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On the J -reflexive operators

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Abstract: A bounded linear operator T on a Banach space X is J -reflexive if every bounded operator on X that leaves invariant the sets $J(T, x)$ for all x is contained in the closure of $orb(T)$ in the strong operator topology. We discuss some properties of J -reflexive operators. We also give and prove some necessary and sufficient conditions under which an operator is J -reflexive. We show that isomorphisms preserve J -reflexivity and some examples are considered. Finally, we extend the J -reflexive property in terms of subsets.

Key words: Orbit of an operator, J -sets, J^{mix} -sets, J -class operator, reflexive operator

1. Introduction

One of the most challenging problems in operator theory is the “invariant subspace problem”, which asks whether every operator on a Hilbert space (more generally, a Banach space) admits a nontrivial invariant subspace. Here, “operator” means “continuous linear transformation” and “invariant subspace” means “closed linear manifold such that the operator maps it to itself”. A subspace is nontrivial if it is neither the zero subspace nor the whole space. An example constructed by Enflo [7] shows that for some Banach spaces there do exist operators with only trivial invariant subspaces. For a Hilbert space, however, the invariant subspace problem remains open. There is a deep connection between invariant subspaces of an operator and its reflexivity. Reflexive operators are those that can be identified by their nontrivial invariant subspaces and they have been studied for a few decades. For a good source on reflexivity, see [12] by Halmos. An operator T is reflexive if any operator that leaves invariant, T -invariant subspaces belongs to the closure of $\{P(T) : P \text{ is a polynomial}\}$ in the weak operator topology [12, 17]. In [11], the authors introduced orbit-reflexivity: an operator T on a Hilbert space is orbit-reflexive if the only operators that leave invariant every norm closed T -invariant subset are contained in the closure of $orb(T)$ in the strong operator topology. For example, compact operators, normal operators, contractions, and weighted shifts on the Hilbert spaces are orbit-reflexive [11]. Hadwin et al. in [10] also introduced and studied the notion of null-orbit reflexivity, which is a slight perturbation of the notion of orbit-reflexivity. The class of null-orbit reflexive operators includes the classes of hyponormal, algebraic, compact, strictly block-upper (lower) triangular operators, and operators whose spectral radius is not 1. They also proved that every polynomially bounded operator on a Hilbert space is both orbit-reflexive and null-orbit reflexive. For further references on these topics, see [8–11, 13, 14].

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In this paper, our purpose is to characterize operators on Banach spaces that can be identified by their J -sets. First, we give some preliminaries that we need to give our results.

Let X be a complex Banach space and denote by $B(X)$ the space of all bounded linear operators on X . For $T \in B(X)$, the set $orb(T) = \{T^n : n \geq 0\}$ is called the orbit of T .

If X is a separable Banach space and $orb(T, x) = \{T^n x : n \geq 0\}$ is dense in X for some x , then T is called a hypercyclic operator and we call x a hypercyclic vector.

The J -set of T under x , $J(T, x)$, is defined by:

$$J(T, x) = \{y : \text{there exists a strictly increasing sequence of positive integers}$$

$$(k_n) \text{ and a sequence } (x_n) \subset X \text{ such that } x_n \rightarrow x, T^{k_n} x_n \rightarrow y\}.$$

The set

$$J^{mix}(T, x) = \{y : \text{there exists a sequence } (x_n) \subset X \text{ such that } x_n \rightarrow x \text{ and}$$

$$T^n x_n \rightarrow y\}$$

is called the J^{mix} -set of T under x . The sets $J(T, x)$ and $J^{mix}(T, x)$ are closed T -invariant subsets of X . If T is invertible, then all J -sets are T^{-1} -invariant. For more details, see [6].

Recall that we say $T \in B(X)$ is power bounded with a power bound $M > 0$ whenever $\|T^n\| \leq M$ for all positive integers n . In [6, Proposition 2.10] it was shown that if T is power bounded we have the following:

$$J(T, x) = \{y : \text{there exists a strictly increasing sequence of positive integers}$$

$$(k_n) \text{ such that } T^{k_n} x \rightarrow y\}.$$

An operator $T \in B(X)$ is called a J -class (J^{mix} -class) operator if $J(T, x) = X$ ($J^{mix}(T, x) = X$) for some $x \in X \setminus \{0\}$. The set of all $x \in X$ satisfying $J(T, x) = X$ ($J^{mix}(T, x) = X$) is denoted by A_T (A_T^{mix}) and its elements are called the J -vectors (J^{mix} -vectors) for T . It is well known that A_T is a closed subset of X and A_T^{mix} is a closed subspace of X . We know that $T \in B(X)$ is hypercyclic if and only if $A_T = X$. For a good source on these topics we refer the reader to [1, 3–6, 15, 16, 18].

Recall that $x \in X \setminus \{0\}$ is called a periodic point for $T \in B(X)$ if $T^n x = x$ for some positive integer n . The smallest such number n is called the period of x .

2. Some properties of J -sets

Here we state and prove some properties of J -sets that will be used in the proof of our main results. The proof of the following lemma is straightforward and so we omit it.

Lemma 2.1 *Suppose that $T \in B(X)$. Then for all $x, y \in X$ we have:*

- a) $J(T, x) \subseteq J(T, Tx) \subseteq J(T, T^2x) \subseteq \dots$,
 - b) $J^{mix}(T, x) \subseteq J^{mix}(T, Tx) \subseteq J^{mix}(T, T^2x) \subseteq \dots$,
- and equality holds if T has a bounded inverse.
- c) $J(T^m, x) \subseteq J(T, x)$ for all $m \in \mathbb{N}$.

Lemma 2.2 *Suppose that $T \in B(X)$. We have:*

- (a) If $(T^n)_n$ is a Cauchy sequence and it has a subsequence that converges to a bounded operator U in the

strong operator topology, then for every x , both sets $J(T, x)$ and $J^{mix}(T, x)$ are equal to the singleton $\{Ux\}$.
 (b) If $(T^n)_n$ converges to an operator U in the strong operator topology, then

$$2J(T, x_1 + x_2) - U(x_1 + x_2) \subseteq J(T, x_1) + J(T, x_2)$$

for all $x_1, x_2 \in X$. The same result holds for J^{mix} -sets.

Proof a) Note that T is power bounded. Since for every $x \in X$, $(T^n x)_n$ is a Cauchy sequence that has a subsequence converging to Ux , $T^n x \rightarrow Ux$. Hence, $J^{mix}(T, x) = J(T, x) = \{Ux\}$.

b) Suppose that $y \in J(T, x_1 + x_2)$. Then there exists a sequence $(w_n)_n$ in X and a strictly increasing sequence of positive integers $(k_n)_n$ such that $w_n \rightarrow x_1 + x_2$ and $T^{k_n} w_n \rightarrow y$. Now we have

$$w_n - x_1 \rightarrow x_2, \quad T^{k_n}(w_n - x_1) \rightarrow y - Ux_1$$

and

$$w_n - x_2 \rightarrow x_1, \quad T^{k_n}(w_n - x_2) \rightarrow y - Ux_2.$$

Hence, we get

$$y - Ux_1 \in J(T, x_2), \quad y - Ux_2 \in J(T, x_1),$$

and so

$$2y - U(x_1 + x_2) \in J(T, x_1) + J(T, x_2),$$

which gives the result. Putting $k_n = n$, by the same method we conclude (b) for J^{mix} -sets. □

Recall that for $T \in B(X)$, by $\{T\}'$ we mean the collection of all bounded operators on X , which commutes with T .

Lemma 2.3 Suppose that $T \in B(X)$ and $U \in \{T\}'$. For all positive integers m and $x \in X$, we have:

a) $U^m(J(T, x)) \subseteq J(T, U^m x)$,

b) $U^m(J^{mix}(T, x)) \subseteq J^{mix}(T, U^m x)$,

and equality holds if U has a bounded inverse. Moreover, if U is surjective, then A_T is a U -invariant subset and A_T^{mix} is a U -invariant subspace of X .

Proof See Lemma 2.6 in [2] for parts (a) and (b). If U has a bounded inverse, then for $z \in J(T, U^m x)$, there exists a strictly increasing sequence of positive integers $(k_n)_n$ and a sequence $(u_n)_n$ in X such that $u_n \rightarrow U^m x$ and $T^{k_n} u_n \rightarrow z$. We therefore get

$$U^{-m} u_n \rightarrow x, \quad T^{k_n}(U^{-m} u_n) \rightarrow U^{-m} z.$$

This yields that $U^{-m} z \in J(T, x)$ and hence $z \in U^m(J(T, x))$. Putting $k_n = n$, we can prove it for J^{mix} -sets. Now if $x \in A_T$, then $J(T, x) = X$, and so we have

$$X = U(X) = U(J(T, x)) \subseteq J(T, Ux) \subseteq X.$$

Hence, $U(A_T) \subseteq A_T$. Similarly, we can prove that A_T^{mix} is a U -invariant subspace. □

A simple consequence of Lemma 2.11 in [6] yields that if x is a J -vector (J^{mix} -vector) for T , then for any nonzero scalar λ , λx is a J -vector (J^{mix} -vector) for T . If $A_T(A_T^{mix})$ is nonempty, then it is an infinite set.

3. On the property of J -reflexive operators

In this section, we first define the J -reflexive property, and then some properties of J -reflexive operators are investigated. We also state and prove necessary and sufficient conditions for an operator to be J -reflexive. Finally, we express J -reflexivity in terms of subsets. From now on, for simplicity, we denote the closure of a subset A in the strong operator topology by \overline{A}^{SOT} .

Definition 3.1 We call $T \in B(X)$ a J -reflexive operator if every bounded operator that leaves invariant $J(T, x)$ (for all x in X) is contained in the $\overline{\text{orb}(T)}^{SOT}$. J^{mix} -reflexivity can also be defined in a similar way.

It is clear that every J -reflexive operator on a Hilbert space is orbit-reflexive, but there exists an orbit-reflexive operator that is not J -reflexive. See the following example.

Example 3.2 Suppose that U is a bilateral shift on $l^2(\mathbb{Z})$, i.e. $Uf(n) = f(n - 1)$ where $f \in l^2(\mathbb{Z})$. Then U is orbit-reflexive, since U is a normal operator [11]. We know that $U^{-1}f(n) = f(n + 1)$, so U^{-1} does not belong to $\overline{\text{orb}(U)}^{SOT}$. By Proposition 2.8 in [6], U^{-1} leaves invariant J -sets of U , so U is not J -reflexive.

It is easy to see that if $T \in B(X)$, and for all $x \in X$, $J(T, x)$ is an empty set or the whole space X or the singleton $\{0\}$, then T is not J -reflexive. For example, hypercyclic operators are not J -reflexive, since their J -sets are the whole space. However, the identity operator I is the only bounded operator for which $J(I, x) = \{x\}$ for all x . Obviously, I is J -reflexive.

Recall that if T is an operator, then the set $\{\lambda \in \mathbb{C} : \lambda I - T \text{ is not invertible}\}$ is called the spectrum of T and it is denoted by $\sigma(T)$. Now we investigate some properties of J -reflexive operators.

Theorem 3.3 Let $T \in B(X)$ be a J -reflexive operator. Then we have the following:

- (a) If T is invertible, then $(T^n)_n$ has a subsequence that converges to I in the strong operator topology.
- (b) If T is not identity and $(T^n)_n$ is a Cauchy sequence, then $0 \in \sigma(T)$.
- (c) If T is not identity and is invertible, then $(T^n)_n$ diverges.

Proof a) Since by Proposition 2.8 in [6], T^{-1} leaves invariant $J(T, x)$ for all x , by the hypothesis there exists a sequence $(n_k)_k$ of positive integers such that $T^{-1}x = \lim_k T^{n_k}x$ for all x . Thus, $T^{n_k+1} \rightarrow I$ in the strong operator topology.

b) On the contrary, suppose that $0 \notin \sigma(T)$, so T is invertible, and by part (a), there exists a sequence of positive integers $(n_k)_k$ such that $T^{n_k} \rightarrow I$ in the strong operator topology. Since $B(X)$ is a Banach space and $(T^n)_n$ is a Cauchy sequence, it follows that $T^n \rightarrow I$ in the strong operator topology. Similarly, since $T^{n_k+1} \rightarrow T$ in the strong operator topology, we have $T^n \rightarrow T$ in the strong operator topology and so $T = I$, which is a contradiction.

c) If $(T^n)_n$ converges, then by part (b), $0 \in \sigma(T)$, which is a contradiction. □

Lemma 3.4 If $T \in B(X)$ is a J -reflexive operator, then $\|T\| \geq 1$.

Proof Suppose that $\|T\| < 1$. Then, since $\|T^n x\| \leq \|T\|^n \|x\|$, we have

$$J^{mix}(T, x) = J(T, x) = \{0\}$$

for all x . Hence, T can not be J -reflexive, which is a contradiction. □

Isomorphisms preserve the J -reflexive property, as follows.

Theorem 3.5 *Suppose that X and Y are Banach spaces and $S : X \rightarrow Y$ is an isomorphism. If T is a J -reflexive operator on X , then STS^{-1} is also a J -reflexive operator on Y .*

Proof First, we show that $J(STS^{-1}, y) = S(J(T, S^{-1}y))$. Note that since S is an isomorphism, S and S^{-1} are bounded. If $z \in J(STS^{-1}, y)$, then there exist $(y_n)_n$ in Y and a strictly increasing sequence of positive integers $(k_n)_n$ such that $y_n \rightarrow y$ and $\lim_n ST^{k_n}S^{-1}y_n = z$. Since $S^{-1}y_n \rightarrow S^{-1}y$, we get $S^{-1}z \in J(T, S^{-1}y)$ and so $z \in S(J(T, S^{-1}y))$. Thus,

$$J(STS^{-1}, y) \subseteq S(J(T, S^{-1}y)).$$

Conversely, if $z \in S(J(T, S^{-1}y))$, then there exist $x \in X$, $(u_n)_n \subseteq X$, and a strictly increasing sequence of positive integers $(k_n)_n$ such that

$$z = Sx, u_n \rightarrow S^{-1}y \text{ and } \lim_n T^{k_n}u_n = x.$$

Putting $v_n = Su_n$, then $v_n \rightarrow y$ and we have

$$z = Sx = \lim_n ST^{k_n}S^{-1}v_n.$$

Hence, $z \in J(STS^{-1}, y)$. Now if $W \in B(Y)$ leaves invariant the sets $J(STS^{-1}, y)$, then for all $y \in Y$, we have

$$WS(J(T, S^{-1}y)) \subseteq S(J(T, S^{-1}y)).$$

Therefore, we get

$$S^{-1}WS(J(T, S^{-1}y)) \subseteq J(T, S^{-1}y).$$

By the J -reflexivity of T , there exists a sequence $(n_k)_k$ of positive integers such that

$$S^{-1}WSx = \lim_k T^{n_k}x$$

for all $x \in X$. Putting $u = Sx$, we have

$$Wu = \lim_k (STS^{-1})^{n_k}u$$

for all $u \in Y$ and the proof is complete. □

In the following theorem, the sufficient conditions for the J -reflexivity of an operator are given.

Theorem 3.6 *Let $T \in B(X)$. If T satisfies one of the following statements, then T is J -reflexive:*

- a) T is a power bounded operator that has no periodic point and $x \in J(T, x)$ for all $x \in X$.
- b) T is power bounded and invertible, and there exists $m \in \mathbb{N}$ such that $T^m x \in J(T, x)$ for all $x \in X$.
- c) T is invertible and there exists positive integer m such that $T^m = I$.

Proof Let T satisfy in (a) and suppose on the contrary that T is not J -reflexive. Then there exists an operator $S \in B(X) \setminus \overline{\text{orb}(T)}^{SOT}$ such that

$$S(J(T, x)) \subseteq J(T, x)$$

for all $x \in X$. Therefore, there exist $x_0 \in X$ and $\delta > 0$ such that $B(Sx_0; \delta) \cap orb(T, x_0)$ is a finite set. Since $x_0 \in J(T, x_0)$, $Sx_0 \in J(T, x_0)$, and this means that there exists a strictly increasing sequence of positive integers $(k_n)_n$ such that

$$Sx_0 = \lim_n T^{k_n} x_0.$$

This is possible only if x_0 is a periodic point for T , which is a contradiction. Now suppose that (b) holds. Let $W \in B(X)$ be such that $W(J(T, x)) \subseteq J(T, x)$ for all x . The surjectivity of T yields the surjectivity of T^m and therefore for every $z \in X$ there exists $x \in X$ such that $z = T^m x$. Since for all x , $T^m x \in J(T, x)$, we have $W(T^m x) \in J(T, x)$ and therefore there exists a strictly increasing sequence of positive integers $(k_n)_n$ such that

$$Wz = W(T^m x) = \lim_n T^{k_n} x = \lim_n T^{k_n - m} z.$$

Thus, $W \in \overline{orb(T)}^{SOT}$ and T is J -reflexive. Finally, assume that (c) holds. Since $T^m = I$, $orb(T)$ is a finite set and therefore T is power bounded. On the other hand, for all x , $Tx \in J(T^m, Tx)$, and then by Lemma 2.1 (c), $Tx \in J(T, Tx)$. Eventually, Lemma 2.1 (a) (equality holds) yields that $Tx \in J(T, x)$. Now by using (b), we get the desired result. \square

Recall that the space of convergent sequences is usually denoted by c . This is a Banach space over \mathbb{C} or \mathbb{R} under the supremum norm.

Example 3.7 Define $T : c \rightarrow c$ by

$$T(x_1, x_2, x_3, \dots) = (x_2, x_1, x_3, \dots).$$

Clearly T is invertible, and $\|T\| = 1$. Since for any $x \in X$ and all $n \in \mathbb{N}$ we have $T^{2n+1}x = Tx$, thus $(T^2)^n(Tx) = Tx$ and so $Tx \in J(T^2, x)$. Now $Tx \in J(T, x)$ by Lemma 2.1 (c). Indeed, $J(T, x) = \{x, Tx\}$ for all x . Hence, T is J -reflexive by Theorem 3.6 (b).

Example 3.8 Let $p(A)$ be a permutation p on a finite set A . Define S_{ij} on c by

$$S_{ij}(x_1, x_2, \dots) = (x_1, \dots, x_{i-1}, p\{x_i, \dots, x_j\}, x_{j+1}, \dots).$$

If m is the smallest positive integer satisfying $p^{m+1} = p$, then $S_{ij}^{m+1}x = S_{ij}x$. Thus, $S_{ij}x \in J(S_{ij}^m, x)$ and by Lemma 2.1 (c), $S_{ij}x \in J(S_{ij}, x)$. Indeed,

$$J(S_{ij}, x) = \{x, S_{ij}x, \dots, S_{ij}^{m-1}x\}.$$

Now by Theorem 3.6 (b), it follows that S_{ij} is J -reflexive.

Example 3.9 Let H be a real Hilbert space and $\Lambda : B(H) \rightarrow B(H)$ be defined by $\Lambda(T) = T^*$. Then $\Lambda^2 = I$ and Λ is J -reflexive by Theorem 3.6 (c).

Next we present some examples of J -reflexive and non- J -reflexive operators on finite dimensional spaces.

Example 3.10 Let T be defined on the space of complex $n \times n$ matrices by $T(A) = A^t$ where A^t is the transpose of A . Since $T^2 = I$, T is J -reflexive by Theorem 3.6 (c).

Example 3.11 Suppose that $\dim X = 1$ and T is defined on X by $Tx = (1/2)x$. Then $\|T\| < 1$, and by Lemma 3.4, it follows that T is not J -reflexive.

The concept of J -reflexivity can be expressed in terms of subsets as follows.

Definition 3.12 Let T be a bounded linear operator on a Banach space X and M be a subset of X . We call T M - J -reflexive if every $W \in B(X)$ that leaves invariant $J(T, x)$ for all x in M is contained in $\overline{\text{orb}(T)}^{SOT}$. M - J^{mix} -reflexivity can also be defined in a similar way.

It is clear that M - J -reflexivity implies J -reflexivity. The converse is true for power bounded operators and dense subsets.

Theorem 3.13 If $T \in B(X)$ is J -reflexive and power bounded and M is a dense subset of X , then T is M - J -reflexive.

Proof Assume that $W \in B(X)$ leaves invariant $J(T, z)$ for all $z \in M$. It is enough to show that W leaves invariant $J(T, x)$ for all $x \in X$. Suppose that $x \in X$ and $y \in J(T, x)$. Then there exists a strictly increasing sequence of positive integers $(k_n)_n$ such that $y = \lim_n T^{k_n}x$. By density of M , we can find sequences $(x_n)_n$ and $(y_n)_n$ in M such that $x_n \rightarrow x$ and $y_n \rightarrow y$. Thus, for every $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $n \geq N$, we have

$$\|x_n - x\| < \epsilon/3L, \|y_n - y\| < \epsilon/3, \|y - T^{k_n}x\| < \epsilon/3$$

, where L is a power bound of T . Now for $m, n \geq N$, we get

$$\|y_m - T^{k_n}x_m\| \leq \|y_m - y\| + \|y - T^{k_n}x\| + \|T^{k_n}\| \|x - x_m\| < \epsilon.$$

Thus, $y_m \in J(T, x_m)$ and therefore $Wy_m \in J(T, x_m)$ for $m \geq N$. Since Wy_m tends to Wy , by applying the same method that has been used in the proof of Lemma 2.5 in [5], we obtain that $Wy \in J(T, x)$. This completes the proof. \square

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