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A novel optimization method for solving constrained and unconstrained problems: modified Golden Sine Algorithm

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Abstract: Recently, the metaheuristic optimization algorithms inspired by nature and different science branches have been powerful solution methods for unconstrained, constrained, and engineering problems. Various metaheuristic optimization algorithms have been proposed and they have been applied to problems in different fields. This paper proposes a novel optimization method based on a modified version of the Golden Sine Algorithm for solving unconstrained, constrained, and engineering problems. The basic idea behind the proposed modified Golden Sine Algorithm (GoldSA-II) depends on finding the optimum solution field in search space by using the decreasing pattern of the sine function and the golden ratio. The performance of the proposed GoldSA-II is evaluated using 19 unconstrained benchmark functions, five constrained optimization test problems, and five real engineering design problems. The results of the proposed GoldSA-II are compared with best-known optimization algorithms using some well-known criteria. The obtained results show that the GoldSA-II converges more accurately to the global solution in many benchmark functions.

Key words: Benchmark testing, Golden Sine Algorithm, metaheuristic algorithms, optimization methods

1. Introduction

Metaheuristic algorithms have proved their potential for finding optimal solutions to real-life problems when classical methods may not achieve the optimal solution within an acceptable computation time, especially when there is a global minimum surrounded by a large number of local minima [1]. Therefore, metaheuristic algorithms are widely used to find the global optimum of real engineering problems. The disadvantages of existing numerical methods, such as simplicity, efficiency, and accuracy, encourage researchers to rely on metaheuristic algorithms based on methods that are inspired by nature or different branches of science to solve engineering optimization problems.

In many complex engineering problems, if there is more than one local optimum, the optimal solution obtained based on the choice of the starting point may not be global optimal [1]. The main reasons for the widespread use of general-purpose heuristics are local optimality avoidance, code simplicity, feasibility, flexibility, robustness, simplicity, analyticity, and derivation [2]. Metaheuristic algorithms usually try to find the right solution by combining the rules imitating natural phenomena and randomness [3]. Another advantage of metaheuristic algorithms is that they are not dependent on the problem. Therefore, these algorithms are general-purpose methods used to solve all kinds of problems, unconstrained or constrained. On the other hand, the accuracy of the results can compromise as much as the minimum error value [4]. Different science branches or their combination inspire general-purpose metaheuristic methods.

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The branches of science that are based on biology, physics, sociology, chemistry, and mathematics inspire the metaheuristic methods. The biological and swarm-based optimization methods are inspired by some successful features such as natural selection process, propagation, nutrition search, and hunting [5]. The most common algorithms in this group are Genetic Algorithm (GA) [6], Differential Evolution (DE) [7], Eco-Inspired Evolutionary Algorithm (ECO) [8], Particle Swarm Optimization (PSO) [9], Convergent Heterogeneous Particle Swarm Optimization (CHPSO) [10], Symbiosis Organisms Search (SOS) algorithm [11], Ant Colony Optimization (ACO) [12], Wolf Search Algorithm (WSA) [13], Firefly Algorithm (FA) [14], and Whale Optimization Algorithm (WOA) [15]. Physics- and chemistry-based optimization methods are inspired by physics and chemistry, include electrical loads, gravity, river systems, perfect harmony, etc., and imitate certain physical and chemical laws. The most common algorithms in this group can be summarized as Artificial Chemical Reaction Optimization Algorithm (ACROA) [16], Electro-Magnetism Optimization (EMO) [17], Gravitational Search Algorithm (GSA) [18], Harmony Search Algorithm (HS) [19], and Water Cycle Algorithm (WCA) [1]. Metaheuristic algorithms based on sociology are inspired by social and competitive behavior of human beings. The most popular algorithms of this group are Anarchic Society Optimization (ASO) [20], Imperialist Competitive Algorithm (ICA) [21], Social-Based Algorithm (SBA) [22], League Championship Algorithm (LCA) [23], and Soccer League Competition Algorithm (SLC) [24]. Mathematical-based optimization algorithms are performed using the combination of metaheuristic and mathematical programming techniques. There are a few algorithms in this field. Base Optimization Algorithm (BaOA) [25], Sine Cosine Algorithm (SCA) [26], and Golden Sine Algorithm (Gold-SA) [27] can be given as examples. Although many optimization algorithms have been presented in the literature, none of the developed optimization algorithms may find an optimum solution for all problems. Therefore, existing algorithms have developed or new algorithms have been proposed in the archival literature.

In the present paper, a novel optimization method (modified Golden Sine Algorithm - GoldSA-II) is proposed. Although the Gold-SA presented by [27] found optimum solutions for unconstrained benchmark functions, it could not show the same success in many constrained problems due to its finding local optimums instead of global optimums. In the proposed optimization method, this problem is eliminated by using a decreased pattern of the sine function and the golden ratio. The GoldSA-II improves the searching facilities to find the global solution. To evaluate the performance of the GoldSA-II, it is applied to unconstrained benchmark functions, constrained test problems, and engineering design problems. The results obtained from the proposed algorithm are compared with those of several well-known optimization algorithms as well as the Gold-SA by means of some performance criteria. It is seen that the proposed GoldSA-II gives satisfactory results.

2. The modified Golden Sine Algorithm

In the literature, population-based algorithms are widely used because of preventing local optimality, exploring the search area, and being more accurate and reliable when compared with global optimum individual-based algorithms [26]. It still has unresolved problems or the idea that earlier problems can be solved better with new algorithms. The GoldSA-II has been developed to eliminate some drawbacks of the Gold-SA. The basic idea of the Gold-SA and GoldSA-II optimization algorithms is the sine function and the golden ratio.

The scan of the unit circle for all values of the sine function is similar to the search space in optimization problems [27]. A trigonometric sine function and its phasor representation can be depicted in Figure 1. When this vector rotates in the anti-clockwise direction, its endpoint will rotate one complete revolution of

2π representing one complete cycle. In the Gold-SA, the entire unit circle can be scanned with a constant radius value and a constant frequency obtained from the sine function. Therefore, the Gold-SA cannot provide a good solution for constrained problems. The reason for this case is that the Gold-SA finds local minima since it cannot meet the constraints depending on objective functions.

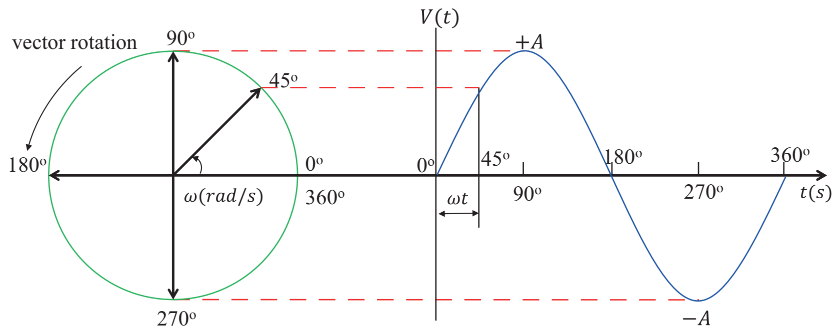

Figure 1. Sinusoidal waveform in the time domain and its phasor representation.

The modified GoldSA-II was developed based on the Gold-SA, which uses the sine function and the golden ratio. The sinusoidal waveform can be presented in (1).

$$V(t) = A.sin(\omega.t), \tag{1}$$

where A is amplitude, ω is angular frequency in rad/s, and t is time in s. As given in (1), angular frequency ω and magnitude A of sine are changed depending on the number of iterations. Golden section search is an optimization technique that can be used to find the maximum or minimum value of unimodal functions [27, 28]. The equations given in (2) and (3) are used to find the optimum value in the golden section search.

$$x_1 = a.(1-\tau) + b.\tau, \tag{2}$$

$$x_2 = a.\tau + b.(1-\tau), \tag{3}$$

where a, b are the interval to be searched, and τ is the golden ratio and its value is approximately 0.618033. The procedure of the golden section search method is given in Figure 2. The modified GoldSA-II aims to improve the search using a reduced pattern of sine function and the golden ratio method. This case is shown in Figure 3.

In general, population-based optimization techniques start with a series of random solutions to the optimization process. This set of random solutions is repeatedly evaluated by the objective function until the stopping criterion is reached to find the optimal solution. It is aimed to find a global optimum by evaluating a sufficient number of sets of random solutions [26]. In all of the population-based optimization algorithms, it is common for the optimization process to be divided into two stages: exploration and exploitation [29]. In the first stage, the optimization algorithm combines the random solutions in the solution domain with a random ratio to find the regions that will reach the global optimum in the search space. However, random variations are less than in the exploration domain and there are gradual changes in random solutions in the second stage. In this study, the position update equation for both steps is given in (4) and (5):

$$X_i^{t+1} = X_i^t - dr_t.sin(\omega.t.r_1).(r_2.x_1.D_p - x_2.X_i^t) \tag{4}$$

$$X_i^{t+1} = X_i^t + dr_t.sin(\omega.t.r_1).(r_2.x_1.D_p - x_2.X_i^t) \tag{5}$$

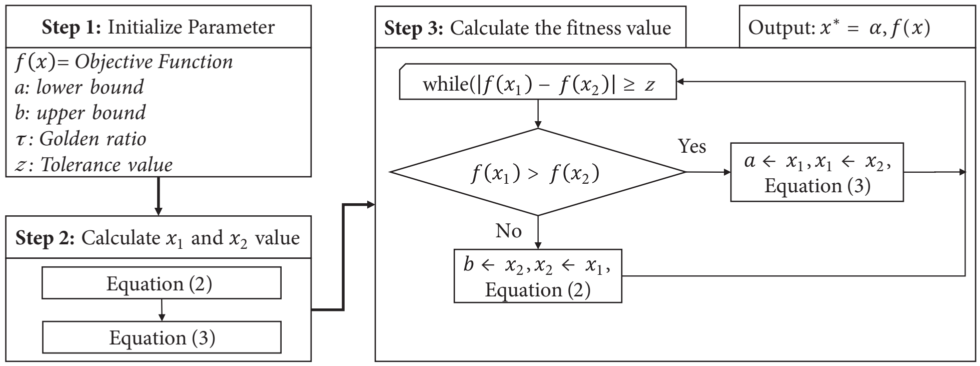

Figure 2. The procedure of the golden section search method.

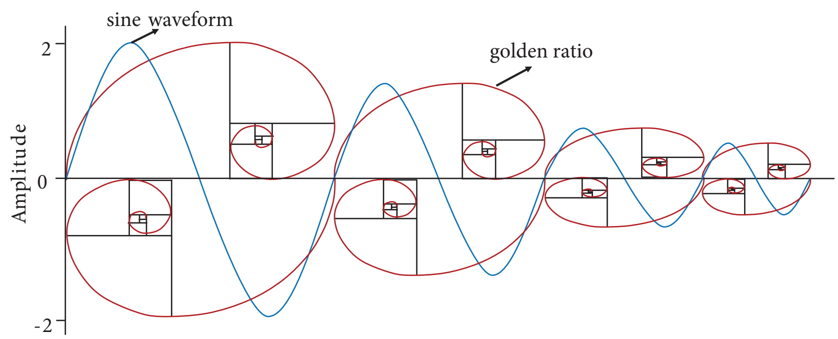

Figure 3. Decreasing ratio of pattern for range of sine and golden ratio.

(4) and (5) will be combined and used as in (6).

$$X_i^{t+1} = \begin{cases} X_i^t - dr_t.sin(\omega.t.r_1).(r_2.x_1.D_p - x_2.X_i^t), & r3 < 0.5 \\ X_i^t + dr_t.sin(\omega.t.r_1).(r_2.x_1.D_p - x_2.X_i^t), & r3 > 0.5, \end{cases} \tag{6}$$

where X_i^t is the t^{th} iteration and i^{th} current resolution position in dimension. r_1, r_2, and r_3 are the random numbers in the range $[0,1]$. dr_t is the amplitude of the sine function in the t^{th} iteration. ω is the angular frequency. x_1 and x_2 are the coefficients obtained by the golden section method. D_p is the determined good global target position value. As shown in (6), there are six main parameters in GoldSA-II. The parameter dr_t indicates the next position region or direction of movement. The new position region may be near or far from the target. If $dr_t > 1$, it represents a region far from the target position; otherwise $dr_t < 1$ represents the target or near region in Figure 4.

The parameters r_1 and ω used in the sine function are as follows: r_1 is a random value that determines how far the movement is to the target or outside of the target for searching the whole region. The parameter r_2 is a random weight value applied to the target to increase or decrease the effect of the target defining the distance. The parameter r_3 is a random value that provides a choice between the two components defined in (4) and (5). The parameters x_1 and x_2 are the values obtained from the golden section search technique, which narrow the search space to bring the current value closer to the target value.

Figures 3 and 4 show how the proposed equations describe the movement in or out of the target region in the search space, using the pattern of decreasing sine function and the decreasing golden section method.

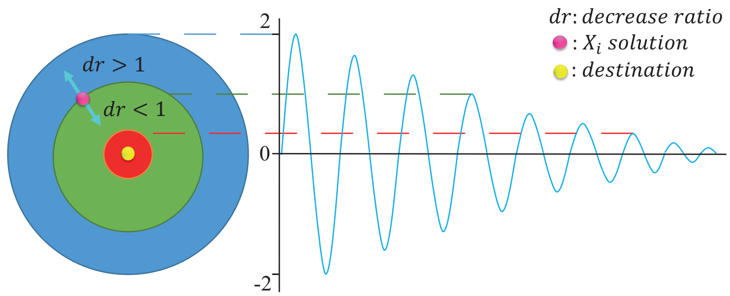

Figure 4. Direction of movement to or from the target region in the search space with decreasing ratio pattern for range of sine.

Movements in the search space ensure that a solution can be repositioned around another solution, exploring the search field and exploiting the domain defined between the two solutions. In order to narrow the solution range, it uses the golden section method as shown in Figure 2. The dr_t, which is used to determine the direction of motion, is given in (7) and the sinusoidal angular frequency ω is given in (8).

$$dr_t = 2.(1-t)/t_{max}, \quad t:0,1,2\ldots t_{max}, \tag{7}$$

where t is the iteration step and t_{max} is the maximum iteration.

$$\omega = 2.\pi.Fc, \tag{8}$$

where Fc is the frequency in Hz. Furthermore, an observation space (P) is formed in the GoldSA-II as a search space. Initially, the search space and observation pool have the same random position values as shown in (9). At each iteration, the best position values to approximate the target are determined and used as the best solution set in the next iteration by using (10). The best optimum value position in this observation pool is taken as the target position (D_p). The optimization procedure of the modified GoldSA-II is shown in Figure 5.

$$X = P = X_{rand} \tag{9}$$

$$P_i^t = \begin{cases} if \quad f(X_i^t) < f()P_i^t), \quad X_i^t \\ else, \quad P_i^t \end{cases} \tag{10}$$

3. Performance evaluation of the proposed algorithm

MATLAB programming software is used for coding and application purposes. The optimization task was performed on an Intel Core (i) i5-4460, 3.2 GHz CPU, 64-bit operating system with 4 GB RAM. In this study, some minimization benchmarking functions, constrained problems, and engineering design problems are investigated. To evaluate the performance of GoldSA-II, 19 benchmark functions, five constrained problems, and five engineering design problems widely used in the literature were tested.

Minimization problems are used instead of maximization problems by transformation $f_{min}(x) = -f_{max}(x)$. The most important thing is that the constraints are not neglected in constrained optimization problems. There are several methods that allow the use of constraints, such as penalty methods, special operators, repair algorithms, separation of objectives and constraints, and hybrid methods [30]. All equality constraints in the function are replaced by inequality constraints, $|h(x)| \leq \varepsilon$ with $\varepsilon = 2e-16$ [31]. All benchmark problems with known global optimizations were run 50 times independently for each optimization method and their results

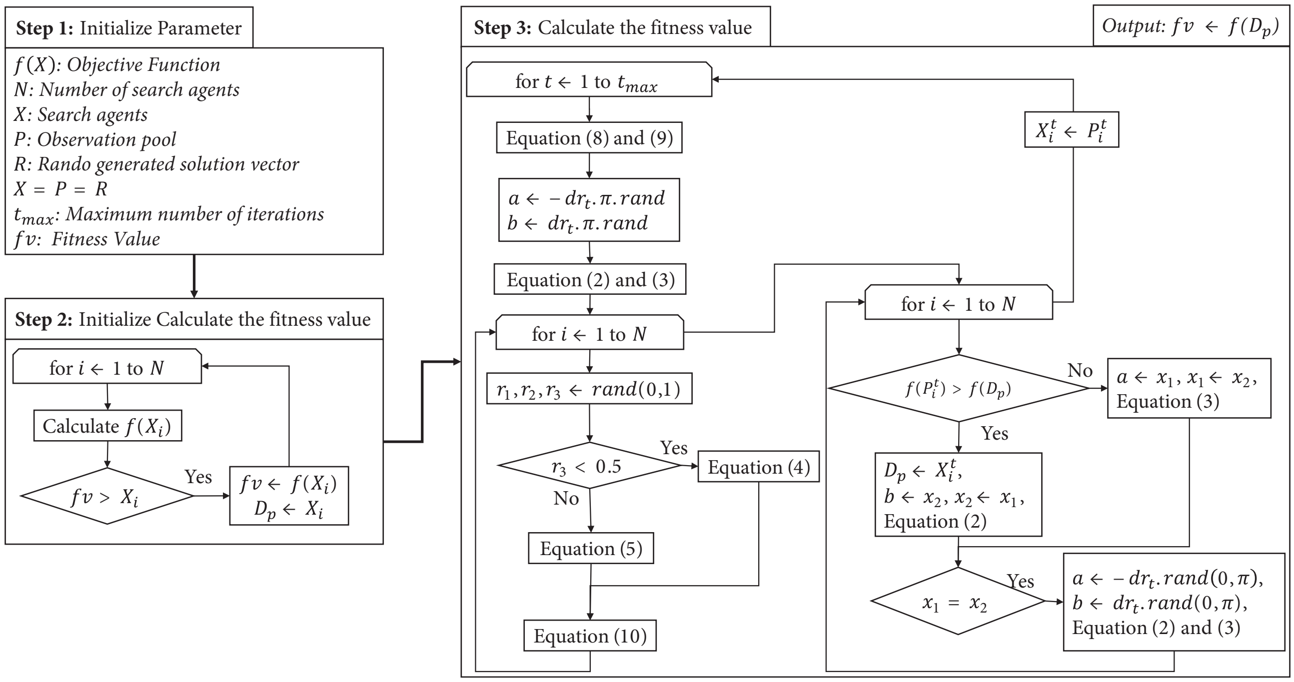

Figure 5. The optimization procedure of the developed GoldSA-II.

were recorded. In each run, if the difference between the best solution found (f_{min}) and the known global optimum (f_{opt}) is less than the predefined value (i.e. less than 0.1% or 0.001% of the optimum), the optimization is interrupted and the values are saved [1]. If the algorithm finds a solution that satisfies $f_{min} - f_{opt} \leq 1e-4$, it is considered successful.

In the present study, statistical results were calculated and recorded for the following performance criteria for each problem:

a) Best value (Best), Mean value (Mean), Worst value (Worst), Standard deviation (Std).

b) Succes Rate(SR-%)= *(Number of successful runs / Total number of runs) × 100*, where *number of successful runs* is $\sum_{i=1}^{n}(|(f_i - f_{opt})| \leq 1e-4)$.

c) Number of function evaluations (NFEs) = *population size* $\times i^{th}$ *iteration.*

d) Average Error (AE) = $\sum_{i=1}^{n}|(f_i - f_{opt})|/n$, where n is total number of runs.

e) Average computational time (ACT) (seconds).

GoldSA-II is compared with the seven well-known metaheuristic algorithms. These are biology-based algorithms: DE, PSO, WOA; physics- and chemistry-based algorithms: GSA, HS, WCA; and a math-based algorithm: Gold-SA. The parameters used in the comparison algorithms are as follows:

- PSO: Inertia Weight Damping Ratio = 0.99, Inertia Weight = 1, Global Learning Coefficient = 2.0, Personal Learning Coefficient = 1.5.
- CHPSO: Inertia Weight Damping Ratio = 0.35, Inertia Weight = 0.9, Acceleration Constants = 1.49445.

- WOA: b = 1.
- GSA: Rnorm = 2, Rpower = 1, Elitist Check = 1.
- HS: Harmony Memory Consideration Rate = 0.9, Fret Width Damp Ratio = 0.995, Fret Width = 0.02 (Upper Bound – Lower Bound), Pitch Adjustment Rate = 0.1.
- DE: Upper Bound of Scaling Factor = 0.8, Lower Bound of Scaling Factor = 0.2, Crossover Probability = 0.2.
- SOS: Benefit factor (BF): random number either 1 or 2.
- WCA: Evaporation condition constant = 1e – 5, The number of rivers = 4.
- GoldSA: Golden section constant = [–pi, pi], Golden ratio (τ) = 0.618033.

All optimizers were run by using equal populations. In addition, the number of iterations was determined based on similar studies for fair comparison in unconstrained and constrained benchmarking problems. For unconstrained benchmark functions, the number of populations is 30 and the maximum number of function evaluations is 3000. For constrained benchmark functions, the number of populations is 200 and the maximum number of function evaluations is $2e + 06$ [1, 8, 25–27].

3.1. Unconstrained benchmark test functions

A total of 19 test functions were selected from the unconstrained benchmark functions commonly used in the literature, including unimodal (F1–F6), multimodal (F7–F14), and fixed-size multimodal functions (F15–F19) [26, 32]. Unimodal functions are functions that are used to test the convergence rate of search algorithms and have a single global optimum. Multimodal functions are very difficult to optimize and have many local minima. In multimodal functions, as the number of problem dimensions increases, the local minimum number also increases. The fixed-size multimodal functions consist of a fixed minimum number of local minimums. Test problems of multimodal functions are very important to evaluate the search capacities of optimization algorithms. In the present study, optimum values for all benchmark functions are given in Table 1.

The statistical results of unimodal, multimodal, and fixed-size multimodal benchmark functions for reported algorithms are given in Table 2, Table 3 and Table 4, respectively. The NFEs given in all tables show the number of function evaluations with the best value.

According to the statistical results of the unimodal benchmark functions in Table 2, only the GoldSA-II and Gold-SA in F1–F3 functions reach the optimum value when the Best results are considered. For F4 and F5 functions, the closest result to the optimum value was also in GoldSA-II and Gold-SA, but better results were obtained in GoldSA-II. In the F6 function, the PSO algorithm gave the best value. Although there are algorithms that give better results than the proposed algorithm according to the number of function evaluations (NFEs) and the average computational time (ACT) for F1–F5 functions, the success rate (SR) of only GoldSA-II is 100%.

The statistical optimization results of the multimodal benchmark functions are given in Table 3. None of the optimizers in function F7 reached the optimum value and the SRs were 0%. However, the results of the GoldSA-II (Best, Mean, Worst, Std) are better than those of the other optimizers. The GoldSA-II, Gold-SA, WOA, and SOS in the F8 function have been the algorithms that optimally converge. As seen in function F8, the SR of these three algorithms is 100%. On the other hand, in terms of the NFEs, the GoldSA-II is more successful than the Gold-SA, SOS, and WOA. The best values for the F9 function are found in the GoldSA-II and Gold-SA. It is seen that NFEs in the GoldSA-II and the ACT in the Gold-SA are better. In the F10 function,

Table 1. Unconstrained benchmark functions.

	Function	**Dimension**	**Range**	**Optimal**
F1	Hyper Sphere	30	[-100, 100]	0
F2	MultiModal	30	[-10, 10]	0
F3	Schwefel (02)	30	[-100, 100]	0
F4	Rosenbrock	30	[-5, 10]	0
F5	Shifted Sphere	30	[-100, 100]	-450
F6	Quartic	30	[-1.28, 1.28]	0
F7	Schwefel (26)	30	[-500, 500]	0
F8	Rastrigin	30	[-5.12, 5.12]	0
F9	Ackley	30	[-32, 32]	0
F10	Griwank	30	[-600, 600]	0
F11	Penalty(01)	30	[-50, 50]	0
F12	Expanded Extended Griewank's plus Rosenbrock's Function (F8F2)	30	[-3,1]	-130
F13	Shifted Rotated Ackley's Function with Global Optimum on Bounds	30	[-32, 32]	-140
F14	Expanded Rotated Extended Scaffe's F6	30	[-100,100]	-300
F15	Kowalik	4	[-5, 5]	0.0003
F16	Branin (01)	2	$x_1\epsilon$[-5, 10], $x_2\epsilon$[0, 15]	0.397887
F17	Hartmann (H6,4)	6	[0, 1]	- 3.32237
F18	Shekel (m=5)	4	[0, 10]	-10.1532
F19	Shekel (m=10)	4	[0, 10]	-10.5363

GoldSA-II, Gold-SA, SOS, PSO, and WOA achieved optimum results when the Best values were examined. However, even at Worst, the GoldSA-II, Gold-SA, and SOS are the only ones that achieve optimum results. According to SR values of the F11 function, only the DE, SOS, and GoldSA-II have the SR of 100%. Given the other performance criteria of the F11 function, DE algorithm achieved better results than the GoldSA-II, except for NFEs and ACT criteria. In none of the F12–F14 functions did optimizers reach the optimum value. GSA in F12 and F13 functions and PSO in F14 function were the most successful algorithms.

When the statistical results of the fixed-size multimodal benchmark functions in Table 4 were examined, none of the optimizers in F17 achieved success. However, in all functions except F17, only the SR of the GoldSA-II is 100%. When the average error (AE) value is examined, in F15 and F19 functions GoldSA-II and in F18 function CHPSO are the most successful algorithms. According to AE value, the PSO, DE, SOS, and GSA algorithms in F16 function and the GSA in F17 function obtained better results than the proposed algorithm, but even in these functions the GoldSA-II reached near optimum values.

The proposed algorithm seems the most outperforming algorithm among other optimizers in all functions. When the obtained statistical results in Table 2, Table 3, and Table 4 are considered together, it is seen that the GoldSA-II is more successful in unimodal and multimodal functions than the other compared optimizers. The proposed algorithm finds the real optimum result for unimodal functions F1–F3, for multimodal functions F8 and F10, and for fixed-size multimodal functions F15, F16, F8, and F19.

3.2. Constrained optimization test problems

The performance of the proposed GoldSA-II was tested, in this section, with 5 of the most frequently used constrained optimization problems (Table 5) [33, 34]. In order to evaluate the proposed algorithm, statistical results obtained from constrained optimization problems were compared with those of other optimizers. The most successful algorithm in P1–P5 problems was the GoldSA-II as shown in Table 6.

Table 2. Statistical results of the unimodal benchmark functions.

		PSO	**WOA**	**DE**	**GSA**	**HS**	**WCA**	**CHPSO**	**SOS**	**Gold-SA**	**GoldSA-II**
F1	Best	3.7354e-18	2.2115e-172	8.9009e-13	5.2473e-17	8.1271	6.7806e-13	1.8631e-09	7.5499e-68	0	0
	Mean	2.2500e-15	2.2161e-147	3.2269e-12	1.2672e-16	23.9406	6.3134e-12	1.1074e-07	3.4513e-66	0	0
	Worst	3.7617e-14	1.0786e-145	1.0671e-11	2.9615e-16	42.4724	2.6878e-11	2.3982e-06	3.1219e-65	0	0
	Std	7.2024e-15	1.5249e-146	1.7731e-12	5.9094e-17	8.1920	5.6952e-12	3.3926e-07	5.8149e-66	0	0
	SR(%)	100	100	100	100	0	100	100	100	100	100
	NFEs	29787	29978	29990	29941	29910	28998	30048	30120	13275	8512
	ACT	1.0585	0.1934	1.1892	1.9722	2.9276	0.6738	1.5237	0.461	0.4976	0.5558
	AE	2.2500e-15	2.2161e-147	3.2269e-12	1.4483e-16	23.9406	6.3134e-12	1.1074e-07	3.5993e-66	2.2500e-15	2.2161e-147
F2	Best	4.3415e-09	8.4336e-113	1.3555e-08	3.2197e-08	0.1702	1.3466e-04	8.4621e-06	4.6346e-35	0	0
	Mean	1.4088e-05	1.0044e-101	3.4703e-08	5.1637e-08	0.4484	4.2324	1.3215e-04	4.0495e-34	1.3973e-288	5.6799e-318
	Worst	5.1589e-04	4.8654e-100	6.5050e-08	8.5641e-08	1.1100	131.517	5.0971e-04	1.4847e-33	6.3149e-287	2.1441e-316
	Std	7.5396e-05	6.8779e-101	1.0765e-08	1.1013e-08	0.1996	18.7362	1.0183e-04	3.3466e-34	0	0
	SR(%)	96	100	100	100	0	0	48	100	100	100
	NFEs	29831	29980	29855	29911	29504	29000	29724	30100	29053	27420
	ACT	1.0747	0.21540	1.2307	1.9331	2.9395	0.75110	1.6153	0.47250	0.53020	0.51870
	AE	1.4088e-05	1.0044e-101	3.4703e-08	5.7315e-08	0.44840	4.2324	1.3215e-04	4.0798e-34	1.3973e-288	5.6799e-318
F3	Best	1.2262	1.3762e+03	1.2072e+04	131.6073	5.0622e+03	0.5326	7.1247e-09	4.53616e-23	0	0
	Mean	6.9551	2.1834e+04	2.4588e+04	441.2274	8.0245e+03	4.4611	5.8335e-07	3.3571e-19	0	0
	Worst	26.5086	4.5115e+04	3.3144e+04	979.2728	1.5586e+04	47.2117	8.2358e-06	1.056e-17	0	0
	Std	4.8504	1.1024e+04	4.7505e+03	172.9949	2.1859e+03	6.5679	1.5097e-06	1.6377e-18	0	0
	SR(%)	0	0	0	0	0	0	100	100	100	100
	NFEs	29507	29990	25111	29971	26833	28992	29764	30064	13441	13826
	ACT	1.7753	0.9653	2.0149	2.6896	3.7037	1.4536	2.3345	1.2135	1.2365	1.743
	AE	6.9551	21834	24588	441.2274	8024.5	4.4611	5.8335e-07	3.357e-19	0	0
F4	Best	0.4971	26.3745	24.4825	25.0627	67.4214	0.0500	28.5569	20.6548	8.8287e-08	6.2071e-10
	Mean	40.7483	27.0401	41.2403	28.8463	178.2338	48.0477	28.6698	22.9747	0.0094	7.5053e-06
	Worst	92.3907	28.7199	102.9401	102.4048	314.3284	206.6126	28.7337	24.8873	0.1136	4.6985e-05
	Std	30.8972	0.4932	24.7859	12.0393	52.6802	42.2518	0.0337	0.7285	0.0206	1.1985e-05
	SR(%)	0	0	0	0	0	0	0	0	20	100
	NFEs	29841	29939	28813	29971	29976	28998	29996	29840	10333	790
	ACT	1.0845	0.2285	1.2103	2.0169	3.465	0.7295	1.6184	0.4991	0.5136	0.5983
	AE	40.7483	27.0401	41.2403	28.8463	178.2338	48.0477	28.6698	22.976	0.0094	7.5100e-06
F5	Best	0.0064	3.1168e-05	0.0098	0.0186	0.0322	0.0142	6.3604e-06	1.5145e-04	6.8535e-07	2.7723e-08
	Mean	0.0157	0.0018	0.0272	0.0650	0.0837	0.0522	9.8525e-04	8.6840e-04	3.4552e-05	2.3313e-05
	Worst	0.0323	0.0136	0.0448	0.1405	0.1954	0.1305	0.0026	0.0023	1.3000e-04	8.2852e-05
	Std	0.0063	0.0025	0.0066	0.0285	0.0288	0.0240	6.5486e-04	4.6719e-04	2.9528e-05	2.0932e-05
	SR(%)	0	4	0	0	0	0	4	0	96	100
	NFEs	20107	27387	28915	15091	15221	25423	24148	27336	13709	19172
	ACT	1.1385	0.3317	1.2827	2.1405	3.0382	0.8149	1.7191	0.5975	0.6320	0.8423
	AE	0.0157	0.0018	0.0272	0.0928	0.0837	0.0522	9.8525e-04	8.6840e-04	3.4552e-05	2.3313e-05
F6	Best	4.3269e-18	308.6220	7.0520e-13	0.0013	7.2013	1.2824e-12	67.2814	2.95018e-10	3.2893e+04	0.3583
	Mean	3.5977e-15	786.0671	2.4201e-12	898.0066	28.4224	1.8620e-08	14.7378	6.4360e-08	6.4290e+04	6.0420
	Worst	5.0535e-14	1.9001e+03	1.0596e-11	3.8631e+03	70.5934	5.4061e-07	1.8223e+03	1.2538e-06	8.3417e+04	95.2080
	Std	8.8616e-15	380.8185	1.7564e-12	751.9577	11.9734	8.4515e-08	2.9429e+02	1.9451e-07	1.0214e+04	15.5689
	SR(%)	100	0	100	0	0	100	0	100	0	0
	NFEs	29888	29834	29943	29971	29750	28998	30072	29992	9981	29968
	ACT	1.7117	0.5545	2.0283	2.2274	4.3413	1.2761	2.0614	0.9404	0.9214	0.9608
	AE	3.5977e-15	786.0671	2.4201e-12	898.0066	28.4224	1.8620e-08	406.9222	6.5781e-08	6.4290e+04	6.0420

According to the optimization results in the Best column in Table 6,the proposed algorithm obtained optimal values in four constrained problems (P1, P3, P4, and P5) and the SR was 100%. The SR of the proposed algorithm in the P4 problem was 2%. While the DE, SOS, and HS algorithms provide the same success rate in the P1 problem, the SOS and DE algorithms complete the faster operation more quickly when looking at the ACT and NFEs respectively. The PSO algorithm in the P2 problem and the GoldSA-II in th P3 problem obtained the most successful solutions.

In the P4 problem, only the GoldSA-II converges to the optimum value when the values in Table 6 are

Table 3. Statistical results of the multimodal benchmark functions.

		PSO	**WOA**	**DE**	**GSA**	**HS**	**WCA**	**CHPSO**	**SOS**	**Gold-SA**	**GoldSA-II**
F7	Best	4.8364e+03	0.0138	0.0013	8.8785e+03	0.8122	2.8684e+03	-2.2542e+05	1.7100e+03	3.8351e-04	3.8233e-04
	Mean	6.2691e+03	1.6711e+03	104.8537	9.9565e+03	8.8813	4.5463e+03	-9.6902e+04	3.6961e+03	0.0507	0.0015
	Worst	7.7976e+03	4.2261e+03	355.3315	1.0609e+04	33.6636	7.1069e+03	-4.1232e+04	5.2077e+03	0.4453	0.0149
	Std	698.2373	1.5548e+03	97.1178	430.5912	8.4141	745.6389	3.8540e+04	7.7611e+02	0.0937	0.0023
	SR(%)	0	0	0	0	0	0	0	0	0	0
	NFEs	22194	29394	29941	910	29663	28014	29812	29740	13490	13916
	ACT	1.1087	0.2542	1.2375	2.0711	2.9735	0.7514	1.6313	0.5508	0.5354	0.6124
	AE	6.2691e+03	1.6711e+03	1.0485e+02	1.1121e+04	8.8813	4.5463e+03	9.6902e+04	3.6961e+03	0.0507	0.0015
F8	Best	21.8891	0	35.7550	15.9193	2.9294	44.7731	1.3114e-09	0	0	0
	Mean	43.9771	0	59.1996	26.3465	6.5239	101.3064	2.2499e-06	0	0	0
	Worst	85.5663	0	72.7832	42.7832	11.1980	175.1120	6.2875e-05	0	0	0
	Std	13.9221	0	7.5681	7.4354	2.0558	36.4931	9.2282e-06	0	0	0
	SR(%)	0	100	0	0	0	0	100	100	100	100
	NFEs	24689	8494	29650	28951	29873	29000	30076	8940	282	158
	ACT	1.1111	0.2007	1.1451	2.0050	2.9909	0.7263	1.5727	0.4721	0.5023	0.5433
	AE	43.9771	0	59.1996	26.3465	6.5239	101.3064	2.2499e-06	0	0	0
F9	Best	1.3454e-09	8.8818e-16	1.9962e-07	5.0011e-09	1.1743	1.3431	4.4959e-06	8.8818e-16	8.8818e-16	8.8818e-16
	Mean	0.9332	3.7303e-15	4.4004e-07	7.5133e-09	2.2467	5.0452	6.4583e-05	4.2988e-15	8.8818e-16	8.8818e-16
	Worst	2.9570	7.9936e-15	6.7643e-07	1.0949e-08	2.9600	17.1473	2.1120e-04	4.4409e-15	8.8818e-16	8.8818e-16
	Std	0.8675	2.2697e-15	1.2694e-07	1.3232e-09	0.4087	4.1105	4.4952e-05	7.0325e-16	0	0
	SR(%)	40	100	100	100	0	0	82	100	100	100
	NFEs	24624	10849	29965	29911	29773	28994	29896	16928	773	464
	ACT	1.1478	0.2269	1.2497	2.0100	2.9974	0.7602	1.6148	0.5066	0.5356	0.5913
	AE	0.9332	3.7303e-15	4.4004e-07	8.2245e-09	2.2467	5.0452	6.4583e-05	4.2988e-15	8.8818e-16	8.8818e-16
F10	Best	0	0	3.4496e-12	2.8178	1.0883	2.8323e-12	1.5266e-09	0	0	0
	Mean	0.0170	0.0015	1.2083e-10	7.9266	1.2266	0.0285	1.7326e-07	0	0	0
	Worst	0.0832	0.0743	3.1097e-09	15.0330	1.5173	0.1419	1.0170e-06	0	0	0
	Std	0.0207	0.0105	4.3973e-10	3.1859	0.0965	0.0300	2.3156e-07	0	0	0
	SR(%)	32	98	100	0	0	22	100	100	100	100
	NFEs	26434	7264	29852	29971	29975	28999	29588	8288	361	153
	ACT	1.2423	0.2586	1.3157	2.0470	3.0106	0.7495	1.6519	0.5363	0.5660	0.6300
	AE	0.0170	0.0015	1.2083e-10	7.9266	1.2266	0.0285	1.7326e-07	0	0	0
F11	Best	5.0298e-19	6.7237e-04	7.4356e-14	2.8738e-19	0.0826	4.4977e-12	0.02367	5.2222e-15	7.7218e-10	4.6908e-15
	Mean	0.1950	0.0052	3.5672e-13	0.1693	0.4850	0.6988	0.0989	1.3831e-12	1.2442e-04	1.4346e-09
	Worst	1.3482	0.0251	1.2283e-12	1.6526	1.1172	12.4488	0.5097	1.6693e-11	0.0017	2.3981e-08
	Std	0.2810	0.0050	2.5657e-13	0.3212	0.2714	1.9432	0.0912	3.1512e-12	2.7003e-04	3.6212e-09
	SR(%)	38	0	100	48	0	46	0	100	76	100
	NFEs	26875	27560	29853	29821	29806	28981	30120	30064	7998	5096
	ACT	1.6922	0.6511	1.9148	2.4240	3.4980	1.1740	2.4136	0.9653	0.9534	1.4433
	AE	0.1950	0.0052	3.5672e-13	0.1693	0.4850	0.6988	0.0989	1.3831e-12	1.2442e-04	1.4346e-09
F12	Best	2.6854	14.6210	7.8357	1.8948	2.0663	4.7615	6.2172	7.9228	23.4494	8.2961
	Mean	4.7997	26.9458	9.7656	3.8569	2.8499	16.2769	13.4331	13.9651	35.3430	13.7406
	Worst	8.4925	44.6340	11.7051	6.2136	4.1258	33.6664	21.7418	19.8509	42.3939	16.4851
	Std	1.5150	6.6722	1.0261	1.0031	0.5017	6.1972	3.5553	2.2104	4.2113	1.6917
	SR(%)	0	0	0	0	0	0	0	0	0	0
	NFEs	29998	29743	29873	29971	28886	28974	30012	29528	9963	29868
	ACT	2.5023	1.6342	2.6820	3.0766	5.4000	2.1574	3.5340	1.8433	1.8921	2.7145
	AE	4.7997	26.9458	9.7656	3.8569	2.8499	16.2769	13.9422	13.9652	23.4494	13.7406
F13	Best	20.8291	20.6630	20.8790	20.0707	20.8138	20.1284	20.9002	20.8970	20.9224	20.3364
	Mean	20.9827	20.8991	21.0361	20.3002	21.0407	20.3277	21.0394	21.0425	21.0746	20.5102
	Worst	21.0864	21.0895	21.1193	20.6474	21.1300	20.5369	21.1479	21.1262	21.1851	20.8060
	Std	0.0642	0.0896	0.0620	0.1087	0.0688	0.0882	0.0512	0.0566	0.0550	0.0901
	SR(%)	0	0	0	0	0	0	0	0	0	0
	NFEs	25545	25155	18246	29971	26226	28999	6556	2632	15911	29900
	ACT	1.8105	0.7846	2.1369	2.3844	4.7894	1.5322	2.5673	1.1361	1.1123	1.4273
	AE	20.9827	20.8991	21.0361	20.3002	21.0407	20.3277	21.0395	21.0425	21.0746	20.5102
F14	Best	11.9236	12.4520	13.0249	13.4780	13.0081	12.7461	12.8184	13.0840	13.4421	12.5242
	Mean	12.8064	13.5799	13.6668	14.3800	13.6767	13.5427	13.2896	13.4809	13.9179	13.0381
	Worst	13.4885	14.0589	13.9961	14.8818	13.9391	14.0663	13.9293	13.8121	14.2244	13.4391
	Std	0.3994	0.3287	0.1950	0.3275	0.1744	0.2935	0.2568	0.1697	0.2107	0.2177
	SR(%)	0	0	0	0	0	0	0	0	0	0
	NFEs	29984	29860	20000	29971	8856	29000	30044	24464	7005	29819
	ACT	7.4952	3.6795	4.5742	4.5306	7.6348	3.9892	5.2678	3.6113	3.6557	5.8144
	AE	12.8064	13.5799	13.6668	14.3641	13.6767	13.5427	13.2897	13.4809	13.9179	13.0381

Table 4. Statistical results of the fixed-size multimodal benchmark functions.

		PSO	**WOA**	**DE**	**GSA**	**HS**	**WCA**	**CHPSO**	**SOS**	**Gold-SA**	**GoldSA-II**
F15	Best	3.0749e-04	3.0952e-04	3.6321e-04	8.5487e-04	5.8868e-04	3.0749e-04	3.0749e-04	3.0749e-04	3.0897e-04	3.0749e-04
	Mean	9.0290e-04	7.8820e-04	7.1181e-04	0.0026	0.0032	4.6331e-04	6.6370e-04	3.1508e-04	4.2335e-04	3.0769e-04
	Worst	0.0204	0.0015	0.0012	0.0164	0.0204	0.0016	0.0010	5.6267e-04	0.0017	3.0947e-04
	Std	0.0028	3.8411e-04	1.7472e-04	0.0022	0.0064	3.5628e-04	3.4259e-04	3.8698e-05	2.3536e-04	3.0297e-07
	SR(%)	80	12	2	0	0	80	46	96	80	100
	NFEs	19337	29486	29987	29971	29975	26564	30012	22192	11550	29948
	ACT	0.9910	0.1523	1.0995	0.8285	1.1639	0.7219	1.5628	0.3866	0.4525	0.4231
	AE	5.9541e-04	4.8071e-04	4.0433e-04	0.0046	0.0028	1.5582e-04	3.5622e-04	7.6014e-06	1.1587e-04	2.0751e-07
F16	Best	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979
	Mean	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3986	0.3979
	Worst	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.4242	0.3979
	Std	3.3645e-16	2.9125e-06	3.3645e-16	3.3645e-16	2.3167e-09	1.3091e-15	9.2429e-14	3.3644e-16	0.0038	4.4942e-10
	SR(%)	100	100	100	100	100	100	100	100	82	100
	NFEs	3321	29959	2618	24121	29104	694	27184	6200	12406	29961
	ACT	0.8561	0.1193	0.7677	0.6367	1.0334	0.6966	1.4605	0.3975	0.3890	0.3082
	AE	0	1.0817e-06	0	0	1.2204e-09	8.5265e-16	3.3040e-14	0	7.2171e-04	3.6219e-10
F17	Best	-3.3220	-3.3220	-3.3220	-3.3220	-3.3220	-3.3220	-3.3220	-3.3220	-3.2864	-3.3220
	Mean	-3.2744	-3.2193	-3.3192	-3.3220	-3.3101	-3.2673	-3.2625	-3.2649	-3.0396	-3.3220
	Worst	-3.2031	-2.4314	-3.2031	-3.3220	-3.2031	-3.2031	-3.2031	-3.2031	-2.6214	-3.3219
	Std	0.0588	0.1521	0.0169	2.0062e-16	0.0360	0.0599	0.0600	0.0600	0.1408	2.4436e-05
	SR(%)	0	0	0	0	0	0	0	0	0	0
	NFEs	6096	28823	10345	28441	29347	21344	29992	10148	8989	29955
	ACT	1.0837	0.2255	1.2031	0.9915	1.3324	0.7251	1.6405	0.5353	0.5209	0.5634
	AE	0.0479	0.1030	0.0031	3.4163e-04	0.0122	0.0550	0.0597	0.0574	0.2827	3.6834e-04
F18	Best	-10.1532	-10.1532	-10.1532	-10.1532	-10.1532	-10.1532	-10.1532	-10.4029	-10.1532	-10.1532
	Mean	-5.7409	-8.3766	-9.6127	-6.2630	-4.5366	-6.4391	-10.1532	-9.30149	-10.1491	-10.1532
	Worst	-2.6305	-0.8820	-2.6829	-2.6829	-2.6305	-2.6305	-10.1532	-5.08767	-10.0732	-10.1532
	Std	3.2124	2.6682	1.8557	3.5374	3.2105	3.3246	4.1618e-09	2.14540	0.0126	3.4442e-06
	SR(%)	32	2	84	44	24	42	100	76	34	100
	NFEs	6287	28547	11388	28591	28822	11772	29912	5508	15445	29949
	ACT	1.2637	0.3464	1.3476	1.0777	1.3622	0.8553	1.8312	0.3249	0.6718	0.8083
	AE	4.4123	1.7766	0.5405	4.1844	5.6166	3.7141	3.2191e-07	1.1014	0.0041	3.2810e-06
F19	Best	-10.5364	-10.5361	-10.5364	-10.5364	-10.5364	-10.5364	-10.5364	-10.5364	-10.5364	-10.5364
	Mean	-5.9937	-7.8931	-10.5364	-10.5364	-7.1133	-8.4522	-10.5364	-10.2119	-10.5328	-10.5364
	Worst	-2.4217	-1.6765	-10.5361	-10.5364	-2.4217	-1.6766	-10.5364	-5.12848	-10.5119	-10.5364
	Std	3.7907	3.2278	4.8849e-05	1.1487e-14	3.7635	3.4189	2.6111e-10	1.2973	0.0057	6.8524e-06
	SR(%)	40	0	98	100	54	72	100	90	8	100
	NFEs	5661	25976	25012	29761	29924	6641	29692	4824	17729	29963
	ACT	1.5532	0.5956	1.5844	1.3316	1.6306	1.0939	2.0292	0.2451	0.9144	1.4469
	AE	4.5427	2.6433	1.6332e-05	9.8167e-06	3.4232	2.0842	9.8165e-06	0.3245	0.0036	5.7766e-06

Table 5. Constrained test problems.

	Problem	**Dimension**	**Optimal**
P1	g01	13	-15
P2	g03	10	-1
P3	g04	5	-30665.539
P4	g09	7	680.6300573
P5	g012	3	-1

examined. With respect to the SR in the P5 problem, almost all the optimization algorithms showed the same success. However, the WOA, CHPSO, and Gold-SA did not reach optimum values. In the P5 problem, the fastest recommended algorithm according to the ACT value of the algorithms that reach the optimum value is the GoldSA-II. Considering the values given in NFEs, the WCA has the smallest number of function evaluations. Another remarkable detail of Table 6 is that the proposed algorithm has the fastest runtime even if the number of function evaluations is high.

Table 6. Statistical results of constrained optimization test problems.

		PSO	WOA	DE	GSA	HS	WCA	CHPSO	SOS	Gold-SA	GoldSA-II
P1	Best	-15	-14.998826	-15	-13.246355	-15	-15	-15.9922	-15	-15	-15
	Mean	-13.278438	-10.928544	-15	-11.850284	-15	-14.8800	-200.56685	-15	-14.9739	-15
	Worst	-10.656250	-6	-15	-8.7445921	-15	-12.0000	-415.34957	-15	-14.8892	-15.0000
	Std	1.1463	2.9968	0	1.0180	0	0.5938	143.2017	0	0.0263	7.1776e-16
	SR(%)	16	0	100	0	100	96	0	100	4	100
	NFEs	394845	1460191	96542	963001	1107623	1599363	2000696	219036	916379	915345
	ACT	79.7169	17.2835	94.8553	486.5377	126.1496	51.9566	112.1804	45.2781	37.1037	39.8732
	AE	1.7216	4.0715	0	3.1653	0	0.1200	185.5668	0	0.0261	2.8422e-16
P2	Best	-0.999999	-0.886629	-0.999993	-0.999972	-0.999994	-0.999999	-0.359145	-0.999999	-0.999999	-0.999997
	Mean	-0.999999	-0.738726	-0.999975	-0.999849	-0.999971	-0.999997	6.5009e-04	-0.999999	-0.999926	-0.999991
	Worst	-0.999999	-0.500953	-0.999956	-0.999583	-0.999908	-0.999999	0.448067	-0.999998	-0.999653	-0.999983
	Std	6.7649e-08	0.0916	7.1778e-06	8.2389e-05	1.9088e-05	4.2009e-07	0.09616	2.7928e-07	7.3182e-05	3.2847e-06
	SR(%)	100	0	100	22	100	100	0	100	74	100
	NFEs	1406849	1998081	1823951	1999801	1525109	1968771	1973776	1600416	1024854	1931091
	ACT	67.3883	12.7396	81.7429	394.5573	105.6845	40.4965	107.4590	37.6378	32.8597	29.4460
	AE	1.5501e-07	0.2613	2.5297e-05	1.6873e-04	2.9162e-05	5.3626e-07	1.0006	7.0128e-07	7.3508e-05	9.0963e-06
P3	Best	-30665.5387	-30665.6296	-30665.5387	-30662.5395	-30659.8290	-30665.5387	4.4445e+19	-30665.5387	-30665.3544	-30665.5387
	Mean	-30665.5387	-30512.4673	-30665.5387	-30618.1665	-30624.9996	-30665.5387	4.4445e+19	-30665.5387	-30611.5479	-30665.5387
	Worst	-30665.5387	-30179.9164	-30665.5387	-30551.3458	-30573.9706	-30665.5387	4.4445e+19	-30665.5387	-30184.0995	-30665.5387
	Std	3.4293e-11	123.3291	3.4813e-11	36.0645	20.8681	4.4614e-06	3.6765e+12	1.0250e-11	122.1702	1.4700e-11
	SR(%)	100	0	100	0	0	100	0	100	0	100
	NFEs	152665	1996029	381878	1013201	1292371	1101463	340524	220012	1158138	467266
	ACT	67.0759	10.5888	81.8975	299.5603	80.3881	49.2998	101.3257	40.3479	32.3201	25.9407
	AE	3.2822e-04	153.0717	2.7430e-11	47.8918	40.5394	8.1047e-07	4.445e+19	9.2404e-12	53.9908	3.6380e-12
P4	Best	680.630467	681.505502	680.632239	681.136046	680.676236	680.630197	2.0014e+03	680.630199	685.424249	680.630081
	Mean	680.633300	685.874020	680.636732	681.594220	681.419542	680.632334	8.6466e+06	680.630462	697.9598843	680.630606
	Worst	680.640361	693.691838	680.645628	682.536594	684.439447	680.641985	1.4455e+08	680.630983	745.495437	680.631494
	Std	0.0020	2.2881	0.0028	0.3411	0.6680	0.0024	2.9107e+07	1.5946e-04	11.1847	2.7544e-04
	SR(%)	0	0	0	0	0	0	0	0	0	2
	NFEs	1784669	1992528	1934697	1999401	1221710	1850379	1996972	1970548	669559	1969688
	ACT	69.1269	12.4243	91.1856	333.6576	91.6404	50.4040	104.0937	39.0303	31.5040	30.4723
	AE	0.0032	5.2440	0.0067	0.9642	0.7895	0.0023	8.646e+06	4.0428e-04	17.3298	5.4844e-04
P5	Best	-1	-0.999999	-1	-1	-1	-1	-0.999897	-1	-0.999999	-1
	Mean	-1	-0.999999	-1	-1	-1	-1	-0.995569	-1	-0.999999	-1
	Worst	-1	-0.999999	-1	-1	-1	-1	-0.984490	-1	-0.999999	-1
	Std	0	1.0751e-12	0	0	0	0	0.00321	0	2.4477e-10	0
	SR(%)	100	100	100	100	100	100	0	100	100	100
	NFEs	21341	1071784	21896	1316775	276059	3781	4	37640	705200	23902
	ACT	81.3462	15.2700	97.3776	244.8613	83.7969	47.6615	117.0028	39.6073	33.2294	27.5108
	AE	0	1.0210e-12	0	0	0	0	0.00443	0	7.8052e-11	0

3.3. Engineering design problems

In this section, five commonly used engineering design problems in the literature were examined, and the performance of the GoldSA-II was compared with that of other optimizers (Table 7) [1, 34]. The statistical results of these design problems are shown in Table 8. In the results shown in Table 8, the GoldSA-II is the most successful algorithm in finding the best solution in two engineering design problems (E2 and E4). The proposed algorithm has the second best solution after the DE algorithm in the E1 problem. The GoldSA-II and SOS algorithms reached the same result with respect to Best, Mean, and Worst values. In the E5 problem, all optimizers except for GSA and CHPSO achieved the same optimal solution and according to NFEs value the GoldSA-II is more successful. According to the ACT value, the WOA found an optimum result in a shorter time.

Table 7. Constrained engineering design problems.

	Engineering design problems	Dimension
E1	Pressure Vessel	4
E2	Tension/Compression Spring	3
E3	Welded Beam	4
E4	Three-Bar Truss	2
E5	Multiple Disk Clutch Brake	5

Table 8. Statistical results of engineering design problems.

		PSO	WOA	DE	GSA	HS	WCA	CHPSO	SOS	Gold-SA	GoldSA-II
E1	Best	5968.816517	5900.546232	5885.332773	6080.059776	6244.435313	5885.332779	5804.376216	6205.866672	5888.210923	5885.332773
	Mean	6156.816293	6435.511820	5885.332773	6237.810896	6784.090788	5887.882955	5543.824694	6744.246213	6522.139856	5885.343590
	Worst	6343.373764	7492.140860	5885.332779	6434.151984	7239.246383	5925.395544	-2731.97105	7057.441306	9467.051057	5885.433602
	Std	107.5655	474.7978	7.9802e-07	79.9211	244.4507	8.4430	1137.2237	2.2605e+02	647.0473	0.0203
	NFEs	1993631	1972750	1942367	1993201	1240444	1885127	1627492	5368	950858	1955314
	ACT	67.0995	10.5732	91.7432	273.0960	74.1577	45.3776	10.4907	18.6262	31.2298	25.9815
E2	Best	0.012666732	0.012672531	0.012665237	0.012742934	0.012761668	0.012665250	6.1660e+15	0.012665266	0.012667386	0.012665233
	Mean	0.012730084	0.013229584	0.012665403	0.013531206	0.014947069	0.012676676	6.1660e+15	0.012665409	0.012881539	0.012665244
	Worst	0.012792826	0.015509274	0.012667137	0.014738941	0.016874415	0.012719054	6.1660e+15	0.012665742	0.013520059	0.012665492
	Std	3.4244e-05	6.5563e-04	2.8822e-07	4.5541e-04	0.0011	1.5432e-05	2.7664	1.1258e-07	2.4959e-04	3.7027e-08
	NFEs	1308974	1939052	1746083	202601	1826138	1207134	677572	1945068	1113484	1846266
	ACT	67.8672	11.1226	97.4728	236.7394	69.8882	49.3988	10.2193	44.1539	32.2848	28.5091
E3	Best	1.724852309	1.731848654	1.724852308	1.795515744	2.047219856	1.724852371	2.6022e+15	1.724852309	1.725790881	1.724852309
	Mean	1.724852309	1.772839744	1.724852427	1.966262395	2.344378728	1.724852795	9.0523e+16	1.724852309	1.783266144	1.724852309
	Worst	1.724852309	1.844834562	1.724853958	2.104010255	2.893052334	1.724853682	1.5516e+17	1.724852309	2.220027121	1.724852309
	Std	1.1215e-15	0.0275	3.4703e-07	0.0687	0.1742	3.1094e-07	5.2001e+16	1.1214e-15	0.0940	9.3670e-16
	NFEs	118631	1887574	1873771	95001	1251075	1220889	2000508	389336	1248478	917060
	ACT	70.7429	13.2415	107.5783	252.6676	78.4343	51.5098	10.6322	47.8059	32.5729	29.7495
E4	Best	263.8958434	263.8958527	263.8958434	263.8967181	263.8958446	263.8958434	1.3333+16	263.8958434	263.8960226	263.8958434
	Mean	263.8958435	263.9222954	263.8958434	263.9062137	263.9292508	263.8958437	1.3333+16	263.8958439	263.8992792	263.8958434
	Worst	263.8958446	264.1594178	263.8958434	263.9285022	264.0392934	263.8958458	1.3333+16	2.638958455	263.9121001	263.8958434
	Std	2.4687e-07	0.0466	1.3274e-09	0.0077	0.0377	4.5721e-07	15.3861	4.0768e-07	0.0030	4.4434e-10
	NFEs	509408	1941947	1802387	1992801	976121	4593	736876	1488340	864191	1503036
	ACT	71.7596	10.3640	85.4953	214.5371	81.7088	45.8156	10.2063	42.7548	31.6452	29.1482
E5	Best	0.313656611	0.313656611	0.313656611	0.313656611	0.313656611	0.313656611	4.0000e+18	0.269467486	0.313656611	0.313656611
	Mean	0.313656611	0.313656611	0.313656611	0.346200934	0.313656611	0.313656611	5.3072e+18	0.296730659	0.313656611	0.313656611
	Worst	0.313656611	0.313656611	0.313656611	0.434830981	0.313656611	0.313656611	6.0581e+18	0.312571830	0.313656611	0.313656611
	Std	1.6822e-16	1.6822e-16	1.6822e-16	0.0311	1.6822e-16	1.6822e-16	9.8664e+17	0.0111582	1.6822e-16	1.6822e-16
	NFEs	275	651	321	28801	2635	14	17040	1512	157	10
	ACT	94.4002	25.5757	104.1501	284.0462	118.8327	74.6891	12.9694	12.3260	44.9855	57.5791

4. Nonparametric statistical analysis results

Statistical tests are used to obtain more reliable results in comparison of metaheuristic algorithms. Wilcoxon's signed rank test is often used as a nonparametric test to compare the performance of algorithms for solving numerical optimization problems [27, 35].

Wilcoxon's signed rank test was performed between the GoldSA-II and the other meta-heuristic algorithms with statistical significance $\alpha = 0.001$. The statistical results are shown in Table 9. P < 0.001 indicates that there is a statistically significant difference between the results obtained from the compared algorithms. Results without significant differences (P > 0.001) are not shown in the table.

In Table 9, the P-value is the average of the significance values in the compared functions. R^+ is the average of the rankings for which the GoldSA-II achieves better results than the compared algorithms. R^- is the average of the rankings in which the GoldSA-II achieves worse results than the other compared algorithms. S^+ is the number of functions for which the GoldSA-II is successful according to the compared algorithms. S^- is the number of functions for which the GoldSA-II is not successful according to the compared algorithms.

From the results given in Table 9, it is clear that the GoldSA-II is superior to other optimizers. According to Wilcoxon's signed rank test comparison results shown in Table 9, over 29 benchmark functions (F1–E5), GoldSA-II outperformed 15/4 against SOS, 19/5 against PSO, 26/0 against WOA, 20/6 against DE, 24/4 against GSA, 24/2 against HS, 23/3 against CHPSO, 19/4 against WCA, and 22/0 against Gold-SA. Looking at the results, the best algorithm after the GoldSA-II is the DE algorithm.

5. Sensitivity analysis

The response of the algorithm with the least precision for the changes made to the control parameters is an important criterion indicating the robustness of the algorithm. In order to test the robustness of the proposed new algorithm, sensitivity analysis was performed by changing the initial population and the iteration of the

Table 9. The results of Wilcoxon's signed rank test.

Benchmark Function	**GoldSA-II & PSO**					**GoldSA-II & WOA**					**GoldSA-II & GSA**				
	P-value*	R+	R-	S+	S-	P-value*	R+	R-	S+	S-	P-value*	R+	R-	S+	S-
F1-F6	7.847e-10	1062.50	212.50	5	1	7.847e-10	1275	0	6	0	7.928e-10	1274.83	0.17	6	0
F7-F14	1.977e-06	922.75	333.88	6	2	6.265e-08	1161.50	0.20	6	0	1.096e-05	927.38	347.63	6	2
F15-F19	7.847e-10	850	425.00	2	1	1.682e-09	1267.60	7.40	5	0	5.650e-08	744.00	531.00	3	2
P1-P5	9.760e-08	693.25	318.75	3	1	7.847e-10	1275	0	5	0	7.847e-10	1275	0	4	0
E1-E5	9.068e-10	1258.33	0	3	0	7.847e-10	1275	0	4	0	7.847e-10	1275	0	5	0
F1-E5	4.153e-07	957.37	258.03	19	5	1.334e-08	1250.82	1.52	26	0	2.203e-06	1099.24	175.76	24	4
Benchmark Function	**GoldSA-II & DE**					**GoldSA-II & WCA**					**GoldSA-II & HS**				
	P-value*	R+	R-	S+	S-	P-value*	R+	R-	S+	S-	P-value*	R+	R-	S+	S-
F1-F6	7.847e-10	1062.50	212.50	5	1	7.847e-10	1062.50	212.50	5	1	1.385e-09	1270.17	4.83	6	0
F7-F14	7.908e-10	956.38	318.63	6	2	7.847e-10	1092.86	182.14	6	1	7.847e-10	1115.63	159.38	7	1
F15-F19	2.408e-05	803.40	471.60	3	2	7.847e-10	850	425.00	2	1	2.505e-05	1068.00	207.00	4	1
P1-P5	1.891e-06	976	0	3	0	7.847e-10	956.25	318.75	3	1	7.847e-10	1275	0	3	0
E1-E5	2.057e-06	844.75	357.00	3	1	7.847e-10	1275	0	3	0	7.847e-10	1275	0	4	0
F1-E5	5.607e-06	928.61	271.95	20	6	7.847e-10	1047.32	227.68	19	4	5.010e-06	1200.76	74.24	24	2
Benchmark Function	**GoldSA-II & SOS**					**GoldSA-II & SOS**					**GoldSA-II & Gold-SA**				
	P-value*	R+	R-	S+	S-	P-value*	R+	R-	S+	S-	P-value*	R+	R-	S+	S-
F1-F6	7.847e-10	1062.50	212.50	5	1	7.847e-10	1275	0	6	0	1.936e-09	809.83	0.17	4	0
F7-F14	1.146e-09	1235.40	0	5	0	8.301e-05	950.86	324.14	5	2	7.945e-10	1275	0	5	0
F15-F19	3.234e-06	484.67	790.33	1	2	2.061e-06	793.67	481.33	2	1	7.847e-10	1275	0	5	0
P1-P5	7.847e-10	0	1275	0	1	7.847e-10	1275	0	5	0	1.043e-09	1272	3.50	4	0
E1-E5	1.821e-09	1267.25	7.75	4	0	7.847e-10	1275	0	5	0	7.847e-10	1275	0	4	0
F1-E5	6.477e-07	809.96	457.12	15	4	1.702e-05	1113.90	161.10	23	3	1.069e-09	1181.27	0.73	22	0

$^{*}P-value<0.001$

control parameters. For sensitivity analysis, each function was run 50 times and the obtained results are given in Table 10.

Table 10. Sensitivity analysis results on control parameters (Npop: Population size, Max: The number of maximum iteration).

	Max	**100**			**200**		
	Npop	**5**	**10**	**20**	**5**	**10**	**20**
F15	Best	3.0791e-04	3.0900e-04	3.0768e-04	3.0773e-04	3.0760e-04	3.0758e-04
	Mean	3.3365e-04	3.2061e-04	3.1307e-04	3.2109e-04	3.1327e-04	3.1113e-04
	Worst	4.8536e-04	3.8476e-04	3.2431e-04	3.9916e-04	3.2636e-04	3.2481e-04
	Std	3.4729e-05	1.4240e-05	4.3903e-06	1.7229e-05	4.4649e-06	3.5265e-06
	SR(%)	96	100	100	100	100	100
	NFEs	488	974	1962	992	1984	3950
	ACT	0.0090	0.0167	0.0316	0.0172	0.0317	0.0616
	AE	2.6160e-05	1.3127e-05	5.5828e-06	1.3602e-05	5.7857e-06	3.6463e-06
P3	Best	-3.0603e+04	-3.0653e+04	-3.0651e+04	-3.0657e+04	-3.0660e+04	-3.0663e+04
	Mean	-3.0223e+04	-3.0371e+04	-3.0511e+04	-3.0416e+04	-3.0565e+04	-3.0634e+04
	Worst	-2.9776e+04	-2.9980e+04	-3.0259e+04	-2.9831e+04	-3.0362e+04	-3.0554e+04
	Std	192.4735	171.8537	107.3814	193.5531	82.0877	29.9069
	SR(%)	0	0	0	0	0	0
	NFEs	482	957	1874	963	1932	3942
	ACT	0.0088	0.0154	0.0290	0.0154	0.0290	0.0564
	AE	442.8686	294.2824	154.6992	249.2049	100.7050	31.6960
E5	Best	0.3137	0.3137	0.3137	0.3137	0.3137	0.3137
	Mean	0.3239	0.3149	0.3144	0.3159	0.3140	0.3137
	Worst	0.3921	0.3372	0.3372	0.3400	0.3333	0.3137
	Std	0.0171	0.0041	0.0040	0.0059	0.0028	1.6822e-16
	SR(%)	56	88	96	84	98	100
	NFEs	43	72	136	16	34	85
	ACT	0.0121	0.0214	0.0419	0.0213	0.0419	0.0811
	AE	0.0102	0.0012	7.8450e-04	0.0023	3.9244e-04	3.8947e-07

When this table is examined, it is seen that obtained values are closer to the optimum result for all the benchmark functions as population or iteration value increases. On the other hand, the difference between them is very small. These results indicate that the algorithm responds with minimal sensitivity to the number of iterations or populations. The convergence graphs for the average fitness values of the F15 benchmark function, P3 test problem, and E5 engineering problem are depicted for different numbers of populations and iterations in Figure 6(a-b), Figure 6(c-d), and Figure 6(e-f), respectively. It is seen in these figures that the proposed algorithm with different iteration and population numbers always tries to converge to the optimum solution.

6. Discussion

When the results of the performance criteria (Best, Mean, Worst, Std, and AE) obtained by the GoldSA-II are examined in constrained and unconstrained problems, it is seen that the proposed algorithm is very successful.

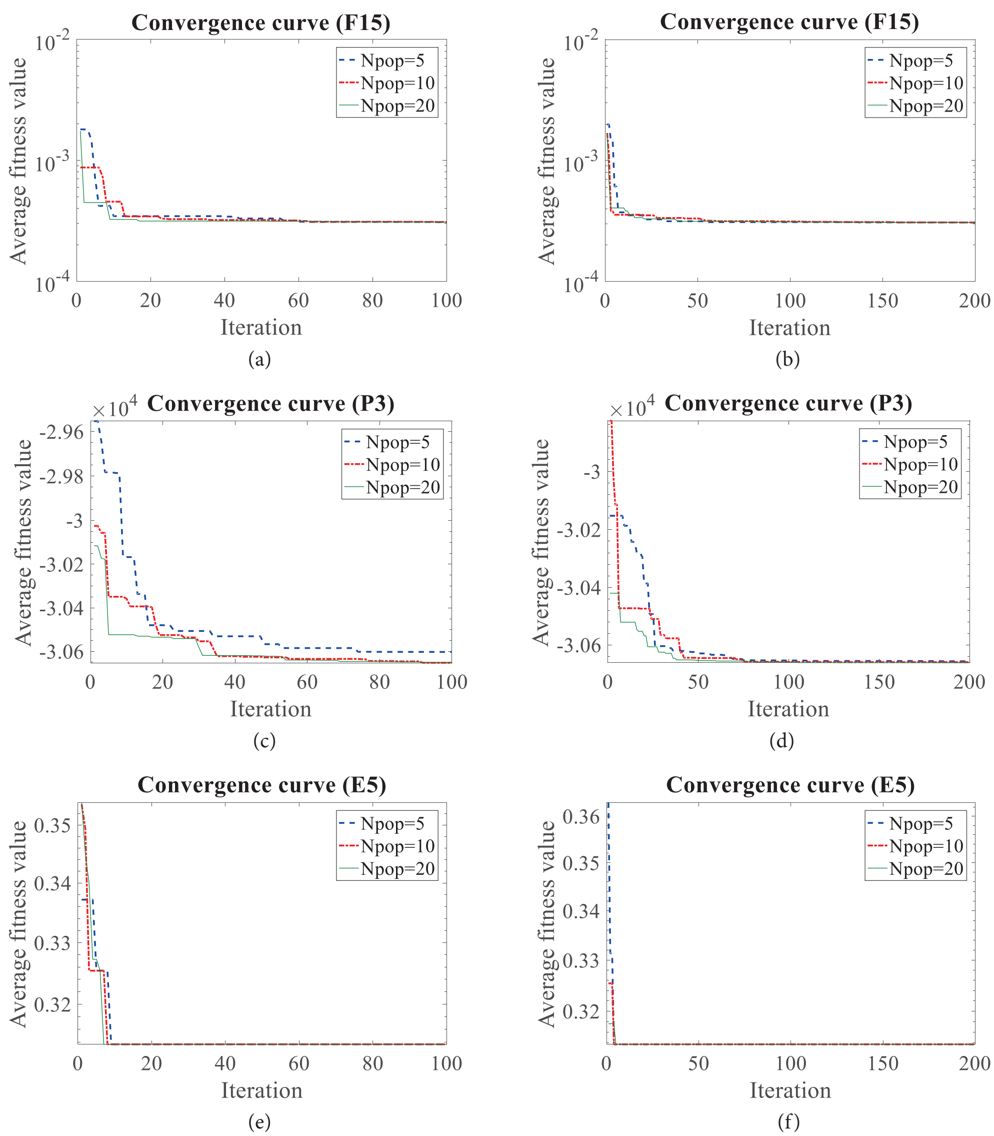

Figure 6. Convergence curves for average fitness values for some benchmark functions. (a) F15 function for 100 iterations. (b) F15 function for 200 iterations. (c) P3 test problem for 100 iterations. (d) P3 test problem for 200 iterations. (e) E5 problem for 100 iterations. (f) E5 problem for 200 iterations.

The proposed GoldSA-II found the best result within nine out of 19 unconstrained benchmark functions, within four out of five constrained benchmark functions, and four out of five real constrained engineering design optimization problems. The obtained results also show that the proposed algorithm works to converge steadily with optimal solution in all problems. Other optimization algorithms do not show the same situation.

Looking at the results of NFEs, the GoldSA-II, in general, has not shown the same success due to scanning the search space based on the maximum number of iterations to find the optimal solution. When the ACT values are examined, the GoldSA-II finds the best optimal solution at much shorter run-time than other algorithms with small NFEs. Even Mean and Worst values of the proposed algorithm obtained from the used problems are better than the Best values obtained by some optimizers. The performance of the proposed algorithm has also been statistically demonstrated according to Wilcoxon's signed rank test. Depending on the performed sensitivity analysis results, it can be concluded that the proposed algorithm always converged to the optimum solution with respect to different numbers of iterations and populations. Better results can be obtained from the proposed GoldSA-II by using optimized parameters, hybridizing with other algorithms, and uniformly distributing the initial candidate solutions in the search space of the problems.

7. Conclusion

In this study, a new search and optimization technique called the GoldSA-II is proposed. The underlying idea behind the method was inspired by the Gold-SA, which uses the sine and the golden ratio functions. The proposed algorithm scans the entire of the search space using the decreasing sine function and the fields that are thought to give better results by narrowing the solution space using the decreasing golden ratio. The proposed novel optimization algorithm has been tested by using 19 unconstrained benchmark functions and 10 constrained problems (five constrained benchmark problems and five engineering design problems) and its results are compared with those of nine metaheuristic algorithms. When the obtained statistical results are investigated, the proposed algorithm provides better solutions for many optimization problems than other methods do. In addition, the proposed GoldSA-II has lower computation cost for almost all optimization problems compared to other optimizers. Thus, this algorithm can be efficiently used to solve real-world optimization problems. However, more research is needed to enhance the performance of the proposed GoldSA-II in large-scale optimization problems.

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