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HAZEM ALI ATTIA

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Hazem A. ATTIA
Department of Mathematics, College of Science, Al-Qasseem University,
P. O. Box 237, Buraidah 81999, SAUDI ARABIA
e-mail: ah1113@yahoo.com

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Abstract

This paper studies unsteady laminar flow of dusty conducting fluid between parallel porous plates with temperature dependent viscosity. The fluid is acted upon by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below. The viscosity of the fluid is assumed to vary exponentially with temperature. The governing nonlinear partial differential equations are solved numerically and some important effects for the variable viscosity and the uniform magnetic field on the transient flow and heat transfer of both the fluid and dust particles are indicated.

Key Words: Fluid mechanics, magnetohydrodynamics, heat transfer, transient state, two-phase flow, dust particles, finite differences.

1. Introduction

The flow of a dusty and electrically conducting fluid through a channel in the presence of a transverse magnetic field has important application in such areas as magnetohydrodynamic generators, pumps, accelerators, cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, and fluid droplets sprays. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators.

The hydrodynamic flow of dusty fluids has been studied by a number of authors [1–5]. Later investigations studied the influence of the magnetic field on the flow of electrically conducting dusty fluids [6–10]. Most of these studies are based on constant physical properties. More accurate prediction for the flow and heat transfer can be achieved by taking into account the variation of these properties, especially the variation of the fluid viscosity with temperature [11]. Klemp et al. [12] has studied the effect of temperature dependent viscosity on the entrance flow in a channel in the hydrodynamic case. Attia and Kotb [13] studied the steady, fully developed MHD flow and heat transfer between two parallel plates with temperature dependent viscosity. Attia [14] extended the problem to the transient state.

In the present work, the effect of variable viscosity on the unsteady laminar flow of an electrically conducting, viscous, incompressible dusty fluid and heat transfer between parallel non-conducting porous plates is studied. The fluid is flowing between two electrically insulating infinite plates maintained at two constant but different temperatures. An external uniform magnetic field is applied perpendicular to the plates. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected. The fluid is acted upon by a constant pressure gradient and its viscosity is assumed to vary exponentially with temperature. The flow and temperature distributions of both the fluid and dust particles are governed by the coupled set of the momentum and energy equations. The Joule and viscous dissipation terms in the energy equation are taken into consideration. The governing coupled nonlinear partial differential equations are solved numerically using the finite difference approximations. The effects of the external uniform magnetic field and the temperature dependent viscosity on the time development of both the velocity and temperature distributions are discussed.

Description of the Problem

The dusty fluid is assumed to be flowing between two infinite horizontal plates located at the $y = \pm h$ planes, as shown in Figure 1. The dusty particles are assumed to be uniformly distributed throughout the fluid. The two plates are assumed to be electrically non-conducting and kept at two constant temperatures: T_1 for the lower plate and T_2 for the upper plate with $T_2 > T_1$. A constant pressure gradient is applied in the x-direction and the parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below. Thus the y component of the velocity is constant and denoted by v_o. A uniform magnetic field B_o is applied in the positive y-direction. By assuming a very small magnetic Reynolds number the induced magnetic field is neglected [15]. The fluid motion starts from rest at $t = 0$, and the no-slip condition at the plates implies that the fluid and dust particles velocities have neither a z nor an x-component at $y = \pm h$. The initial temperatures of the fluid and dust particles are assumed to be equal to T_1 and the fluid viscosity is assumed to vary exponentially with temperature. Since the plates are infinite in the x and z-directions, the physical variables are invariant in these directions. The flow of the fluid is governed by the Navier-Stokes equation [15]

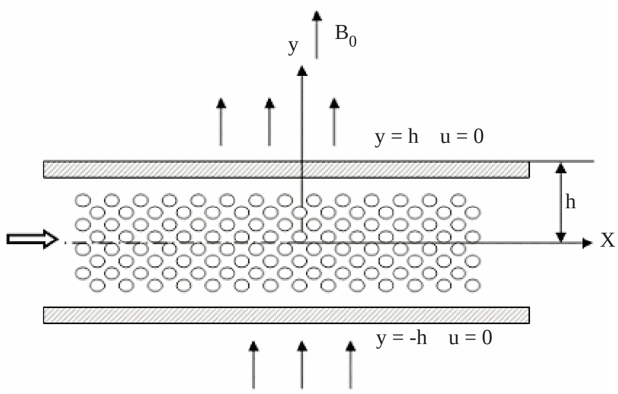

Figure 1. The geometry of the problem.

$$\rho \frac{\partial u}{\partial t} + \rho v_o \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \sigma B_o^2 u - KN(u - u_p), \tag{1}$$

where ρ is the density of clean fluid, μ is the viscosity of clean fluid, u is the velocity of fluid, u_p is the velocity of dust particles, σ is the electric conductivity, p is the pressure acting on the fluid, N is the number of dust particles per unit volume, and K is a constant. The first three terms in the right hand side are, respectively, the pressure gradient, viscous force and Lorentz force terms. The last term represents the force term due to the relative motion between fluid and dust particles. It is assumed that the Reynolds number of the relative velocity is small. In such a case the force between dust and fluid is proportional to the relative velocity [1]. The motion of the dust particles is governed by Newton's second law [1] via

$$m_p \frac{\partial u_p}{\partial t} = KN(u - u_p), \tag{2}$$

where m_p is the average mass of dust particles. The initial and boundary conditions on the velocity fields are respectively given by

$$t = 0; \quad u = u_p = 0. \tag{3}$$

For $t > 0$, the no-slip condition at the plates implies that

$$y = -h: \quad u = u_p = 0; \tag{4}$$

$$y = h: \quad u = u_p = 0. \tag{5}$$

Heat transfer takes place from the upper hot plate towards the lower cold plate by conduction through the fluid. Also, there is a heat generation due to both the Joule and viscous dissipations. The dust particles gain heat energy from the fluid by conduction through their spherical surface. Two energy equations are required which describe the temperature distributions for both the fluid and dust particles and are respectively given by [16]

$$\rho c \frac{\partial T}{\partial t} + \rho c v_o \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \sigma B_o^2 u^2 + \frac{\rho_p C_s}{\gamma_T}(T_p - T), \tag{6}$$

$$\frac{\partial T_p}{\partial t} = -\frac{1}{\gamma_T}(T_p - T), \tag{7}$$

where T is the temperature of the fluid, T_p is the temperature of the particles, c is the specific heat capacity of the fluid at constant pressure, C_s is the specific heat capacity of the particles, k is the thermal conductivity of the fluid, γ_T is the temperature relaxation time ($=3\,\Pr\,\gamma_p C_s/2c$), γ_p is the velocity relaxation time ($=2\rho_s \mathrm{D}^2/9\mu$), ρ_s is the material density of dust particles ($= 3\rho_p/4\pi \mathrm{D}^3\mathrm{N}$), D is the average radius of dust particles. The three terms on the right-hand side of Eq. (6) represent the viscous dissipation, the Joule dissipation, and the heat conduction between the fluid and dust particles, respectively. The initial and boundary conditions on the temperature fields are given as

$$t \le 0 : T = T_p = 0, \tag{8}$$

$$t > 0, y = -h : T = T_p = T_{1,} \tag{9}$$

$$t > 0, y = h : T = T_p = T_2. \tag{10}$$

The viscosity of the fluid is assumed to depend on temperature and is defined as $\mu = \mu_o f(T)$. By assuming the viscosity to vary exponentially with temperature, the function $f(T)$ takes the form [13, 14] $f(T)$=$\mathrm{e}^{-b(T-T1)}$, where the parameter b has the dimension of T^{-1} and such that at $T = T_1$, $f(T_1) = 1$ and then $\mu = \mu_o$. This means that μ_o is the viscosity coefficient at $T = T_1$. The parameter b may take positive values for liquids such as water, benzene or crude oil. In some gases like air, helium or methane $a_1$may be negative, i.e. the coefficient viscosity increases with temperature [13, 14].

The problem is simplified by writing the equations in dimensionless form. The characteristic length is taken to be h, and the characteristic time is $\rho h^2/\mu_o$, while the characteristic velocity is $\mu_o/h\rho$. Thus we define the following non-dimensional quantities:

$(\hat{x}, \hat{y}, \hat{z}) = (x, y, z)/h, \quad \hat{t} = t\mu_o/\rho h^2, \quad \hat{P} = P\rho h^2/\mu_o^2, \quad \alpha = -\dfrac{d\hat{p}}{d\hat{x}}$

$(\hat{u}, \hat{v}, \hat{w}) = (u, v, w)\rho h/\mu_o, \quad (\hat{u}_p, \hat{v}_p, \hat{w}_p) = (u_p, v_p, w_p)\rho h/\mu_o$

$\hat{T} = \frac{T-T_1}{T_2-T_1}, \hat{T}_p = \frac{T_p-T_1}{T_2-T_1},$

$f(\hat{T}) = e^{-b(T_2-T_1)\hat{T}} = e^{-a\hat{T}}$, where a is the viscosity parameter,

$H_a^2 = \sigma B_o^2 h^2/\mu_o$, where H_a is the Hartmann number,

and

$R = KNh^2/\mu_o$ is the particle concentration parameter,

$G = m_p\mu_o/\rho h^2 K$ is the particle mass parameter,

$\xi = \rho h v_o/\mu_o$ is the suction parameter,

$\mathrm{Pr} = \mu_o c/k$ is the Prandtl number,

$E_c = \mu_o^2/(h^2 c\rho^2(T_2 - T_1))$ is the Eckert number,

$L_o = \rho h^2/\mu_o\gamma_T$ is the temperature relaxation time parameter.

In terms of the above dimensionless variables and parameters, equations (1)–(6) take the following form (where we have dropped the hats for convenience):

$$\frac{\partial u}{\partial t} + \xi\frac{\partial u}{\partial y} = \alpha + f(T)\frac{\partial^2 u}{\partial y^2} + \frac{\partial f(T)}{\partial y}\frac{\partial u}{\partial y} - H_a^2 u - R(u - u_p) \tag{11}$$

$$G\frac{\partial u_p}{\partial t} = (u - u_p) \tag{12}$$

$$t \le 0; \quad u = u_p = 0. \tag{13}$$

$$t > 0, \quad y = -1; \quad u = u_p = 0, \tag{14}$$

$$t > 0, \quad y = 1; \quad u = u_p = 0, \tag{15}$$

$$\frac{\partial T}{\partial t} + \xi \frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + Ecf(T)\left(\frac{\partial u}{\partial y}\right)^2 + EcH_a^2 u^2 + \frac{2R}{3\Pr}(T_p - T), \tag{16}$$

$$\frac{\partial T_p}{\partial t} = -L_o(T_p - T), \tag{17}$$

$$t \le 0; \quad T = T_p = 0, \tag{18}$$

$$t > 0, \quad y = -1; \quad T = T_p = 0, \tag{19}$$

$$t > 0, \quad y = 1; \quad T = T_p = 1. \tag{20}$$

Equations (11)–(20) represent a system of coupled and nonlinear partial differential equations which must be solved numerically under the initial and boundary conditions (13)–(15) and (18)–(20) using finite difference approximations. The system is solved using the Crank-Nicolson implicit method [17]. Finite difference equations relating the variables are obtained by writing the equations at the mid-point of the computational cell and then replacing the different terms by their second order central difference approximations in the y-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. The nonlinear terms are first linearized and then an iterative scheme is used at every time-step to solve the linearized system of difference equations.

2. Results and Discussions

The exponential dependence of viscosity on temperature results in decomposing the viscous force term $(=\partial/\partial y(\mu \partial u/\partial y))$ in Eq. (1) into two terms. The variations of these resulting terms with the viscosity parameter a and their relative magnitudes have an important effect on the flow and temperature fields in the absence or presence of the applied uniform magnetic field. In the following discussion selected parameters are given the following fixed values: $R = 0.5$, $G = 0.8$, $\alpha = 5$, Pr=1, $E_c = 0.2$, and $L_o = 0.7$.

Figures 2 and 3 indicate the variations of the velocities u and u_p at the centre of the channel ($y = 0$) with time for different values of the viscosity parameter a and for $H_a = 0$ and $\xi = 0$. The figures show that increasing a increases the velocity and the time required to approach the steady state. This implies that higher velocities are obtained at lower viscosities. The effect of the parameter a on the steady state time is more pronounced for positive values of a than for negative values. Notice that u reaches the steady state more quickly than u_p. This is because the fluid velocity is the source for the dust particles velocity. Figure 2 shows also that the influence of a on u_p is negligible for some time and then increases as the time develops.

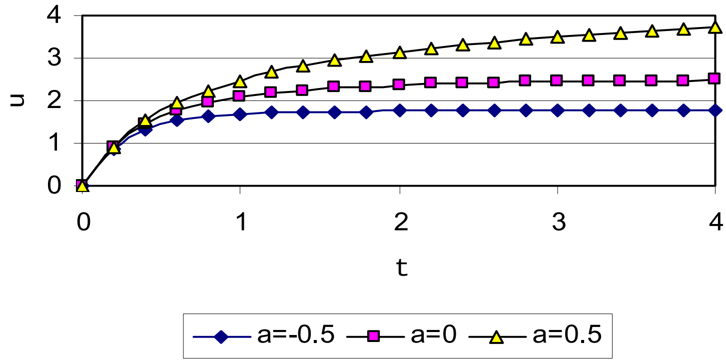

Figure 2. The evolution of u for different values of a ($H_a = 0$, ξ =0).

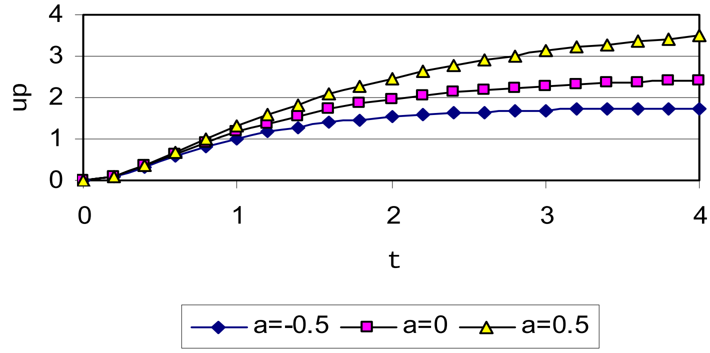

Figure 3. The evolution of u_p for different values of a ($H_a = 0$, ξ =0).

Figures 4 and 5 show the variations of the temperatures T and T_p at the centre of the channel ($y = 0$) with time for different values of the viscosity parameter a for $H_a = 0$ and $\xi = 0$. The figures show that increasing a increases the temperatures and the steady state times. Tables 1 and 2 show the evolution of T and T_p at the centre of the channel ($y = 0$) for different values of a and for $H_a = 0$ and $\xi = 0$. Increasing the positive values of a decreases the temperature for small times, but increases it as time develops. Thus, increasing a increases the steady state value of the temperature with the appearance of cross-over of the temperature curves corresponding to different values of a. The time at which the curves intersect increases with the increment in a and is longer for T than for T_p, as T_p always follows T. It is noticed that the steady state values of T_p coincide with the corresponding steady state values of T, and the time required for T_p to reach the steady state, which depends on a, is longer than that for T. The reduction in temperature with increasing the viscosity exponent a that occurs at small time can be attributed to the fact that the only source term is the viscous dissipation (since $H_a = 0$). At small time the velocity gradient is small and an increase in $a$decreases the viscous dissipation as a result of decreasing viscosity and, in turn, decreases T.

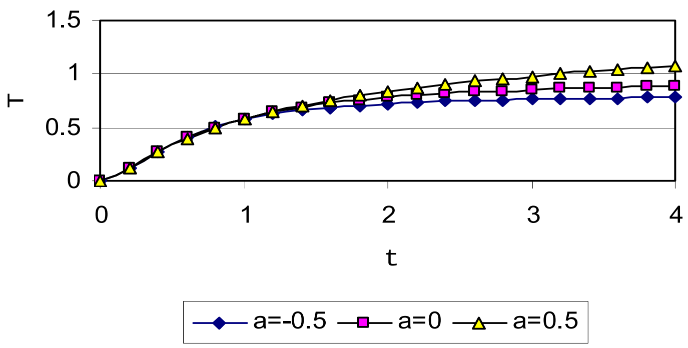

Figure 4. The evolution of T for different values of a ($H_a = 0$, ξ =0).

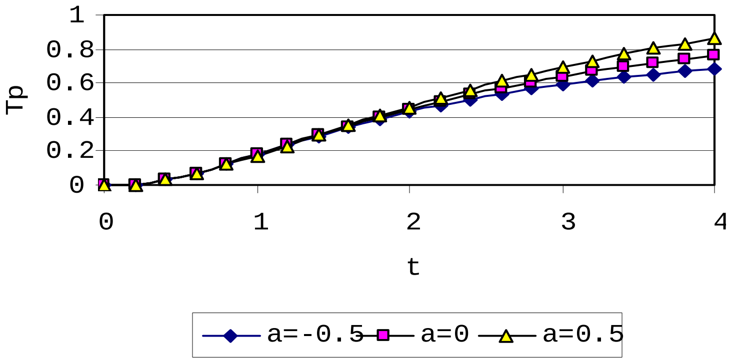

Figure 5. The evolution of T_p for different values of a ($H_a = 0$, ξ =0).

Table 1. The evolution of T for different values of a, $H_a = 0$.

	$a = 0$	$a = 0.5$	$a = 1$	$a = -0.1$	$a = -0.5$
t=0	0	0	0	0	0
t=0.5	0.352	0.346	0.341	0.353	0.359
t=1	0.584	0.583	0.577	0.584	0.581
t=2	0.781	0.839	0.877	0.768	0.721
t=3	0.853	0.979	1.093	0.832	0.762
t=4	0.888	1.066	1.273	0.862	0.780

Table 2. The evolution of H_p for different values of a, $H_a = 0$.

	$a = 0$	$a = 0.5$	$a = 1$	$a = -0.1$	$a = -0.5$
t=0	0	0	0	0	0
t=0.5	0.005	0.005	0.005	0.005	0.005
t=1	0.181	0.178	0.176	0.181	0.183
t=2	0.448	0.462	0.468	0.445	0.431
t=3	0.638	0.694	0.737	0.627	0.590
t=4	0.757	0.864	0.968	0.739	0.682

The application of the uniform magnetic field adds one resistive term to the momentum equation and the Joule dissipation term to the energy equation. Figures 6 and 7 show the influence of the viscosity parameter a on the evolution of both the velocities u and u_p at the centre of the channel, respectively for $H_a = 1$ and $\xi = 0$. The presence of the magnetic field results in a reduction in the velocities and the steady state time for all values of a, and is thus a damping effect. This implies lower velocities at higher magnetic fields.

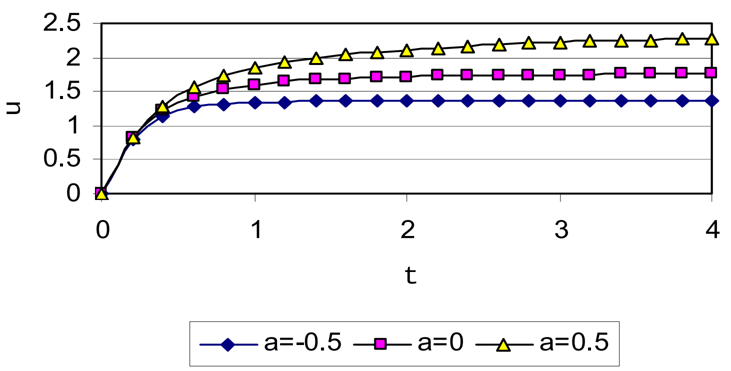

Figure 6. The evolution of u for different values of a ($H_a = 1$, ξ =0).

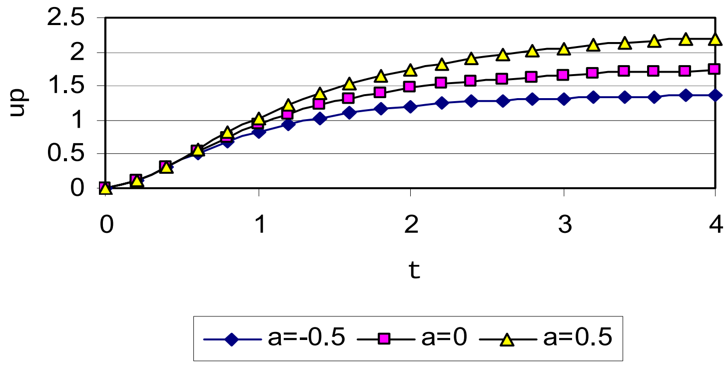

Figure 7. The evolution of u_p for different values of a ($H_a = 1$, ξ =0).

Figures 8 and 9 show the influence of the viscosity parameter a on the evolution of temperatures T and T_p at the centre of the channel, respectively for $H_a = 1$ and $\xi = 0$. Increasing the magnetic field increases the temperatures for all positive values of a due to the effect of Joule dissipation, which increases the dissipation and therefore increases temperature. However, for constant, negative values of a, increasing the magnetic field increases the temperatures for some time then decreases as the time develops; this effect can be seen in Tables 3 and 4. The effect arises from the resistive effect of the magnetic field, and becomes more pronounced as time develops, especially with the case of negative a, which exhibits the same resistive effect.

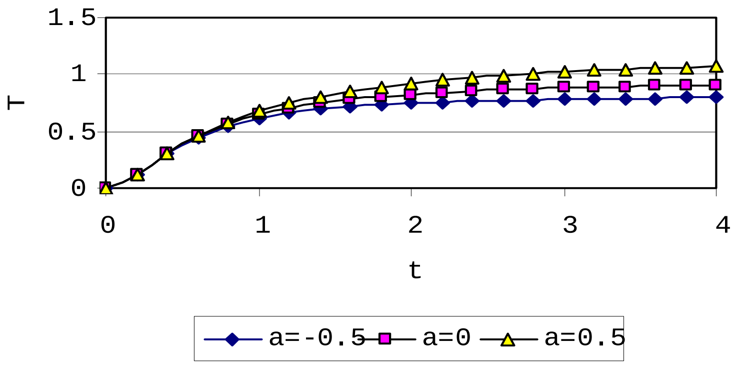

Figure 8. The evolution of T for different values of a ($H_a = 1$, ξ =0).

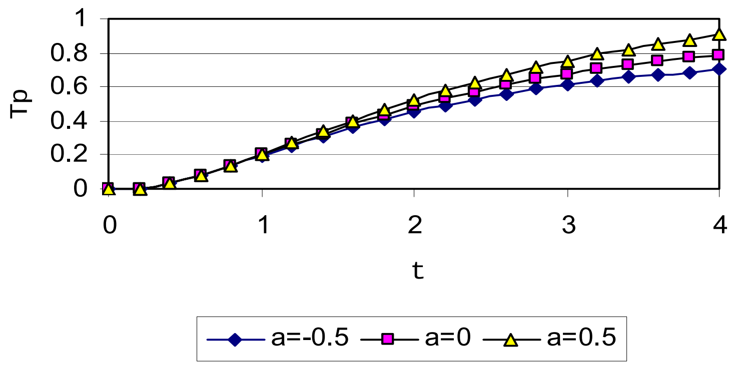

Figure 9. The evolution of T_p for different values of a ($H_a = 1$, ξ =0).

Figures 10 and 11 indicate the variations of the velocities u and u_p at the centre of the channel ($y = 0$) with time for different values of the viscosity parameter a and for $H_a = 0$ and $\xi = 1$. It is clear that the suction velocity decreases both u and $u_p$and their steady state times as a result of pumping the fluid from the slower lower half region to the centre of the channel. This implies higher velocities at lower suction. The influence of suction on u and $u_p$is more pronounced for higher values of the parameter a.

Table 3. The evolution of T for different values of a, $H_a = 1$.

	$a = 0$	$a = 0.5$	$a = 1$	$a = -0.1$	$a = -0.5$
t=0	0	0	0	0	0
t=0.5	0.398	0.401	0.404	0.397	0.395
t=1	0.651	0.681	0.709	0.644	0.621
t=2	0.823	0.922	1.031	0.806	0.744
t=3	0.880	1.021	1.200	0.857	0.780
t=4	0.907	1.071	1.304	0.881	0.797

Table 4. The evolution of T_p for different values of a, $H_a = 1$.

	$a = 0$	$a = 0.5$	$a = 1$	$a = -0.1$	$a = -0.5$
t=0	0	0	0	0	0
t=0.5	0.006	0.006	0.006	0.006	0.006
t=1	0.203	0.208	0.212	0.202	0.198
t=2	0.486	0.524	0.563	0.479	0.453
t=3	0.674	0.755	0.849	0.659	0.611
t=4	0.786	0.904	1.057	0.767	0.702

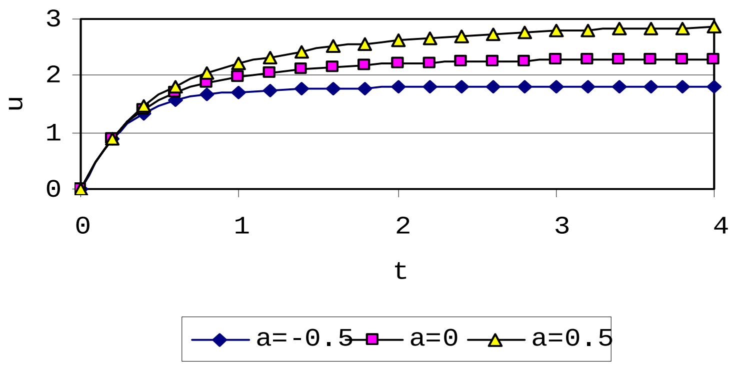

Figure 10. The evolution of u for different values of a ($H_a = 0$, ξ =1).

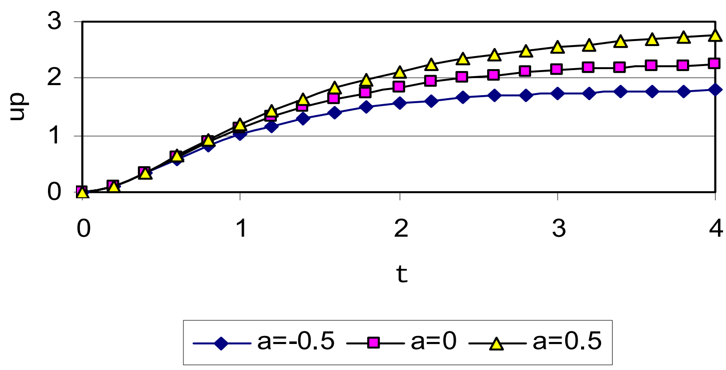

Figure 11. The evolution of u_p for different values of a ($H_a = 0$, ξ =1).

Figures 12 and 13 present the influence of the viscosity parameter a on the evolution of the temperatures T and T_p at the centre of the channel, respectively for $H_a = 0$ and $\xi = 1$. It is shown that increasing suction velocity decreases both T and T_p and their steady state times. This results from pumping the fluid from colder lower half region to the centre of the channel which decreases temperatures. The effect of suction on T and T_p is more apparent for higher values of a.

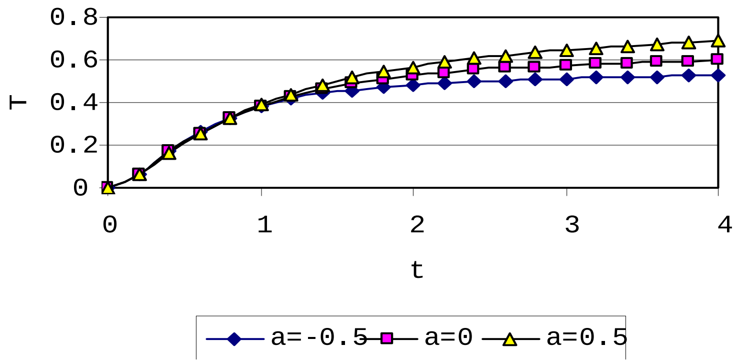

Figure 12. The evolution of T for different values of a ($H_a = 0$, ξ =1).

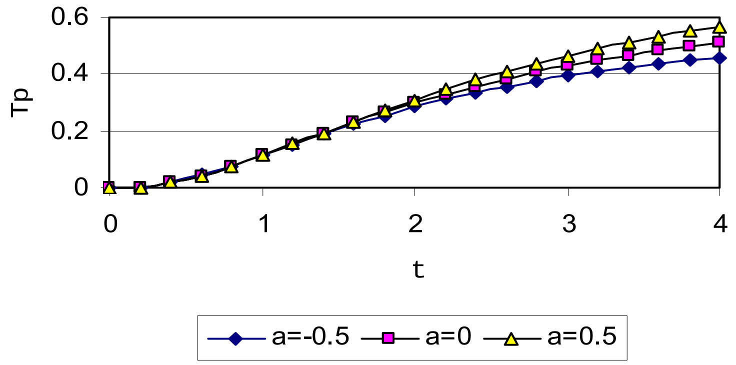

Figure 13. The evolution of T_p for different values of a ($H_a = 0$, ξ =1).

3. Conclusions

In this paper the effect of a temperature dependent viscosity, suction and injection velocity and an external uniform magnetic field on the unsteady laminar flow and temperature distributions of an electrically conducting viscous incompressible dusty fluid between two parallel porous plates has been studied. The viscosity was assumed to vary exponentially with temperature and the Joule and viscous dissipations were taken into consideration. The most interesting result was the cross-over of the temperature curves due to the variation of the parameter a and the influence of the magnetic field in the suppression of such cross-over. On the other hand, changing the magnetic field results in the appearance of cross-over in the temperature curves for a given negative value of a. Also, changing the viscosity parameter a leads to asymmetric velocity profiles about the central plane of the channel ($y = 0$), which is similar to the effect of variable percolation perpendicular to the plates. The effect of the suction velocity on both the velocity and temperature of the fluid and particles is more pronounced for higher values of the parameter a.

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