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Effects of adding crown variables in stem taper and volume predictions for black pine

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Abstract: Crown characteristics are an important component of growth and yield models. The stem form of a tree depends to a large extent on the tree crown dimensions. However, there is no unified agreement on whether crown variables (crown ratio and crown length) should be incorporated into taper models for accurate predictions of diameter and volume. The purpose of this study was to investigate the level of improvement that the incorporation of crown variables into taper equations for black pine (*Pinus nigra* J.F.Arnold subsp. *pallasiana* (Lamb.) Holmboe) gives to diameter and stem volume predictions.. Linear and nonlinear crown variable functions were incorporated into a compatible volume and taper equation to evaluate their effect in model prediction accuracy. The inclusion of crown variables provided significantly different parameter estimates but only resulted in a modest reduction of fitting statistics for both diameter and volume with lower average bias and lower mean squared errors for all modified models, although the improvement in stem volume prediction was minimal. Based on the results, the improvement obtained by the inclusion of crown variables is not enough to justify the additional costs in measuring crown variables of standing trees for volume estimation.

Key words: Crown ratio, crown length, diameter, segmented taper model

1. Introduction

Stem taper and stem volume are the 2 most valuable tree characteristics for determining the value of a tree and are the primary interest of most growth-model users (Weiskittel et al. 2011). The stem form of a tree is strongly influenced by its crown size and position (crown length [CL, m], crown ratio [CR], and crown height [CH, m]). The crown contains the foliage, the photosynthetic structure that provides carbohydrates for the growth and development of the whole tree (Larson 1963; Leites and Robinson 2004). The changes in the size of the live crown, the distribution of branches within the crown, and the length of the branch-free bole are tree attributes that create variations in stem taper (Larson 1963).

Generally, stem form has been modeled using taper models. These models are the mathematical expression of the change in stem diameter as a function of stem height and are particular for tree species, stand age, density, and the many factors that affect site quality (Muhairwe 1994). Taper equations are usually used in forestry practice to estimate tree diameter in-bark (*dib*, cm) or over-bark (*dob*, cm) at any given height (h_i, m). By integrating the stem profile model between 2 given heights, it is possible to estimate the total volume contained in that portion of stem, thereby allowing a flexible size classification for the products of the stem (Calama and Montero 2006). However, stem boles cannot be completely described in simple mathematical models. Numerous models of varying complexity have been proposed in an attempt to describe tree taper (Kozak et al. 1969; Max and Burkhart 1976; Fang et al. 2000). In most mathematical models, taper is modeled in terms of diameter at breast height (*dbh*, cm) and total tree height (*h*, m).

Analyzing and comparing the influences of site conditions and stand treatments on stem formation is becoming increasingly important (Lee et al. 2003). Several attempts have been made using auxiliary variables to increase the accuracy of existing stem profile models, including crown variables, stand and site variables, and upper stem diameter measurements (Trincado and Burkhart 2006).

Earlier studies have demonstrated different results on the benefit of incorporating crown variables in stem profile equations. Some studies (e.g., Matney and Sullivan 1979; Valenti and Cao 1986) showed that the addition of a CR term to the model leads to a significant improvement. However, other studies (e.g., Burkhart and Walton 1985; Muhairwe et al. 1994) found no improvement for several

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species studied. Valentine and Gregoire (2001) included CH information in modeling the taper of sweet gum and noted improved predictive accuracy, but found no improvement for slash pine and ponderosa pine. Leites and Robinson (2004) found a significant relationship between crown variables (CL and CR) and the estimated random effects parameters in the Max and Burkhart (1976) stem-profile model for loblolly pine; they also found that accuracy of the original equation was improved. Jiang et al. (2007), using data from yellow poplar trees, tested the validity of adding CR as a predictor variable in segmented taper and volume models. The results demonstrated that inclusion of the CR variable provided significantly different parameter estimates but only modest improvement in the prediction of stem diameter and volume. Jiang and Liu (2011) modified Max and Burkhart's (1976) taper equation to include functions of crown ratio and stand density for Dahurian larch in northeastern China. They found that the inclusion of these variables provided more accurate predictions for both over-bark and in-bark diameters.

As can be seen in the above-mentioned studies, CR is the crown variable most widely used. However, CL is an interesting variable that may influence the prediction of diameter and volume. As indicated by Mäkelä (2002), differences in the form of the main stem have been associated mostly with differences in CL. For this reason, some forms of CR and CL functions were incorporated into the tree-stem taper and volume prediction models.

The aims of this study were to investigate the relationship between crown variables (crown ratio and crown length) and parameter estimates in a segmented taper model for black pine (*Pinus nigra* J.F.Arnold subsp. *pallasiana* (Lamb.) Holmboe) and to develop compatible taper and volume equations that incorporate crown variables in their formulation.

2. Materials and methods

The data used in this study were collected for black pine (*Pinus nigra* J.F.Arnold subsp. *pallasiana* (Lamb.) Holmboe) from even-aged managed stands of Beyşehir Forest Enterprise, Turkey. The trees were felled throughout the clear-cutting areas of Beyşehir Forest Enterprise and were subjectively selected to provide representative information for variety in density, height, stand structure, age, and site condition throughout the clear-cutting areas. Trees possessing multiple stems, broken tops, large knots, stem deformations, obvious cankers, or crooked boles were not included in the sample. Total height (*h*) and height to the live crown (*CH*) were measured to the nearest 0.05 m on felled trees. Diameter over-bark and in-bark at breast height (1.3 m) was measured and recorded to the nearest 0.1 cm using digital calipers. Diameters (*dob* and *dib*) at 1-m intervals from stump height to the top of the tree were measured. Diameter records were the average of 2 measurements taken at a perpendicular position to each other along the axis of the tree bole. The arithmetic mean was chosen rather than the geometric mean, since our main interest was in predicting diameter at a given height rather than sectional area, as indicated by Leites and Robinson (2004). The height to the base of the live crown was determined by identifying the point along the bole where the lowest live branch or branch whorl was attached to the main bole, as indicated by Jiang et al. (2007). Finally, CL and CR were derived from crown measurements. CR was defined as the ratio between the length of the live crown and the total tree height. Actual volume for each section and tree was calculated by applying the overlapping bolts method as described by Bailey (1995).

All data were plotted by relative diameter over relative height to investigate possible anomalies in the data (Figure 1). The systematic approach proposed by Bi (2000) for

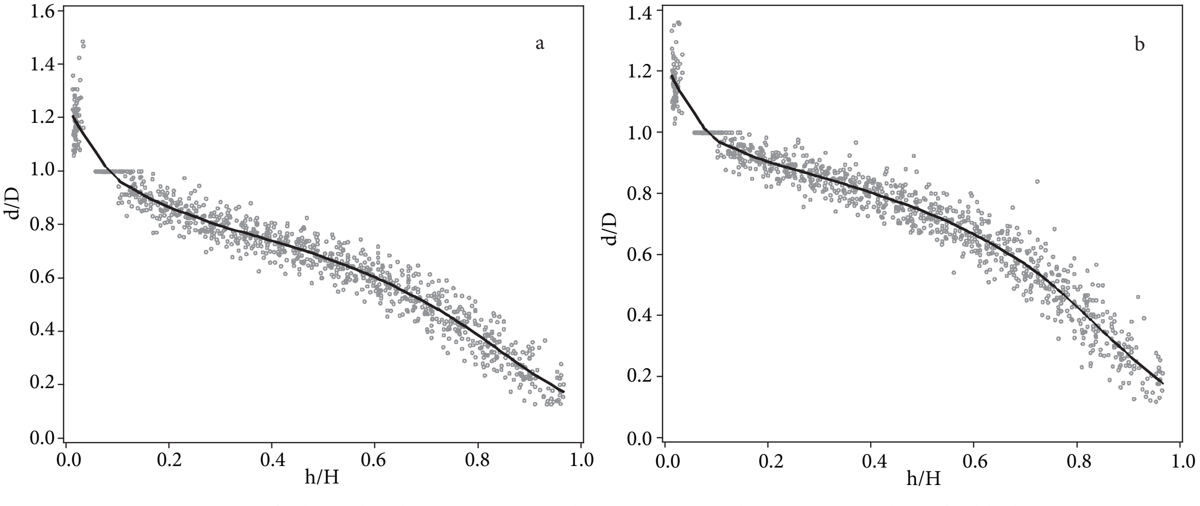

Figure 1. Plot of relative height versus relative diameter over-bark (a) and in-bark (b) for black pine.

detecting abnormal data points was applied to increase the efficiency of the process. For this reason, a nonparametric taper curve was fitted by local regression, using the LOESS procedure. This involved local quadratic fitting with a smoothing parameter of 0.3 for the dataset, which was selected after iterative fitting and visual examination of the smoothed taper curves overlaid on the data. The extreme values accounted for less than 0.16% of the whole dataset. All trees with total height of less than 5.3 m were eliminated, as they could not be used to fit the selected segmented taper equation, as explained later. Sample trees were selected to adequately represent the distribution of these trees in the population in terms of their respective diameter and height classes. Descriptive statistics for the data sets are presented in Table 1.

2.1. Taper and volume equations

As stated by Li and Weiskittel (2010), the effects of crown variables on taper equations depend on species, site quality, the range of crown size in fitting the dataset, and the particular taper equation used. Although many different equations were considered and explored, a modified form of the segmented taper model published by Clark et al. (1991) was used in this study, as it showed better-fitting statistics than others (e.g., Max and Burkhart 1976; Fang et al. 2000; Li and Weiskittel 2010). Earlier research by Jiang et al. (2005) showed that this equation appears to be more accurate than other model formulations for estimating diameter, height, and volume.

Table 1. Summary statistics for black pine for model fitting data.

	Model growth data (70 trees)			
	Mean	SD	Minimum	Maximum
Diameter at breast height (D), (*dob*), (cm)	31.32	9.29	10.90	65.00
Total height (H), (m)	16.42	3.12	8.90	22.50
Disk diameter (*dob*), (cm)	21.35	10.11	4.30	85.00
Disk diameter (*dib*), (cm)	17.99	8.29	3.40	72.00
Disk height (m)	7.59	4.86	0.30	21.30
F_{dob}	24.37	8.24	6.70	53.70
F_{dib}	20.83	7.08	5.40	46.90
Crown length (CL), (m)	8.43	1.75	3.70	12.00
Crown ratio (CR)	0.51	0.05	0.40	0.64

dob: diameter over-bark; *dib*: diameter in-bark; SD: standard deviation.

$$d = \left\{ \begin{aligned} & I_S D^2 \left\{ 1 + \left[\frac{\left(1 - \frac{h}{H}\right)^{b_1} - \left(1 - \frac{1.3}{H}\right)^{b_1}}{1 - \left(1 - \frac{1.3}{H}\right)^{b_1}} \right] \right\} \\ & + I_B \left\{ D^2 - \left(D^2 - F^2\right) \left[\frac{\left(1 - \frac{1.3}{H}\right)^{b_2} - \left(1 - \frac{h}{H}\right)^{b_2}}{\left(1 - \frac{1.3}{H}\right)^{b_2} - \left(1 - \frac{5.3}{H}\right)^{b_2}} \right] \right\} \\ & + I_T \left\{ F^2 \left[b_4 \left(\frac{h - 5.3}{H - 5.3} - 1 \right)^2 + I_M \left(\frac{1 - b_4}{b_2^3} \right) \left(b_3 - \frac{h - 5.3}{H - 5.3} \right)^2 \right] \right\} \end{aligned} \right\}^{0.5} \tag{1}$$

The volume equation derived through integration of Eq. (1) is of the form:

$$V = k\left\{\begin{array}{l} I_1 D^2\left[(1-GW)(U_1-L_1)+W\left[\dfrac{\left(1-\dfrac{L_1}{H}\right)^{b_1}(H-L_1)-\left(1-\dfrac{U_1}{H}\right)^{b_1}(H-U_1)}{(b_1+1)}\right]\right] \\ +I_2 I_3\left[T(U_2-L_2)+Z\left[\dfrac{\left(1-\dfrac{L_2}{H}\right)^{b_2}(H-L_2)-\left(1-\dfrac{U_2}{H}\right)^{b_2}(H-U_2)}{(b_2+1)}\right]\right] \\ +I_4 F^2\left[\begin{array}{l} b_4(U_3-L_3)-\dfrac{b_4((U_3-5.30)^2-(L_3-5.30)^2)}{(H-5.30)}+\dfrac{\dfrac{b_4}{3}((U_3-5.30)^3-(L_3-5.30)^3)}{(H-5.30)^2} \\ +\dfrac{I_5\left(\dfrac{1}{3}\right)(1-\dfrac{b_4}{b_3^2})(b_3(H-5.30)-(L_3-5.30))^3}{(H-5.30)^2} \\ -\dfrac{I_6\left(\dfrac{1}{3}\right)(1-\dfrac{b_4}{b_3^2})(b_3(H-5.30)-(U_3-5.30))^3}{(H-5.30)^2} \end{array}\right] \end{array}\right\} \quad (2)$$

where D is diameter at breast height over-bark (1.3 m above ground, cm); d is diameter over-bark (cm) to the measurement point at height h; H is total tree height (m); h is height above the ground to the measurement point (m); V is total stem volume over-bark and in-bark from stump (m^3); b_1 is the regression coefficient for stem height below 1.3 m; b_2 is the regression coefficient for stem height between 1.3 and 5.3 m; b_3, b_4 are the regression coefficients for stem height above 5.3 m; F is diameter over-bark (cm) at 5.3 m above ground; U is the upper height of interest in cm; L is the lower height of interest in cm; L_1, L_2, L_3, U_1, U_2, U_3 are combined variables; and $k = 0.0000785$. For further details on definitions of basic symbols, indicator variables, parameters, and combined variables used in the modified segmented taper model, see the papers by Clark et al. (1991) and Jiang et al. (2005).

$$I_S = \begin{cases} 1 & h < 1.30 \\ 0 & \text{otherwise} \end{cases} \qquad I_B = \begin{cases} 1 & 1.30 \le h\ 55.30 \\ 0 & \text{otherwise} \end{cases}$$

$$I_T = \begin{cases} 1 & h > 5.30 \\ 0 & \text{otherwise} \end{cases} \qquad I_M = \begin{cases} 1 & h < (55.30 + b_3(H - 5.30) \\ 0 & \text{otherwise} \end{cases}$$

$$G = (1 - \frac{1.30}{H})^{b_1} \qquad W = \frac{1}{1 - G}, \qquad x = \left(\frac{1 - 1.30}{H}\right)^{b_2}, \qquad y = \left(\frac{5.30}{H}\right)^{b_2}, \qquad z = \frac{(D^2 - F^2)}{(x - y)}$$

$T = D^2 - ZX$, $L_1 = \max(L, 0.30)$, $L_2 = \max(L_2, 1.30)$, $L_3 = \max(L, 5.30)$,

$U_1 = \min(U, 1.30)$, $U_2 = \min(U, 5.30)$, $U_3 = \min(U, H)$.

$$I_1 = \begin{cases} 1 & L < 1.30 \\ 0 & \text{otherwise} \end{cases} \qquad I_2 = \begin{cases} 1 & L < 5.30 \\ 0 & \text{otherwise} \end{cases} \qquad I_3 = \begin{cases} 1 & U > 1.30 \\ 0 & \text{otherwise} \end{cases} \qquad I_4 = \begin{cases} 1 & U > 5.30 \\ 0 & \text{otherwise} \end{cases},$$

$$I_5 = \begin{cases} 1 & (L_3 - 5.30) < b_3(H - 5.30) \\ 0 & \text{otherwise} \end{cases}, \qquad I_6 = \begin{cases} 1 & (U_3 - 5.30) < b_3(H - 5.30) \\ 0 & \text{otherwise} \end{cases}$$

Two crown variables, CL and CR, were included in the best-fitting model identified for black pine. Burkhart and Walton (1985) evaluated both linear and nonlinear CR functions in loblolly pine. The forms of CR and CL functions used in this study are given below.

$$\lambda_1 + \lambda_2\,(CR) \tag{3}$$

$$\lambda_1 + \lambda_2\,(CL) \tag{4}$$

$$\lambda_1 + \lambda_2\,(CL) + \lambda_2\,(CR) \tag{5}$$

$$\lambda_1 + \lambda_2\,(CR)^{\lambda_3} \tag{6}$$

$$\lambda_1 + \lambda_2\,(CL)^{\lambda_3} \tag{7}$$

Here, λ_1 are the parameters to be estimated from the data. Eqs. (3) through (7) were incorporated into the existing taper (Eq. (1)) and volume (Eq. (2)) models in order to ascertain the effects of incorporating crown variables into the existing model forms for black pine.

2.2. Model comparison

As recommended by Kozak and Smith (1993), 4 statistical criteria obtained from residuals were examined to compare the performance of tested models: the mean bias (B), the standard error of the estimate (SEE), the mean absolute error (MAE), and a fit index (FI). The average B and SEE were used as lack-of-fit statistics and calculated by relative height class for diameter over-bark and in-bark and volume estimations.

$$B = \frac{\sum_{i=1}^{n} (Y_i - Y_i)}{n} \tag{8}$$

$$SEE = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - Y_i)^2}{df}} \tag{9}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n} \left| Y_i - \widehat{Y}_i \right| \tag{10}$$

$$FI = 1 - \left[\frac{\sum_{i=1}^{n} (Y_i - \widehat{Y}_i)^2}{\sum_{i=1}^{n} (Y_i - \overline{Y}_i)^2} \right] \tag{11}$$

Here, Y_i = observed value for the i*th* observation, Y_i = predicted value for the i*th* observation, Y_i = mean of the Y_i, df = degrees of freedom of the model, and *n* = number of observations in the dataset.

Seemingly unrelated regression (SUR) was used to estimate the coefficients for both equations. SUR can be used to simultaneously estimate equation parameters while concurrently minimizing both diameter and volume predictions in the presence of cross-correlated error structures. The models were independently fitted to both over-bark and in-bark data using the SAS/ETS (SAS Institute, 2002) model procedure. Although measurements within the same tree are not independent, autocorrelation of errors was not taken into account because prediction accuracy is little affected by the correlated error structure and the correlated errors structure is accounted for in the equation fitting process (Kozak 1997; Williams and Reich 1997; Jiang et al. 2007). As stated by Jiang and Liu (2011), autocorrelation is generally ignored and has no use in forestry applications.

The nonlinear extra sum of squares procedure, as demonstrated by Bates and Watts (1988), was used in this study to determine whether the overall addition of the CR and CL significantly influences the taper equation (Neter et al. 1996). This method is based on the likelihood ratio test for detecting simultaneous homogeneity among parameters, and it requires the fitting of both a full and a reduced model. In this study, the full model corresponds to different sets of parameters for CR, CL, or the combination of both. The reduced model corresponds to the model without crown variables. The appropriate test statistic is an F-test given by Bates and Watts (1998):

$$F = \frac{(SSE_R - SSE_F)/(df_R - df_F)}{SSE_F/df_F} \tag{12}$$

where SSE_R is the error sum of squares associated with the reduced model and its degrees of freedom, written as df_R. SSE_F is the error sum of squares associated with the full model and its degrees of freedom, written as df_F. Generally, the F-test is significant if the P-value for the test is less than 0.05.

3. Results

First, a full model considering crown variables in all parameters was fitted, but convergence was not achieved when employing either the linear crown variables in Eqs. (3)–(5) or the nonlinear form of the crown variables in Eqs. (6) and (7). All combinations of CR, CL, and both within each parameter were then tested, but only the replacement of b_4 with linear Eqs. (3), (4), and (5) resulted in significant parameter estimates ($P < 0.0001$). Parameter estimates for the different taper and volume equations are presented in Tables 2 and 3 for over-bark and in-bark diameters, respectively.

Model *OM* represents the original model forms without the addition of the crown variable function, while models *MCR*, *MCL*, and *MCRCL* represent the modified model incorporating Eqs. (3), (4), and (5) for the b_4 parameter, respectively. The performance of each model for describing tree taper and volume prediction was evaluated (Tables 4 and 5). Based on the fitting statistics, inclusion of the linear CR and linear CR with CL functions improved the fit for *dob* and *dib* taper equations for black pine. Slightly better results were obtained for *dob*, although the overall improvements were small.

The inclusion of crown variables had a positive effect for all fitting statistics except average bias in the *MCRCL* model for *dib* prediction (Tables 4 and 5) when using the actual upper diameter measurement at 5.30 m. Models including crown variables, as compared with the original model form, showed an average decrease of 1.8% in MAE (std. dev. = 0.6), 1.1% in SEE (std. dev. = 0.4), and 64% in B (std. dev. = 37.4).

The additions of linear functions of CR, CL, and both improved model performance by reducing SEE 0.9% and 1.2%, MAE 2.0% and 1.5%, and E 69.2% and 58.7% for *dob* and *dib* estimation, respectively. The *MCR*, *MCL*, and *MCRCL* models explained more than 98% of the total variation for predicting upper stem diameter for black pine. Most residuals clustered around 0 (Figure 2), and the residual variance along the stem was less heterogeneous in models containing crown variables.

Table 2. Parameter estimates for over-bark taper and volume equations for black pine.

Models	Parameter						
	b_1	b_2	b_3	b_4	λ_1	λ_2	λ_3
OM	38.6483	6.0012	0.8767	4.8199			
MCR	38.6740	6.0004	0.8768		4.7567	0.1322	
MCL	38.6107	6.0022	0.8771		4.2421	0.0636	
MCRCL	38.7278	5.9980	0.8772		4.9123	0.1462	–2.9166

OM, the original model forms for Eqs. (1) and (2); *MCR*, the model form with CR; *MCL*, the model form with CL; *MCRCL*, the model form with CR and CL.

Table 3. Parameter estimates for in-bark taper and volume equations for black pine.

Models	Parameter						
	b_1	b_2	b_3	b_4	λ_1	λ_2	λ_3
OM	40.5778	4.9663	0.8335	4.1451			
MCR	40.5718	4.9663	0.8336		3.9622	0.3754	
MCL	40.5672	4.9657	0.8346		3.7276	0.0469	
MCRCL	40.8159	4.9618	0.8355		4.0828	0.0919	–1.5438

OM, the original model forms for Eqs. (1) and (2); *MCR*, the model form with CR; *MCL*, the model form with CL; *MCRCL*, the model form with CR and CL.

Table 4. Fit statistics for Eqs. (1) and (2) fitted for over-bark for black pine trees.

Models	Taper (cm)				Volume (m^3)			
	B	SEE	MAE	FI	B	SEE	MAE	FI
OM	–0.0549	1.3848	0.9239	0.9813	0.0005	0.0048	0.0027	0.9859
MCR	–0.0445	1.3786	0.9129	0.9813	0.0005	0.0047	0.0026	0.9860
MCL	–0.0101	1.3687	0.9026	0.9816	0.0005	0.0047	0.0026	0.9860
MCRCL	0.0039	1.3678	0.8998	0.9817	0.0005	0.0047	0.0026	0.9861

OM, the original model forms for Eqs. (1) and (2); *MCR*, the model form with CR; *MCL*, the model form with CL; *MCRCL*, the model form with CR and CL.

Table 5. Fit statistics for Eqs. (1) and (2) for in-bark black pine trees.

Models	Taper (cm)				Volume (m^3)			
	B	SEE	MAE	FI	B	SEE	MAE	FI
OM	–0.0627	1.1595	0.7390	0.9804	0.0004	0.0035	0.0019	0.9845
MCR	–0.0522	1.1504	0.7315	0.9808	0.0004	0.0035	0.0018	0.9846
MCL	–0.0089	1.1434	0.7283	0.9810	0.0004	0.0034	0.0018	0.9845
MCRCL	–0.0165	1.1413	0.7245	0.9811	0.0004	0.0034	0.0018	0.9846

OM, the original model forms for Eqs. (1) and (2); *MCR*, the model form with CR; *MCL*, the model form with CL; *MCRCL*, the model form with CR and CL.

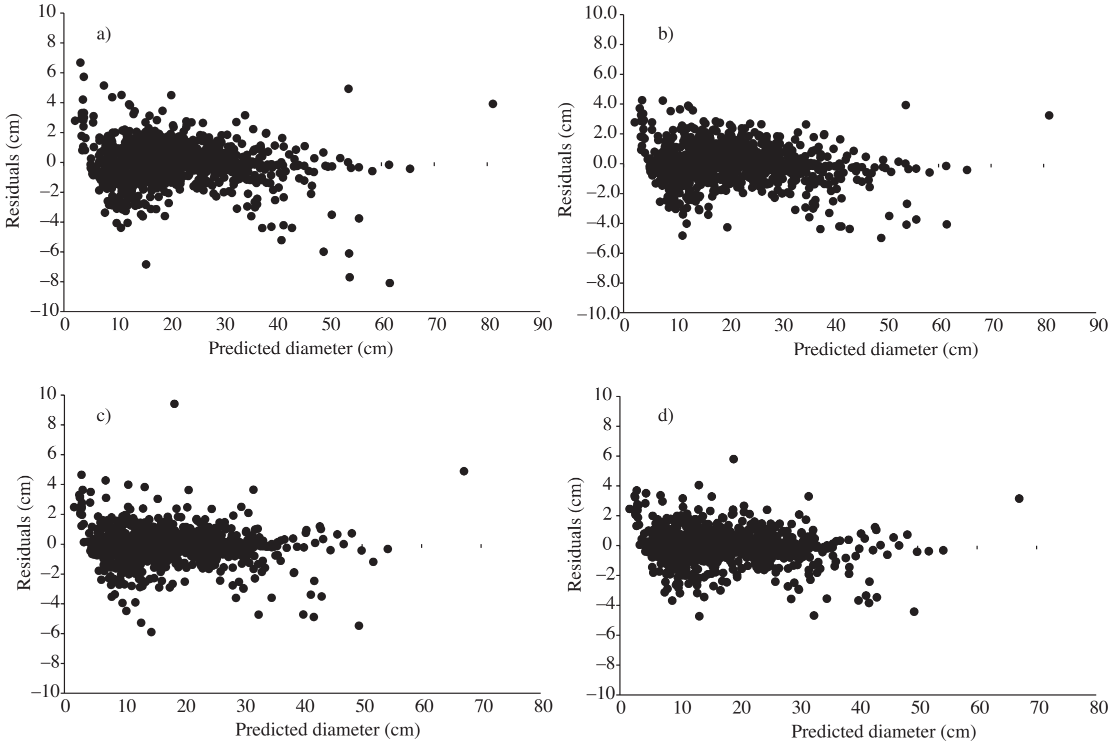

Figure 2. Residual plots for original taper model (a and c) and for taper model with CR and CL (b and d) for *dob* (a and b) and *dib* (c and d).

Fitting statistics (average B, SEE, MAE, and FI) for over-bark and in-bark volumes are presented in Tables 4 and 5, respectively. Significant improvements were observed due to the inclusion of CR, CL, and CR with CL for Eq. (2). For the volume function, the *MCRCL* model performed slightly better than the *MCR* and *MCL* models (Tables 4 and 5). All models overestimated volume; however, the modified models showed a lower average B for volume estimations. For total bole volume, the *MCRCL* equation had the highest FI and the lowest SEE values for both *dob* and *dib*.

To evaluate diameter and cubic meter volume prediction accuracy at different sections of the tree bole, the fit statistics were examined for 10 relative height (RH) classes. Average bias and SEE were calculated for each equation at different stem sections by relative height (10% interval) along the merchantable stem. The *MCR*, *MCL*, and *MCRCL* models showed consistent and good performance for estimating diameter by each stem section and did not exhibit large variation in any section (Tables 6 and 7). The modified models displayed similar bias and SEE trends for all sections. The SEE of the modified models was slightly lower than the original model for the bottom section. The SEE values for the middle section (from 30% to 60% of relative height) were improved by adding crown variables (Tables 6 and 7; *MCR*, *MCL*, and *MCRCL* models). For both Eqs. (1) and (2), the *MCR*, *MCL*, and *MCRCL* model forms performed better than the *OM* model above 50% of total height, with differences being similar in magnitude to those listed for the middle section. The *MCRCL* model performed slightly better than the *MCR* and the *MCL* models at the lower section (relative heights of <20%), middle section (relative heights of 30%–60%), and top section (relative heights of >70%) for *dob* and *dib* estimations (Tables 6 and 7). For the top section (relative heights of >80%), the inclusion of crown variables increased the SEE values.

Differences in cubic meter volume error by relative height class were also evaluated (Tables 8 and 9). The *MCR*, *MCL*, and *MCRCL* models showed slightly better performance than the *OM* model for over-bark and in-bark volume estimation of different stem sections. The *MCR*, *MCL*, and *MCRCL* models exhibited a similar upper stem volume prediction error for all sections. The equations with added crown variables were then more precise and less biased than the original model form for relative heights of 40%–70% of stem and relative heights of 50%–80% for over-bark volume and in-bark volume, respectively.

Results of the nonlinear least square fits for the full and reduced taper equations on the data are provided in Table 10. The F-test showed that there were significant differences for all the models when compared (P < 0.0001). Based on these results, crown variables clearly influence the taper of the trees used in this study.

Table 6. Bias and standard error of the estimate for over-bark diameter estimation by relative height (RH).

RH	*n*	*OM*		*MCR*		*MCL*		*MCRCL*	
		B (cm)	SEE (cm)	B (cm)	SEE (cm)	B (cm)	SEE (cm)	B (cm)	SEE (cm)
0.0–0.1	125	–0.5328	2.1879	–0.5326	2.1878	–0.5271	2.1851	–0.5274	2.1852
0.1–0.2	110	–0.2231	0.8822	–0.2231	0.8822	–0.2238	0.8825	–0.2237	0.8824
0.2–0.3	109	0.2565	0.6542	0.2565	0.6524	0.2522	0.6509	0.2530	0.6504
0.3–0.4	112	0.1901	0.6407	0.1954	0.6361	0.1839	0.6325	0.1881	0.6317
0.4–0.5	107	0.2002	0.7709	0.2214	0.7503	0.2451	0.7355	0.2490	0.7316
0.5–0.6	108	0.1544	0.8904	0.1737	0.9005	0.2415	0.8792	0.2365	0.8878
0.6–0.7	111	–0.0338	1.1880	–0.0093	1.1553	0.1177	1.1370	0.1037	1.1274
0.7–0.8	110	–0.3096	1.6188	–0.2867	1.6167	–0.1301	1.5747	–0.1505	1.5807
0.8–0.9	101	–0.7131	1.8770	–0.6973	1.8658	–0.5540	1.8278	–0.5750	1.8299
0.9–1.0	44	1.2885	2.6224	1.2646	2.6127	1.3137	2.6731	1.2940	2.6564
All	1037	–0.0549	1.3848	–0.0445	1.3786	–0.0100	1.3687	0.0039	1.3678
		FI	0.9813	FI	0.9813	FI	0.9816	FI	0.9817

OM, the original model forms for Eqs. (1) and (2); *MCR*, the model form with CR; *MCL*, the model form with CL; *MCRCL*, the model form with CR and CL.

Table 7. Bias and standard error of the estimate for in-bark diameter estimation by relative height (RH).

RH	*n*	*OM*		*MCR*		*MCL*		*MCRCL*	
		B (cm)	SEE (cm)	B (cm)	B (cm)	B (cm)	SEE (cm)	B (cm)	SEE (cm)
0.0–0.1	125	–0.5281	1.6076	–0.5243	1.6054	–0.5115	1.5979	–0.5116	1.5980
0.1–0.2	110	–0.1805	0.5978	–0.1806	0.5978	–0.1811	0.5981	–0.1811	0.5980
0.2–0.3	109	0.1041	0.4187	0.1040	0.4185	0.1005	0.4171	0.1014	0.4173
0.3–0.4	112	0.1281	0.4835	0.1339	0.4837	0.1263	0.4774	0.1308	0.4796
0.4–0.5	107	0.0962	0.7088	0.1183	0.7087	0.1421	0.6798	0.1444	0.6891
0.5–0.6	108	0.0502	0.7134	0.0706	0.7168	0.1327	0.7137	0.1237	0.7163
0.6–0.7	111	–0.0674	0.9809	–0.0432	0.9956	0.0649	0.9584	0.0446	0.9741
0.7–0.8	110	–0.2254	1.6438	–0.2061	1.5977	–0.0822	1.5908	–0.1090	1.5731
0.8–0.9	102	–0.4487	1.7052	–0.4336	1.6886	–0.3470	1.6740	–0.3653	1.6710
0.9–1.0	43	1.3387	2.1874	1.3109	2.1736	1.3312	2.2121	1.3135	2.1945
All	1037	–0.0627	1.1595	–0.0522	1.1504	–0.0089	1.1434	–0.0165	1.1413
		FI	0.9804	FI	0.9808	FI	0.9810	FI	0.9811

OM, the original model forms for Eqs. (1) and (2); *MCR*, the model form with CR; *MCL*, the model form with CL; *MCRCL*, the model form with CR and CL.

Table 8. Bias and standard error of the estimate for over-bark volume estimation by relative height (RH).

RH	*n*	*OM*		*MCR*		*MCL*		*MCRCL*	
		B (m^3)	SEE (m^3)	B (m^3)	SEE (m^3)	B (m^3)	SEE (m^3)	B (m^3)	SEE (m^3)
0.0–0.1	125	0.0015	0.0101	0.0015	0.0101	0.0015	0.0101	0.0015	0.0101
0.1–0.2	110	–0.0002	0.0042	–0.0002	0.0042	–0.0002	0.0042	–0.0002	0.0042
0.2–0.3	109	0.0013	0.0038	0.0013	0.0038	0.0013	0.0038	0.0013	0.0038
0.3–0.4	112	0.0007	0.0028	0.0007	0.0028	0.0007	0.0028	0.0007	0.0027
0.4–0.5	107	0.0011	0.0038	0.0012	0.0038	0.0013	0.0036	0.0013	0.0036
0.5–0.6	108	0.0007	0.0033	0.0008	0.0032	0.0009	0.0032	0.0009	0.0032
0.6–0.7	111	0.0003	0.0035	0.0003	0.0034	0.0005	0.0034	0.0005	0.0035
0.7–0.8	110	–0.0006	0.0036	–0.0006	0.0035	–0.0003	0.0036	–0.0004	0.0036
0.8–0.9	101	–0.0009	0.0031	–0.0009	0.0031	–0.0007	0.0031	–0.0008	0.0031
0.9–1.0	44	0.0008	0.0021	0.0008	0.0021	0.0007	0.0021	0.0007	0.0021
All	1037	0.0005	0.0048	0.0005	0.0047	0.0005	0.0047	0.0005	0.0047
		FI	0.9859	FI	0.9860	FI	0.9860	FI	0.9861

OM, the original model forms for Eqs. (1) and (2); *MCR*, the model form with CR; *MCL*, the model form with CL; *MCRCL*, the model form with CR and CL.

Table 9. Bias and standard error of the estimate for in-bark volume estimation by relative height (RH).

RH	*n*	*OM*		*MCR*		*MCL*		*MCRCL*	
		B (m^3)	SEE (m^3)	B (m^3)	SEE (m^3)	B (m^3)	SEE (m^3)	B (m^3)	SEE (m^3)
0.0–0.1	125	0.0014	0.0066	0.0014	0.0066	0.0014	0.0067	0.0014	0.0067
0.1–0.2	110	0.0001	0.0032	0.0001	0.0032	0.0001	0.0032	0.0001	0.0032
0.2–0.3	109	0.0009	0.0032	0.0009	0.0032	0.0009	0.0032	0.0009	0.0032
0.3–0.4	112	0.0005	0.0024	0.0006	0.0024	0.0005	0.0024	0.0005	0.0024
0.4–0.5	107	0.0008	0.0030	0.0009	0.0030	0.0009	0.0029	0.0009	0.0029
0.5–0.6	108	0.0004	0.0025	0.0004	0.0024	0.0005	0.0024	0.0005	0.0024
0.6–0.7	111	0.0003	0.0032	0.0003	0.0031	0.0005	0.0032	0.0004	0.0032
0.7–0.8	110	–0.0003	0.0034	–0.0003	0.0033	–0.0001	0.0033	–0.0002	0.0033
0.8–0.9	101	–0.0003	0.0023	–0.0003	0.0022	–0.0002	0.0022	–0.0003	0.0022
0.9–1.0	44	0.0006	0.0013	0.0006	0.0013	0.0006	0.0013	0.0006	0.0013
All	1037	0.0004	0.0035	0.0004	0.0035	0.0004	0.0035	0.0004	0.0034
		FI	0.9845	FI	0.9846	FI	0.9845	FI	0.9846

OM, the original model forms for Eqs. (1) and (2); *MCR*, the model form with CR; *MCL*, the model form with CL; *MCRCL*, the model form with CR and CL.

Table 10. F-test for modified Clark et al. (1991) taper model with and without CR, CL, and CR with CL.

Model pairs	Full model		Reduced model		F-value	P-value
	df$_F$	SSE$_F$	df$_R$	SSE$_R$		
Over-bark						
MCR-OM	1869.9	1032	1889.1	1033	10.596	<0.0001
MCL-OM	1845.7	1032	1889.1	1033	24.266	<0.0001
MCRCL-OM	1842.1	1031	1889.1	1033	13.150	<0.0001
In-bark						
MCR-OM	1302.5	1032	1323.6	1033	16.718	<0.0001
MCL-OM	1289.0	1032	1323.6	1033	27.701	<0.0001
MCRCL-OM	1283.3	1031	1323.6	1033	16.188	<0.0001

OM, the original model forms; *MCR*, the model form with CR; *MCL*, the model form with CL; *MCRCL*, the model form with CR and CL.

4. Discussion

The accurate estimation of over-bark and in-bark diameter and stem volume is crucial for the efficient management of forest resources. As stated by Li and Weiskittel (2011), previous research suggests that the capability of crown variables to improve the performance of taper equations is species-specific and possibly also study-specific. Their results indicated that inclusion of crown variables essentially improved stem volume predictions for 3 species, but had minimal impact on stem diameter predictions. The level of improvement is likely a function of natural variation in crown variables and the adaptability of a particular model form to additional auxiliary variables. In this study, the incorporation of crown variables to taper and volume equations showed a clear improvement in diameter and stem volume predictions. Slightly better results were obtained for stem form predictions than stem volume predictions. As indicated by Weiskittel et al. (2011), crown variables often explain a very small amount of variation in stem volume. Prediction improvements for upper stem diameter and volume were greater for model forms with CR and CL than for model forms with CR or CL, though the overall improvements were small. However, the goal of adding additional covariates to a taper equation should not necessarily be to improve model fit, but rather to ensure biologically reasonable extrapolations (Weiskittel et al. 2011).

CR has been used successfully in taper models and improvement was observed in model performance for natural shortleaf pine in Louisiana (Farrar and Murphy 1987), planted longleaf pine in Texas and Louisiana (Baldwin and Polmer 1981), natural longleaf stands in the eastern Gulf region (Farrar 1987), and natural longleaf pine in northwest Florida, southwest Georgia, central and southern Alabama, and southern Mississippi (Shaw et al. 2003). Although Hann et al. (1987) found that crown variables explained a very small amount of variation in stem volume, their inclusion in taper models can be important because of their documented influence on tree form.

Evaluation of the fit statistics by relative height class showed an improvement in prediction accuracy in those relative height classes of over 50% of total height. As indicated by Jiang and Liu (2011), this is not a surprising result given the fact that only the b_4 parameter was changed. The equations with added crown variables were then more precise and less biased for the middle section of the stem in both *dob* and *dib* predictions. The standard error of estimates for the modified equations was lower than that of the original model form for all the relative height sections. For the upper stem section (relative heights 80%–100%), the inclusion of crown variables increased the bias and SEE values, suggesting slight over-fitting, but because this is the smallest section, we did not view it as a problem for the model. For relative heights between 0%–10% and 70%–100%, tested models showed bigger SEE values than at other height intervals. As indicated by Jiang et al. (2007), this may be caused by the larger variation for butt and upper sections of the black pine trees tested.

One problem of the modified segmented taper model used in this study is that it requires as inputs the diameter at breast height, the total tree height, and the diameter at 5.3 m (Özçelik and Brooks 2012). Although it is now easier to estimate the last variable in the field using equipment such as the Criterion RD 1000, a prediction equation system was proposed by Clark et al. (1991) to estimate this variable if this measurement is not done.

In order to determine the best method to apply, the discussed accuracy/convenience trade-off has to be considered seriously. When prediction accuracy is the most important element in a survey, the modified taper models including crown variables are the best option. However, when convenience is the limiting factor that plays the most vital role in a survey, or when additional accuracy is not of utmost importance, the basic taper model can be safely applied. The determination of the proper approach should take into account both the advantages and limitations of each. In our opinion, the improvement obtained by the inclusion of crown variables is not enough to justify the additional cost of taking extra field measurements.

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