogonal and tangent to the geodesic trajectory (9) of the zero approximation. The amplitudes of these oscillations are asymptotically small where $T >> 1$ and the amplitudes of the longitudinal oscillations are essentially smaller than the amplitudes of the transversional oscillations.

4. Conclusion

Here we discuss the problem of approximate solution of the string equation in Schwarzschild spacetime. It is shown that the perturbative string equations for the Schwarzschild spacetime are reduced to the linear system of the exactly solvable modified Bessel equations. The proposed approximation selfconsistently describes the string dynamics on the scale of large values for the worldsheet time in the fixed gauge. The asymptotic non-trivial string motion has the character of damped oscillations with the amplitudes falling as a power of the slow worldsheet time. An interesting peculiarity of this perturbative description is the asymptotic stability of the string dynamics in the Schwarzschild background.

The string motion in a Schwarzschild background was previously considered also in the paper [17] and found to be unstable in some particular appropriate regime. This result was based on the paper [6]. The base idea of the paper [6] is an expansion of string solutions around the geodesic line of the string mass center described by a mass parameter m. An extensive application of this perturbative approach necessitates its further investigations. In particular, the nature of a small perturbative parameter and the procedure of its bringing into the string equations and constraints are important points for study. Moreover, it is appropriate to find a mechanism for fixing arbitrariness in the choice of the phenomenological mass parameter m, to define a relevant scale for measuring the world-sheet parameter τ and σ which are the arguments of perturbative functions. In principle a well defined mass parameter attributed to the center mass trajectory may be absent. While studying this matter, a new representation for the string action including kinetic and potential terms of the string Lagrangian as independent additive terms was considered in [13]. The perturbative string equations [13], were shown to be transformed into the perturbative equations [6] after rescaling the world-sheet parameter σ (or τ) and fixing the phenomenological mass parameter by the value $m = 0$. These results point out to the existence of different realizations of the considered perturbative approach. So it becomes important to establish the regions for applicability of the different realizations and to understand the physical effects connected with them. In particular, it may occur that the perturbative string dynamics critically depends on the value of the phenomenological mass parameter m for some type of the curved space-time. The de Sitter space just belongs to this case. Actually, as shown in [6, 7], the perturbative string frequency modes in the de Sitter space are defined as $\omega_n = \sqrt{n^2 - (\alpha' Hm)^2}$ and become imaginary for large value of the Hubble constant H. This results in instabilities of the string dynamics in the realization of the perturbative approach considered in [6, 7]. It follows from the above formula for ω_n that the instabilities must disappear, acquires zero value. This value is in exact accordance with the restriction of the perturbative scheme realized in [13]. Therefore it seems important to present a rigorous verification of the absent of instabilities in the realization of the perturbative approach proposed in [13], as well as to develop and substantiate this perturbative scheme itself.

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References

[1] T.W.B. Kibble, *J. Phys.,* **A9**, (1976) 1387.

[2] Ya.B. Zel'dovich, *Mon. Not. Roy. Astro. Soc.,* **192,** (1980) 663.

[3] N. Turok, R. Branderberger, *Phys. Rev.,* **D33,** (1986) 663.

[4] H. Sato, *Mod. Phys. Lett.,* **A1,** (1986) 9.

[5] C.J. Hogan, *Phys. Lett.,* **143B,** (1984) 87.

[6] H.J. de Vega, N. Sanchez, *Phys. Lett.,* **B197,** (1987) 320.

[7] N. Sanchez, G. Veneziano, *Nucl. Phys.,* **B333,** (1990) 253.

[8] M. Gasperini, N. Sanchez and G. Veneziano, *Nucl. Phys.,* **B364,** (1991) 265.

[9] M. Gasperini, G. Veneziano, *Phys. Rev.,* **D50,** (1994) 2519.

[10] I.A. Bandos, A.A. Zheltukhin, *Fortschr. Phys.,* **41,** (1993) 612.

[11] M.P. Dobrowski, A.L. Larsen, *Null string in Schwarzschild spacetime*, hep-th/9610243.

[12] S.N. Roshchupkin, A.A. Zheltukhin, *Class. Quant. Grav.,* **12**, (1995) 2519.

[13] A.A. Zheltukhin, *Class. Quant. Grav.,* **12**, (1996) 2357.

[14] A.A. Zheltukhin, S.N. Roshchupkin, *Sov. J. Teor. Mat. Fiz.,* **111**, (1997) 402.

[15] S.N. Roshchupkin, *A.A. Zheltukhin, Nucl. Phys.,* **B543,** (1999) 365.

[16] W. Rindler, Essential. *Relativity, New York: Van Nostrand*, 1969.

[17] M. Gasperini, *Phys. Lett.* **258B** (1991), 70.