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AKBAR ALIYEV

SEVDA ISAYEVA

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Existence and nonexistence of global solutions for nonlinear transmission acoustic problem

Akbar ALIEV¹ , Sevda ISAYEVA^{2,*}

¹Department of Differential Equations, Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan, Baku, Azerbaijan

²Department of Higher Mathematics, Baku State University, Baku, Azerbaijan

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Abstract: In this paper we consider a mixed problem for the nonlinear wave equations with transmission acoustic conditions, that is, the wave propagation over bodies consisting of two physically different types of materials, one of which is clamped. We prove the existence of a global solution. Under the condition of positive initial energy we show that the solution for this problem blows up in finite time.

Key words: Nonlinear wave equation, transmission acoustic conditions, locally reacting boundary, existence of global solutions, blow up result

1. Introduction

Let Ω be a bounded domain in R^n ($n \geq 1$) with smooth boundary Γ_1 , $\Omega_2 \subset \Omega$ is a subdomain with smooth boundary Γ_2 , and $\Omega_1 = \Omega \setminus \Omega_2$ is a subdomain with boundary $\Gamma = \Gamma_1 \cup \Gamma_2$, $\Gamma_1 \cap \Gamma_2 = \emptyset$. The nonlinear transmission acoustic problem considered here is

$$u_{tt} - \Delta u + |u_t|^{q_1-1} u_t = |u|^{p-1} u \text{ in } \Omega_1 \times (0, \infty), \quad (1.1)$$

$$v_{tt} - \Delta v + |v_t|^{q_2-1} v_t = |v|^{p-1} v \text{ in } \Omega_2 \times (0, \infty), \quad (1.2)$$

$$M\delta_{tt} + D\delta_t + K\delta = -u_t \text{ on } \Gamma_2 \times (0, \infty), \quad (1.3)$$

$$u = 0 \text{ on } \Gamma_1 \times (0, \infty), \quad (1.4)$$

$$u = v, \delta_t = \frac{\partial u}{\partial \nu} - \frac{\partial v}{\partial \nu} \text{ on } \Gamma_2 \times (0, \infty), \quad (1.5)$$

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), x \in \bar{\Omega}_1, \quad (1.6)$$

$$v(x, 0) = v_0(x), v_t(x, 0) = v_1(x), x \in \bar{\Omega}_2, \quad (1.7)$$

*Correspondence: isayevasevda@rambler.ru

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$$\delta(x, 0) = \delta_0(x), \delta_t(x, 0) = \frac{\partial u_0}{\partial \nu} - \frac{\partial v_0}{\partial \nu} \equiv \delta_1, x \in \bar{\Gamma}_2, \quad (1.8)$$

where ν is the unit outward normal vector to Γ ; $M, D, K : \bar{\Gamma}_2 \rightarrow R$, $u_0, u_1 : \bar{\Omega}_1 \rightarrow R$, $v_0, v_1 : \bar{\Omega}_2 \rightarrow R$, $\delta_0 : \bar{\Gamma}_2 \rightarrow R$ are given functions; $p > 1$, $q_i > 1$, $i = 1, 2$ are constants.

Transmission problems arise in several applications of physics and biology. For example, problems like (1.1)–(1.8), called transmission acoustic problems, are related to the problem of two wave equations, which model the transverse acoustic vibrations of a membrane composed of two different materials Ω_1 and Ω_2 .

Transmission problems were studied, for example, in [1, 2, 4, 13, 35, 37, 40]. The transmission problem for hyperbolic equations was investigated by Dautray and Lions [13], who proved the existence and regularity of solutions for the linear problem. Rivera and Oquendo [40] considered the transmission problem of viscoelastic waves and proved that the dissipation produced by the viscoelastic part can produce exponential decay of the solution. Bae [2] studied the transmission problem, in which one component is clamped and the other is in a viscoelastic fluid producing a dissipation mechanism on the boundary, and established a decay result. Bastos and Raposo [4] investigated the transmission problem with frictional damping and showed the well-posedness and exponential stability of the total energy.

Aliev and Mammadhasanov [1] studied the initial boundary value problem on longitudinal impact on a composite linear viscoelastic bar and established a well-posedness result by the method of dynamic regularization of transmission and boundary conditions.

In [19] the authors made a comparison of several boundary conditions, among which were the acoustic boundary conditions.

The acoustic boundary conditions were introduced by Beale and Rosencrans [7] and studied in [5, 6, 9–12, 14–18, 20, 22–24, 27, 30, 39, 41–43]. In [7] the authors derived

$$\begin{aligned} u_{tt} - \Delta u &= 0 \quad \text{in } \Omega \times (0, \infty), \\ \frac{\partial u}{\partial \nu} &= \delta_t \quad \text{on } \Gamma \times (0, \infty), \\ m\delta_{tt} + d\delta_t + k\delta &= -\rho_0 u_t \quad \text{on } \Gamma \times (0, \infty) \end{aligned}$$

as a theoretical model for describing the acoustic wave motion into a fluid in $\Omega \times R^3$; here ρ_0, m, d, k are physical known quantities. The function $u(x, t)$ is the velocity potential of a fluid and $\delta(x, t)$ models the normal displacement of the point $x \in \Gamma$ at time t . To obtain the model, Beale and Rosencrans assumed that each point of surface Γ acts like a spring in response to excess pressure from the fluid in the interior and that each point of Γ does not influence the others. Surfaces of such type are called locally reacting; see Morse and Ingard [38]. See also [15] for a related model.

Similarly, in [12, 14] acoustic boundary conditions were coupled with homogeneous Dirichlet condition on a portion of the boundary. In [15, 18, 39] acoustic boundary conditions were imposed in the whole boundary. In [14] Frota et al. obtained decay results to a nonlinear wave equation when $n = 1$; Cousin et al. [12] and Park and Park [42] obtained decay results when the coefficient in front of δ is zero.

In [17] Frota et al. considered acoustic boundary conditions in domains with nonlocally reacting boundary. A mixed problem for wave equations with nonlinear acoustic boundary conditions was considered by Gao et al.

[20] and Graber [22, 23]. In [27] Jeong et al. studied the global nonexistence of solutions for a quasilinear wave equation with acoustic boundary conditions.

Graber and Said-Houari [24] studied the stability of a structural acoustic wave equation with semilinear porous acoustic boundary conditions and obtained several results in local existence, global existence, the decay rate, and blow up results.

Bucci and Lasiecka [11] studied uniform stability properties of a strongly coupled system of partial differential equations of hyperbolic/parabolic type, which arises from the analysis and control of acoustic models with structural damping on an interface. In [30] Lasiecka obtained results on uniform stabilizability of a three-dimensional structural acoustic model describing the pressure in an acoustic chamber with flexible walls.

For the one wave equation

$$u_{tt} - \Delta u + |u_t|^{m-2} u_t = |u|^{p-2} u \text{ in } (0, T) \times \Omega, \quad (1.9)$$

with Dirichlet boundary conditions on $\partial\Omega$ in the absence of the damping term $|u_t|^{m-2} u_t$, the source term $|u|^{p-2} u$ causes finite-time blow-up of solutions with negative initial energy (see [3, 8, 28]). In contrast, in the absence of the source term, the damping term assures global existence for arbitrary initial data (see [25, 29]). The interaction between the damping and source terms was first considered by Levine [31, 32] for linear damping ($m = 2$). Levine showed that solutions with negative initial energy blow up in finite time. The main tool used in [25] and [31] is the “concavity method”, which fails in the case of a nonlinear damping term ($m > 2$). Georgiev and Todorova [21] extended Levin’s result to nonlinear damping case $m > 2$. In their work, the authors introduced a new method and determined relations between m and p for which there is a global existence and the relations between m and p for which there is a finite time blow up. Specifically, they showed that solutions with negative initial energy continue to exist globally if $m \geq p$ and blow up in finite time if $p > m$ and the initial energy is negative. Their method is based on the construction of a function L that satisfies the differential inequality

$$L'(t) \geq \omega L^{1+\nu}(t) \quad (1.10)$$

in $[0, \infty)$, where $\omega > 0$ and $\nu > 0$. The inequality (1.10) leads to a blow up of the solutions in finite time, provided that $L(0) > 0$.

The result in [21] was later generalized to an abstract setting and to unbounded domains by Levine and Serrin [34] and Levine and Park [33]. Vitillaro in [44] combined the arguments in [21] and [34] to extend these results to situations where damping is nonlinear and the solution has positive initial energy.

Our main goal in this paper is to extend the above results on the wave equation (1.9) to our system (1.1)–(1.8). We study a mixed problem for the nonlinear wave equations with transmission acoustic conditions. To the best of our knowledge, there are no results on nonlinear wave equations with transmission acoustic conditions. We prove the existence of a global solution for the problem (1.1)–(1.8) under the condition $p \leq \min\{q_1, q_2\}$. For positive initial energy and the condition $p > \max\{q_1, q_2\}$ we give the blow up result.

Our paper is organized as follows. In Section 2 we introduce some notations, preliminaries, and a statement of main results; in Section 3 we prove the existence of a global solution, and the blow up result is proved in Section 4.

2. Preliminaries and main results

The inner product and norm in $L^2(\Omega_i)$, $i = 1, 2$, and $L^2(\Gamma_2)$ are denoted, respectively, by

$$(u, v)_i = \int_{\Omega_i} u(x) v(x) dx, \quad \|u\|_i = \left(\int_{\Omega_i} (u(x))^2 dx \right)^{\frac{1}{2}}, \quad i = 1, 2,$$

$$(\delta, \theta)_{\Gamma_2} = \int_{\Gamma_2} \delta(x) \theta(x) d\Gamma_2, \quad \|\delta\|_{\Gamma_2} = \left(\int_{\Gamma_2} (\delta(x))^2 d\Gamma_2 \right)^{\frac{1}{2}}.$$

We define a closed subspace of the $H^1_{\Gamma_1}(\Omega_1)$ as

$$H^1_{\Gamma_1}(\Omega_1) = \{ u \in H^1(\Omega_1) : \gamma_0(u) = 0 \text{ a. e. on } \Gamma_1 \},$$

where $\gamma_0 : H^1(\Omega_1) \rightarrow H^{1/2}(\Gamma)$ is the trace map of order zero and $H^{1/2}(\Gamma)$ is the Sobolev space of order $\frac{1}{2}$ defined over Γ , as introduced by Lions and Magenes [36]. Observe that the norm in $H^1_{\Gamma_1}(\Omega_1)$,

$$\|u\|_{H^1_{\Gamma_1}(\Omega_1)} = \left(\sum_{i=1}^n \int_{\Omega_1} \left(\frac{\partial u}{\partial x_i} \right)^2 dx \right)^{\frac{1}{2}},$$

and the norm of the real Sobolev space $H^1(\Omega_1)$ are equivalent, because Poincaré’s inequality holds in $H^1_{\Gamma_1}(\Omega_1)$.

Thus, we consider $H^1_{\Gamma_1}(\Omega_1)$ with the above gradient norm.

In this section we give our main results on the existence and nonexistence of global solutions. First of all, we give the theorem on local existence of solutions and the regularity theorem for the problem (1.1)–(1.8), which were proved in [26] by combining the Galerkin method and the fixed point method (see [21]).

Theorem 2.1 (*local existence*) *Assume that*

$$M, D, K \in C(\bar{\Gamma}_2), \quad M \geq 0, \quad D > 0, \quad K \geq 0 \quad \text{for } \forall x \in \bar{\Gamma}_2, \tag{2.1}$$

$$p > 1 \quad \text{if } n = 1, 2, \quad 1 < p \leq \frac{n}{n-2} \quad \text{if } n \geq 3, \tag{2.2}$$

$$q_i > 1 \quad \text{if } n = 1, 2, \quad 1 < q_i \leq \frac{n+2}{n-2} \quad \text{if } n \geq 3, \tag{2.3}$$

and

$$(u_0, v_0, \delta_0) \in H^1_{\Gamma_1}(\Omega_1) \times H^1(\Omega_2) \times L^2(\Gamma_2),$$

$$(u_1, v_1, \delta_1) \in L^2(\Omega_1) \times L^2(\Omega_2) \times L^2(\Gamma_2),$$

$$u_0 = v_0 \text{ and } u_1 = v_1 \text{ on } \Gamma_2.$$

Then there exists $T > 0$ such that the problem (1.1)–(1.8) has a unique solution (u, v, δ) , which satisfies

$$(u, v, \delta) \in L^\infty(0, T; H^1_{\Gamma_1}(\Omega_1) \times H^1(\Omega_2) \times L^2(\Gamma_2)),$$

$$u_t \in L^\infty(0, T; L^2(\Omega_1)) \cap L^{q_1+1}(\Omega_1 \times [0, T]),$$

$$v_t \in L^\infty(0, T; L^2(\Omega_2)) \cap L^{q_2+1}(\Omega_2 \times [0, T]), \delta_t \in L^\infty(0, T; L^2(\Gamma_2)).$$

Moreover, at least one of the following statements holds:

- 1) $\lim_{t \rightarrow T-0} \left(\|u_t\|_1^2 + \|v_t\|_2^2 + \|\nabla u\|_1^2 + \|\nabla v\|_2^2 + \|\sqrt{M}\delta_t\|_{\Gamma_2}^2 + \|\sqrt{K}\delta\|_{\Gamma_2}^2 \right) = +\infty;$
- 2) $T = +\infty.$

Theorem 2.2 (regularity result). Suppose that (2.1)–(2.3) hold. Let

$$u_0 \in H_{\Gamma_1}^1(\Omega_1) \cap H^2(\Omega_1), u_1 \in H_{\Gamma_1}^1(\Omega_1) \cap L^{2q_1}(\Omega_1), v_0 \in H^2(\Omega_2), v_1 \in H^1(\Omega_2) \cap L^{2q_2}(\Omega_2),$$

$$u_0 = v_0 \text{ and } u_1 = v_1 \text{ on } \Gamma_2, \delta_0 \in L^2(\Gamma_2).$$

Then there exists $T > 0$ such that the problem (1.1)–(1.8) has a unique solution (u, v, δ) , which satisfies

$$u, u_t \in L^\infty(0, T; H_{\Gamma_1}^1(\Omega_1)), u_{tt} \in L^\infty(0, T; L^2(\Omega_1)),$$

$$v, v_t \in L^\infty(0, T; H^1(\Omega_2)), v_{tt} \in L^\infty(0, T; L^2(\Omega_2)),$$

$$u(t) \in H(\Delta, \Omega_1), v(t) \in H(\Delta, \Omega_2) \text{ a.e. in } (0, T),$$

$$\delta, \delta_t \in L^\infty(0, T; L^2(\Gamma_2)), \delta_{tt} \in L^2(0, T; L^2(\Gamma_2)),$$

and

$$u_{tt} - \Delta u + |u_t|^{q_1-1} u_t = |u|^{p-1} u \text{ a. e. in } \Omega_1 \times (0, T),$$

$$v_{tt} - \Delta v + |v_t|^{q_2-1} v_t = |v|^{p-1} v \text{ a. e. in } \Omega_2 \times (0, T),$$

$$M\delta_{tt} + D\delta_t + K\delta = -u_t, u = v \text{ a. e. on } \Gamma_2 \times (0, T),$$

$$\langle \gamma_1(u(t) - v(t)), \gamma_0(\varphi) \rangle_{H^{-1/2}(\Gamma_2) \times H^{1/2}(\Gamma_2)} = (\delta_t(t), \gamma_0(\varphi))_{\Gamma_2}$$

for $\forall \varphi \in H(\Delta, \Omega_1) \cup H(\Delta, \Omega_2)$ a. e. in $(0, T)$,

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x) \text{ a. e. in } \Omega_1,$$

$$v(x, 0) = v_0(x), v_t(x, 0) = v_1(x) \text{ a. e. in } \Omega_2,$$

$$\delta(x, 0) = \delta_0(x) \text{ a. e. on } \Gamma_2.$$

The following theorem shows that the solution obtained in Theorem 2.1 is a global solution if $p \leq \min\{q_1, q_2\}$.

Theorem 2.3 (global existence) If the assumptions of Theorem 2.1 hold and

$$p \leq \min\{q_1, q_2\}, \tag{2.4}$$

then the local solution (u, v, δ) is a global solution and T may be taken arbitrarily large.

Next we consider the problem (1.1)–(1.8) under the assumption

$$p > \max \{q_1, q_2\}, \tag{2.5}$$

in which case we show that the solution blows up in finite time.

Let s be a number with $2 < s < +\infty$ if $n = 1, 2$ and $2 \leq s \leq \frac{2n}{n-2}$ if $n \geq 3$. Then there exists the constant B_1 depending on Ω_1 and s such that

$$\|u\|_{L^s(\Omega_1)} \leq B_1 \|\nabla u\|_{L^2(\Omega_1)} \text{ for } \forall u \in H^1_{\Gamma_1}(\Omega_1). \tag{2.6}$$

By this and Phriedrich’s inequality there exists the constant B depending on Ω_1, Ω_2 , and p such that

$$\|u\|_{L^{p+1}(\Omega_1)}^{p+1} + \|v\|_{L^{p+1}(\Omega_2)}^{p+1} \leq B \left(\|\nabla u\|_1^2 + \|\nabla v\|_2^2 \right)^{\frac{p+1}{2}} \tag{2.7}$$

for $\forall u \in H^1_{\Gamma_1}(\Omega_1)$ and $\forall v \in H^1(\Omega_2)$, which satisfy the condition (1.5)₁.

Now we define the following energy function associated with the solution (u, v, δ) of the problem (1.1)–(1.8):

$$E(t) = \frac{1}{2} \left[\|u_t\|_1^2 + \|\nabla u\|_1^2 + \|v_t\|_2^2 + \|\nabla v\|_2^2 + \left\| \sqrt{M} \delta_t \right\|_{\Gamma_2}^2 + \left\| \sqrt{K} \delta \right\|_{\Gamma_2}^2 \right] - \frac{1}{p+1} \left(|u|^{p+1}, 1 \right)_1 - \frac{1}{p+1} \left(|v|^{p+1}, 1 \right)_2. \tag{2.8}$$

Multiplying equation (1.1) by u_t , (1.2) by v_t , and (1.3) by δ_t and integrating over $\Omega_1 \times [0, t], \Omega_2 \times [0, t], \Gamma_2 \times [0, t]$, respectively, then summing them and integrating by parts (we will do this in Section 3), we can obtain

$$\frac{d}{dt} E(t) = - \left(\|u_t\|_{L^{q_1+1}(\Omega_1)}^{q_1+1} + \|v_t\|_{L^{q_2+1}(\Omega_2)}^{q_2+1} + \left\| \sqrt{D} \delta_t \right\|_{\Gamma_2}^2 \right) \leq 0 \tag{2.9}$$

or

$$E(t) - E(0) = - \int_0^t \left(\|u_t\|_{L^{q_1+1}(\Omega_1)}^{q_1+1} + \|v_t\|_{L^{q_2+1}(\Omega_2)}^{q_2+1} + \left\| \sqrt{D} \delta_t \right\|_{\Gamma_2}^2 \right) d\tau. \tag{2.10}$$

Thus, if (u, v, δ) is a solution of the problem (1.1)–(1.8) then $E(t)$ is a nonincreasing function for $t > 0$. It follows from (2.6), (2.7), and (2.8) that

$$\begin{aligned} E(t) &\geq \frac{1}{2} \left(\|\nabla u\|_1^2 + \|\nabla v\|_2^2 \right) - \frac{1}{p+1} \int_{\Omega_1} |u|^{p+1} dx - \frac{1}{p+1} \int_{\Omega_2} |v|^{p+1} dx = \\ &= \frac{1}{2} \left(\|\nabla u\|_1^2 + \|\nabla v\|_2^2 \right) - \frac{1}{p+1} \|u\|_{L^{p+1}(\Omega_1)}^{p+1} - \frac{1}{p+1} \|v\|_{L^{p+1}(\Omega_2)}^{p+1} \geq \\ &\geq \frac{1}{2} \left(\|\nabla u\|_1^2 + \|\nabla v\|_2^2 \right) - \frac{B}{p+1} \left(\|\nabla u\|_1^2 + \|\nabla v\|_2^2 \right)^{\frac{p+1}{2}} = g(\alpha), \end{aligned} \tag{2.11}$$

where

$$\alpha = \left(\|\nabla u\|_1^2 + \|\nabla v\|_2^2 \right)^{\frac{1}{2}}, \tag{2.12}$$

$$g(\alpha) = \frac{1}{2} \alpha^2 - \frac{B}{p+1} \alpha^{p+1}. \tag{2.13}$$

Therefore,

$$g(0) = 0, \quad g'(\alpha_1) = 0,$$

where $\alpha_1 = B^{-\frac{1}{p-1}}$, and since $p > 1$ then

$$g_{\max}(\alpha_1) = B^{-\frac{2}{p-1}} \left(\frac{1}{2} - \frac{1}{p+1} \right) = \alpha_1^2 \left(\frac{1}{2} - \frac{1}{p+1} \right) = E_1 > 0. \tag{2.14}$$

Applying the idea of Vitillaro [44] we have the following lemma.

Lemma 2.4 *Let (u, v, δ) be a solution of the problem (1.1)–(1.8). Assume that (2.1) and (2.2) hold. If*

$$0 < E(0) < E_1 \tag{2.15}$$

and

$$\left(\|\nabla u_0\|_1^2 + \|\nabla v_0\|_2^2 \right)^{\frac{1}{2}} > \alpha_1, \tag{2.16}$$

then there exists $\alpha_2 > \alpha_1$ such that

$$\left(\|\nabla u\|_1^2 + \|\nabla v\|_2^2 \right)^{\frac{1}{2}} \geq \alpha_2 \tag{2.17}$$

and

$$\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \geq B\alpha_2^{p+1} \tag{2.18}$$

for $t \geq 0$.

Proof. Let $E(0) < E_1$. Then by (2.14) there exists a number α_2 such that $\alpha_2 > \alpha_1$ and $g(\alpha_2) = E(0)$. On the other hand, by (2.11), $g(\alpha) \leq E(t)$, and hence

$$g(\alpha_0) \leq E(0) = g(\alpha_2), \tag{2.19}$$

where $\alpha_0 = \alpha|_{t=0} = \left(\|\nabla u_0\|_1^2 + \|\nabla v_0\|_2^2 \right)^{\frac{1}{2}}$. Since by (2.16), $\alpha_0 > \alpha_1$ and $g'(\alpha) < 0$ if $\alpha > \alpha_1$, we obtain from (2.19) that $\alpha_0 \geq \alpha_2$.

Now, to establish (2.17) for such α_2 , we suppose by contradiction that

$$\left(\|\nabla u(t_0, \cdot)\|_1^2 + \|\nabla v(t_0, \cdot)\|_2^2 \right)^{\frac{1}{2}} < \alpha_2 \tag{2.20}$$

for some $t_0 > 0$. By the continuity of $\left(\|\nabla u(t, \cdot)\|_1^2 + \|\nabla v(t, \cdot)\|_2^2 \right)^{\frac{1}{2}}$ we can suppose that

$$\alpha_1 < \left(\|\nabla u(t_0, \cdot)\|_1^2 + \|\nabla v(t_0, \cdot)\|_2^2 \right)^{\frac{1}{2}} < \alpha_2.$$

From (2.11) and (2.20) we obtain

$$E(t_0) \geq g(\alpha|_{t_0}) = g\left(\left(\|\nabla u(t_0, \cdot)\|_1^2 + \|\nabla v(t_0, \cdot)\|_2^2 \right)^{\frac{1}{2}} \right) > g(\alpha_2) = E(0),$$

but it is impossible by (2.9). Therefore, (2.17) is proved.

Using (2.8), (2.9), (2.10), (2.12), (2.13), and (2.17) we get

$$\begin{aligned} & \frac{1}{p+1} \int_{\Omega_1} |u|^{p+1} dx + \frac{1}{p+1} \int_{\Omega_2} |v|^{p+1} dx = \\ & = \frac{1}{2} \left(\|u_t\|_1^2 + \|\nabla u\|_1^2 + \|v_t\|_2^2 + \|\nabla v\|_2^2 + \left\| \sqrt{M} \delta_t \right\|_{\Gamma_2}^2 + \left\| \sqrt{K} \delta \right\|_{\Gamma_2}^2 \right) - E(t) \geq \\ & \geq \frac{1}{2} \left(\|\nabla u\|_1^2 + \|\nabla v\|_2^2 \right) - E(t) = \frac{1}{2} \alpha^2 - E(t) \geq \frac{1}{2} \alpha_2^2 - E(t) \geq \frac{1}{2} \alpha_2^2 - E(0) = \\ & = \frac{1}{2} \alpha_2^2 - g(\alpha_2) = \frac{B}{p+1} \alpha_2^{p+1}, \end{aligned}$$

from which we obtain (2.18).

Lemma 2.4 is proved.

Theorem 2.5 Assume that (2.1)–(2.2) and (2.5) hold and $K \geq 1$. If

$$(u_0, v_0, \delta_0) \in H_{\Gamma_1}^1(\Omega_1) \times H^1(\Omega_2) \times L^2(\Gamma_2),$$

$$(u_1, v_1, \delta_1) \in L^2(\Omega_1) \times L^2(\Omega_2) \times L^2(\Gamma_2),$$

then the solution of the problem (1.1)–(1.8) with initial data satisfying (2.15) and (2.16) blows up in finite time.

3. Proof of Theorem 2.3

Approximating the initial data with the smooth ones and using Theorems 2.1 and 2.2, we can show that all the operations performed below are valid.

Let (u, v, δ) be a weak solution to the transmission acoustic problem (1.1)–(1.8). Multiplying equation (1.1) by u_t , (1.2) by v_t , and (1.3) by δ_t and integrating over $\Omega_1, \Omega_2, \Gamma_2$, respectively, then summing them and integrating by parts, we have

$$\frac{1}{2} \frac{d}{dt} \|u_t\|_1^2 - \left(\frac{\partial u}{\partial \nu}, u_t \right)_{\Gamma_2} + \frac{1}{2} \frac{d}{dt} \|\nabla u\|_1^2 + \left(|u_t|^{q_1+1}, 1 \right)_1 = \frac{1}{p+1} \frac{d}{dt} \left(|u|^{p+1}, 1 \right)_1,$$

$$\frac{1}{2} \frac{d}{dt} \|v_t\|_2^2 + \left(\frac{\partial v}{\partial \nu}, v_t \right)_{\Gamma_2} + \frac{1}{2} \frac{d}{dt} \|\nabla v\|_2^2 + \left(|v_t|^{q_2+1}, 1 \right)_2 = \frac{1}{p+1} \frac{d}{dt} \left(|v|^{p+1}, 1 \right)_2,$$

$$\frac{1}{2} \frac{d}{dt} \left\| \sqrt{M} \delta_t \right\|_{\Gamma_2}^2 + \left\| \sqrt{D} \delta_t \right\|_{\Gamma_2}^2 + \frac{1}{2} \frac{d}{dt} \left\| \sqrt{K} \delta \right\|_{\Gamma_2}^2 = - (u_t, \delta_t)_{\Gamma_2},$$

from which, using (1.5)₁, we obtain

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} \left(\|u_t\|_1^2 + \|\nabla u\|_1^2 + \|v_t\|_2^2 + \|\nabla v\|_2^2 + \left\| \sqrt{M} \delta_t \right\|_{\Gamma_2}^2 + \left\| \sqrt{K} \delta \right\|_{\Gamma_2}^2 \right) - \\ & - \frac{1}{p+1} \frac{d}{dt} \left[\left(|u|^{p+1}, 1 \right)_1 + \left(|v|^{p+1}, 1 \right)_2 \right] - \left(\frac{\partial u}{\partial \nu} - \frac{\partial v}{\partial \nu}, u_t \right)_{\Gamma_2} + \end{aligned}$$

$$+(u_t, \delta_t)_{\Gamma_2} + (|u_t|^{q_1+1}, 1)_1 + (|v_t|^{q_2+1}, 1)_2 + \|\sqrt{D}\delta_t\|_{\Gamma_2}^2 = 0$$

or by (1.5)₂

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} \left(\|u_t\|_1^2 + \|\nabla u\|_1^2 + \|v_t\|_2^2 + \|\nabla v\|_2^2 + \|\sqrt{M}\delta_t\|_{\Gamma_2}^2 + \|\sqrt{K}\delta\|_{\Gamma_2}^2 \right) - \\ & - \frac{1}{p+1} \frac{d}{dt} \left[(|u|^{p+1}, 1)_1 + (|v|^{p+1}, 1)_2 \right] + \\ & + (|u_t|^{q_1+1}, 1)_1 + (|v_t|^{q_2+1}, 1)_2 + \|\sqrt{D}\delta_t\|_{\Gamma_2}^2 = 0. \end{aligned} \tag{3.1}$$

Note that, using (2.8), equality (3.1) can be written as (2.9).

Integrating (3.1) from 0 to t we obtain

$$\begin{aligned} & \frac{1}{2} \left(\|u_t\|_1^2 + \|\nabla u\|_1^2 + \|v_t\|_2^2 + \|\nabla v\|_2^2 + \|\sqrt{M}\delta_t\|_{\Gamma_2}^2 + \|\sqrt{K}\delta\|_{\Gamma_2}^2 \right) + \\ & + \frac{1}{p+1} (|u|^{p+1}, 1)_1 + \frac{1}{p+1} (|v|^{p+1}, 1)_2 + \\ & + \int_0^t \left[(|u_t|^{q_1+1}, 1)_1 + (|v_t|^{q_2+1}, 1)_2 + \|\sqrt{D}\delta_t\|_{\Gamma_2}^2 \right] d\tau = \\ & = \frac{1}{2} \left(\|u_1\|_1^2 + \|\nabla u_0\|_1^2 + \|v_1\|_2^2 + \|\nabla v_0\|_2^2 + \|\sqrt{M}\delta_1\|_{\Gamma_2}^2 + \|\sqrt{K}\delta_0\|_{\Gamma_2}^2 \right) + \\ & + \frac{1}{p+1} (|u_0|^{p+1}, 1)_1 + \frac{1}{p+1} (|v_0|^{p+1}, 1)_2 + \\ & + 2 \int_0^t (|u|^{p-1} u, u_t)_1 d\tau + 2 \int_0^t (|v|^{p-1} v, v_t)_2 d\tau. \end{aligned} \tag{3.2}$$

Let us estimate the last two terms of (3.2). Using the Hölder inequality with exponents $\rho = \frac{q_1+1}{q_1}$ and $\rho' = q_1 + 1$ ($\frac{1}{\rho} + \frac{1}{\rho'} = 1$), we have

$$\begin{aligned} & \int_0^t (|u|^{p-1} u, u_t)_1 d\tau \leq \int_0^t \int_{\Omega_1} |u|^p u_t dx d\tau \leq \\ & \leq \left(\int_0^t \int_{\Omega_1} |u|^{\frac{p(q_1+1)}{q_1}} dx d\tau \right)^{\frac{q_1}{q_1+1}} \left(\int_0^t \int_{\Omega_1} |u_t|^{q_1+1} dx d\tau \right)^{\frac{1}{q_1+1}}, \end{aligned}$$

from which, using the Young inequality ($ab \leq \frac{a^\rho}{\rho\eta^\rho} + \frac{\eta^{\rho'} b^{\rho'}}{\rho'}$, $\frac{1}{\rho} + \frac{1}{\rho'} = 1$) with the parameter $\eta = \mu_1^{\frac{1}{q_1+1}}$, we obtain

$$\int_0^t (|u|^{p-1} u, u_t)_1 d\tau \leq \frac{q_1}{(q_1 + 1) \mu_1^{\frac{1}{q_1}}} \int_0^t \int_{\Omega_1} |u|^{\frac{p(q_1+1)}{q_1}} dx d\tau +$$

$$+ \frac{\mu_1}{q_1 + 1} \int_0^t \int_{\Omega_1} |u_t|^{q_1+1} dx d\tau. \tag{3.3}$$

By the condition (2.4) and the Young inequality with exponents $\rho = \frac{(p+1)q_1}{p(q_1+1)}$, $\rho' = \frac{(p+1)q_1}{q_1-p}$ ($\frac{1}{\rho} + \frac{1}{\rho'} = 1$) we have

$$|u|^{\frac{p(q_1+1)}{q_1}} \leq \frac{p(q_1+1)}{(p+1)q_1} |u|^{p+1} + \frac{q_1-p}{(p+1)q_1}.$$

Using this in (3.3) we get

$$\begin{aligned} \int_0^t (|u|^{p-1} u, u_t)_1 d\tau &\leq \frac{p\mu_1^{-\frac{1}{q_1}}}{p+1} \int_0^t \int_{\Omega_1} |u|^{p+1} dx d\tau + \frac{(q_1-p)\mu_1^{-\frac{1}{q_1}} \text{mes } \Omega_1}{(q_1+1)(p+1)} + \\ &+ \frac{\mu_1}{q_1+1} \int_0^t \int_{\Omega_1} |u_t|^{q_1+1} dx d\tau. \end{aligned} \tag{3.4}$$

Similarly, we have

$$\begin{aligned} \int_0^t (|v|^{p-1} v, v_t)_2 d\tau &\leq \frac{p\mu_2^{-\frac{1}{q_2}}}{p+1} \int_0^t \int_{\Omega_2} |v|^{p+1} dx d\tau + \frac{(q_2-p)\mu_2^{-\frac{1}{q_2}} \text{mes } \Omega_2}{(q_2+1)(p+1)} + \\ &+ \frac{\mu_2}{q_2+1} \int_0^t \int_{\Omega_2} |v_t|^{q_2+1} dx d\tau. \end{aligned} \tag{3.5}$$

From (3.2), (3.4), and (3.5) we conclude that

$$\begin{aligned} &\frac{1}{2} \left(\|u_t\|_1^2 + \|\nabla u\|_1^2 + \|v_t\|_2^2 + \|\nabla v\|_2^2 + \|\sqrt{M}\delta_t\|_{\Gamma_2}^2 + \|\sqrt{K}\delta\|_{\Gamma_2}^2 \right) + \\ &+ \frac{1}{p+1} \left[(|u|^{p+1}, 1)_1 + (|v|^{p+1}, 1)_2 \right] + \left(1 - \frac{2\mu_1}{q_1+1} \right) \int_0^t (|u_t|^{q_1+1}, 1)_1 d\tau + \\ &+ \left(1 - \frac{2\mu_2}{q_2+1} \right) \int_0^t (|v_t|^{q_2+1}, 1)_2 d\tau + \int_0^t \|\sqrt{D}\delta_t\|_{\Gamma_2}^2 d\tau \leq \\ &\leq \frac{1}{2} \left(\|u_1\|_1^2 + \|\nabla u_0\|_1^2 + \|v_1\|_2^2 + \|\nabla v_0\|_2^2 + \|\sqrt{M}\delta_1\|_{\Gamma_2}^2 + \|\sqrt{K}\delta_0\|_{\Gamma_2}^2 \right) + \\ &+ \frac{1}{p+1} \left[(|u_0|^{p+1}, 1)_1 + (|v_0|^{p+1}, 1)_2 \right] + \frac{T}{p+1} \sum_{i=1}^2 \frac{(q_i-p)\mu_i^{-\frac{1}{q_i}} \text{mes } \Omega_i}{q_i+1} + \\ &+ \frac{p}{p+1} \max \left\{ \mu_1^{-\frac{1}{q_1}}, \mu_2^{-\frac{1}{q_2}} \right\} \int_0^t \left[(|u|^{p+1}, 1)_1 + (|v|^{p+1}, 1)_2 \right] d\tau \end{aligned} \tag{3.6}$$

and μ_1, μ_2 are chosen such that

$$1 - \frac{2\mu_1}{q_1 + 1} > 0, \quad 1 - \frac{2\mu_2}{q_2 + 1} > 0.$$

By Gronwall's inequality and (3.6) we obtain

$$\begin{aligned} & \frac{1}{2} \left(\|u_t\|_1^2 + \|\nabla u\|_1^2 + \|v_t\|_2^2 + \|\nabla v\|_2^2 + \|\sqrt{M}\delta_t\|_{\Gamma_2}^2 + \|\sqrt{K}\delta\|_{\Gamma_2}^2 \right) + \\ & + \frac{1}{p+1} \left[\left(|u|^{p+1}, 1 \right)_1 + \left(|v|^{p+1}, 1 \right)_2 \right] + \left(1 - \frac{2\mu_1}{q_1 + 1} \right) \int_0^t \left(|u_t|^{q_1+1}, 1 \right)_1 d\tau + \\ & + \left(1 - \frac{2\mu_2}{q_2 + 1} \right) \int_0^t \left(|v_t|^{q_2+1}, 1 \right)_2 d\tau + \int_0^t \|\sqrt{D}\delta_t\|_{\Gamma_2}^2 d\tau \leq C_T, \end{aligned}$$

where C_T depends on $\|u_1\|_1, \|v_1\|_2, \|\nabla u_0\|_1, \|\nabla v_0\|_2, \|\delta_0\|_{\Gamma_2}, \|\delta_1\|_{\Gamma_2}$ and on $T > 0$, which is arbitrary. By the standard continuation argument, the local solution (u, v, δ) obtained in Theorem 2.1 is global.

Theorem 2.3 is proved.

4. Proof of Theorem 2.5

Let

$$H(t) = E_1 - E(t), \quad t \geq 0. \tag{4.1}$$

From (2.9) we have

$$H'(t) = -E'(t) = \|u_t\|_{L^{q_1+1}(\Omega_1)}^{q_1+1} + \|v_t\|_{L^{q_2+1}(\Omega_2)}^{q_2+1} + \|\sqrt{D}\delta_t\|_{\Gamma_2}^2 \geq 0, \quad t \geq 0. \tag{4.2}$$

Therefore, from (2.15), (4.1), and (4.2) we obtain that

$$0 < H(0) \leq H(t), \quad t \geq 0. \tag{4.3}$$

By (2.8) and (4.1) we have

$$\begin{aligned} & H(t) = \\ & = E_1 - \frac{1}{2} \left(\|u_t\|_1^2 + \|\nabla u\|_1^2 + \|v_t\|_2^2 + \|\nabla v\|_2^2 + \|\sqrt{M}\delta_t\|_{\Gamma_2}^2 + \|\sqrt{K}\delta\|_{\Gamma_2}^2 \right) + \\ & + \frac{1}{p+1} \left(|u|^{p+1}, 1 \right)_1 + \frac{1}{p+1} \left(|v|^{p+1}, 1 \right)_2 \leq E_1 - \frac{1}{2} \left(\|\nabla u\|_1^2 + \|\nabla v\|_2^2 \right) + \\ & + \frac{1}{p+1} \left[\left(|u|^{p+1}, 1 \right)_1 + \left(|v|^{p+1}, 1 \right)_2 \right], \end{aligned}$$

from which, using (2.12) and (2.17),

$$H(t) \leq E_1 - \frac{1}{2}\alpha_2^2 + \frac{1}{p+1} \left[\left(|u|^{p+1}, 1 \right)_1 + \left(|v|^{p+1}, 1 \right)_2 \right]$$

or since $\alpha_2 > \alpha_1$, then by (2.14)

$$H(t) \leq \frac{1}{p+1} \left[\left(|u|^{p+1}, 1 \right)_1 + \left(|v|^{p+1}, 1 \right)_2 \right]. \tag{4.4}$$

We define $\Theta(t)$ as follows:

$$\Theta(t) = H^{1-\sigma}(t) + \varepsilon \left(\int_{\Omega_1} u u_t dx + \int_{\Omega_2} v v_t dx - \int_{\Gamma_2} u \delta d\Gamma_2 \right) \tag{4.5}$$

for $\varepsilon > 0$ to be chosen later and for σ satisfying

$$0 < \sigma \leq \min \left\{ \frac{p-1}{2(p+1)}, \frac{p-q_1}{q_1(p+1)}, \frac{p-q_2}{q_2(p+1)} \right\}. \tag{4.6}$$

Note that $p-1 > 0$, $p-q_1 > 0$, $p-q_2 > 0$ by (2.2) and (2.5).

Differentiating (4.5) and taking into account (1.1) and (1.2) yields

$$\begin{aligned} \Theta'(t) = & (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon \int_{\Omega_1} u_t^2 dx + \varepsilon \int_{\Omega_2} v_t^2 dx + \\ & + \varepsilon \left(\int_{\Gamma_2} \frac{\partial u}{\partial \nu} u d\Gamma_2 - \int_{\Omega_1} |\nabla u|^2 dx \right) - \\ & - \varepsilon \int_{\Omega_1} |u_t|^{q_1-1} u_t u dx + \varepsilon \int_{\Omega_1} |u|^{p+1} dx + \varepsilon \left(- \int_{\Gamma_2} \frac{\partial v}{\partial \nu} v d\Gamma_2 - \int_{\Omega_2} |\nabla v|^2 dx \right) - \\ & - \varepsilon \int_{\Omega_2} |v_t|^{q_2-1} v_t v dx + \varepsilon \int_{\Omega_2} |v|^{p+1} dx - \varepsilon \int_{\Gamma_2} u \delta_t d\Gamma_2 - \varepsilon \int_{\Gamma_2} \delta u_t d\Gamma_2, \end{aligned}$$

or since by (1.5),

$$\int_{\Gamma_2} \frac{\partial u}{\partial \nu} u d\Gamma_2 - \int_{\Gamma_2} \frac{\partial v}{\partial \nu} v d\Gamma_2 = \int_{\Gamma_2} \left(\frac{\partial u}{\partial \nu} - \frac{\partial v}{\partial \nu} \right) u d\Gamma_2 = \int_{\Gamma_2} \delta_t u d\Gamma_2,$$

then we obtain

$$\begin{aligned} \Theta'(t) = & (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon \int_{\Omega_1} u_t^2 dx + \varepsilon \int_{\Omega_2} v_t^2 dx - \\ & - \varepsilon \int_{\Omega_1} |\nabla u|^2 dx - \varepsilon \int_{\Omega_2} |\nabla v|^2 dx - \\ & - \varepsilon \int_{\Omega_1} |u_t|^{q_1-1} u_t u dx - \varepsilon \int_{\Omega_2} |v_t|^{q_2-1} v_t v dx + \varepsilon \int_{\Omega_1} |u|^{p+1} dx + \\ & + \varepsilon \int_{\Omega_2} |v|^{p+1} dx - \varepsilon \int_{\Gamma_2} \delta u_t d\Gamma_2. \end{aligned} \tag{4.7}$$

Using the Hölder inequality with exponents $\rho = \frac{q_1+1}{q_1}$ and $\rho' = q_1 + 1$ ($\frac{1}{\rho} + \frac{1}{\rho'} = 1$) we have

$$\int_{\Omega_1} |u_t|^{q_1-1} u_t u dx \leq \int_{\Omega_1} |u_t|^{q_1} |u| dx \leq$$

$$\leq \left(\int_{\Omega_1} |u_t|^{q_1+1} dx \right)^{\frac{q_1}{q_1+1}} \left(\int_{\Omega_1} |u|^{q_1+1} dx \right)^{\frac{1}{q_1+1}},$$

from which, by the Young inequality $\left(ab \leq \frac{a^\rho}{\rho\eta^\rho} + \frac{\eta^{\rho'} b^{\rho'}}{\rho'} , \frac{1}{\rho} + \frac{1}{\rho'} = 1 \right)$ with parameter $\eta = [P_1 H^{-\sigma}(t)]^{-\frac{q_1}{q_1+1}}$ (P_1 is a large constant to be fixed later), we obtain

$$\begin{aligned} \left| \int_{\Omega_1} |u_t|^{q_1-1} u_t u dx \right| &\leq \frac{q_1}{q_1+1} P_1 H^{-\sigma}(t) \int_{\Omega_1} |u_t|^{q_1+1} dx + \\ &+ \frac{1}{(q_1+1) (P_1 H^{-\sigma}(t))^{q_1}} \int_{\Omega_1} |u|^{q_1+1} dx. \end{aligned} \tag{4.8}$$

Similarly, we have

$$\begin{aligned} \left| \int_{\Omega_2} |v_t|^{q_2-1} v_t v dx \right| &\leq \frac{q_2}{q_2+1} P_2 H^{-\sigma}(t) \int_{\Omega_2} |v_t|^{q_2+1} dx + \\ &+ \frac{1}{(q_2+1) (P_2 H^{-\sigma}(t))^{q_2}} \int_{\Omega_2} |v|^{q_2+1} dx. \end{aligned} \tag{4.9}$$

By (4.2) from (4.8) and (4.9), we get

$$\begin{aligned} &\left| \varepsilon \int_{\Omega_1} |u_t|^{q_1-1} u_t u dx + \varepsilon \int_{\Omega_2} |v_t|^{q_2-1} v_t v dx \right| \leq \\ &\leq \varepsilon P H^{-\sigma}(t) \left[\int_{\Omega_1} |u_t|^{q_1+1} dx + \int_{\Omega_2} |v_t|^{q_2+1} dx \right] + \\ &+ \frac{\varepsilon}{(P_1 H^{-\sigma}(t))^{q_1}} \int_{\Omega_1} |u|^{q_1+1} dx + \frac{\varepsilon}{(P_2 H^{-\sigma}(t))^{q_2}} \int_{\Omega_2} |v|^{q_2+1} dx \leq \\ &\leq \varepsilon P H^{-\sigma}(t) H'(t) + \varepsilon P_1^{-q_1} H^{\sigma q_1}(t) \int_{\Omega_1} |u|^{q_1+1} dx + \\ &+ \varepsilon P_2^{-q_2} H^{\sigma q_2}(t) \int_{\Omega_2} |v|^{q_2+1} dx, \end{aligned} \tag{4.10}$$

where $P = \max \{P_1, P_2\}$.

Taking into account (4.4) and the obvious inequalities

$$\begin{aligned} \int_{\Omega_1} |u|^{q_1+1} dx &\leq k_1 \left(\int_{\Omega_1} |u|^{p+1} dx \right)^{\frac{q_1+1}{p+1}} \leq \\ &\leq k_1 \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right)^{\frac{q_1+1}{p+1}}, \end{aligned}$$

$$\begin{aligned} \int_{\Omega_2} |v|^{q_2+1} dx &\leq k_2 \left(\int_{\Omega_2} |v|^{p+1} dx \right)^{\frac{q_2+1}{p+1}} \leq \\ &\leq k_2 \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right)^{\frac{q_2+1}{p+1}}, \end{aligned}$$

with $k_1 = |\Omega_1|^{\frac{p-q_1}{p+1}}$, $k_2 = |\Omega_2|^{\frac{p-q_2}{p+1}}$, from (4.10) we get

$$\begin{aligned} \left| \varepsilon \int_{\Omega_1} |u_t|^{q_1-1} u_t u dx + \varepsilon \int_{\Omega_2} |v_t|^{q_2-1} v_t v dx \right| &\leq \varepsilon P H^{-\sigma}(t) H'(t) + \\ &+ \frac{\varepsilon P_1^{-q_1} k_1}{(p+1)^{\sigma q_1}} \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right)^{\sigma q_1 + \frac{q_1+1}{p+1}} + \\ &+ \frac{\varepsilon P_2^{-q_2} k_2}{(p+1)^{\sigma q_2}} \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right)^{\sigma q_2 + \frac{q_2+1}{p+1}}. \end{aligned} \tag{4.11}$$

By (2.8) and (4.1) we have

$$\begin{aligned} -\|\nabla u\|_1^2 - \|\nabla v\|_2^2 &= 2H(t) - 2E_1 + \|u_t\|_1^2 + \|v_t\|_2^2 + \left\| \sqrt{M} \delta_t \right\|_{\Gamma_2}^2 + \left\| \sqrt{K} \delta \right\|_{\Gamma_2}^2 - \\ &- \frac{2}{p+1} \int_{\Omega_1} |u|^{p+1} dx - \frac{2}{p+1} \int_{\Omega_2} |v|^{p+1} dx. \end{aligned} \tag{4.12}$$

From (2.18) we get

$$-2\varepsilon E_1 \geq -\frac{2\varepsilon E_1}{B\alpha_2^{p+1}} \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right). \tag{4.13}$$

Since $g'(\alpha_1) = 0$, then by (2.13), we obtain that $\alpha_1 - B\alpha_1^p = 0$ or

$$\alpha_1^2 = B\alpha_1^{p+1}. \tag{4.14}$$

Taking into account (2.14) and (4.14) in (4.13) yields

$$\begin{aligned} -2\varepsilon E_1 &\geq -2\varepsilon \alpha_1^2 \left(\frac{1}{2} - \frac{1}{p+1} \right) \frac{1}{B\alpha_2^{p+1}} \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right) = \\ &= -\frac{\varepsilon(p-1)}{p+1} \left(\frac{\alpha_1}{\alpha_2} \right)^{p+1} \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right). \end{aligned} \tag{4.15}$$

Multiplying (4.12) by ε and using (4.15) we have

$$\begin{aligned} -\varepsilon \|\nabla u\|_1^2 - \varepsilon \|\nabla v\|_2^2 &\geq \\ &\geq 2\varepsilon H(t) - \left[\frac{\varepsilon(p-1)}{p+1} \left(\frac{\alpha_1}{\alpha_2} \right)^{p+1} + \frac{2\varepsilon}{p+1} \right] \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right) + \\ &+ \varepsilon \|u_t\|_1^2 + \varepsilon \|v_t\|_2^2 + \varepsilon \left\| \sqrt{M} \delta_t \right\|_{\Gamma_2}^2 + \varepsilon \left\| \sqrt{K} \delta \right\|_{\Gamma_2}^2. \end{aligned}$$

Using this and (4.11) from (4.7) we get

$$\begin{aligned}
 \Theta'(t) &\geq (1 - \sigma - \varepsilon P) H^{-\sigma}(t) H'(t) + 2\varepsilon H(t) + \\
 &\quad + 2\varepsilon \int_{\Omega_1} u_t^2 dx + 2\varepsilon \int_{\Omega_2} v_t^2 dx - \varepsilon \int_{\Gamma_2} \delta u_t d\Gamma_2 + \\
 &\quad + \varepsilon \left(1 - \frac{p-1}{p+1} \left(\frac{\alpha_1}{\alpha_2} \right)^{p+1} - \frac{2}{p+1} \right) \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right) + \\
 &\quad + \varepsilon \left\| \sqrt{M} \delta_t \right\|_{\Gamma_2}^2 + \varepsilon \left\| \sqrt{K} \delta \right\|_{\Gamma_2}^2 - \\
 &\quad - \frac{\varepsilon k_1}{P_1^{q_1} (p+1)^{\sigma q_1}} \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right)^{\sigma q_1 + \frac{q_1+1}{p+1}} - \\
 &\quad - \frac{\varepsilon k_2}{P_2^{q_2} (p+1)^{\sigma q_2}} \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right)^{\sigma q_2 + \frac{q_2+1}{p+1}}.
 \end{aligned} \tag{4.16}$$

Since $\alpha_2 > \alpha_1$ and $p > 1$, then

$$1 - \frac{p-1}{p+1} \left(\frac{\alpha_1}{\alpha_2} \right)^{p+1} - \frac{2}{p+1} = \frac{p-1}{p+1} \left(1 - \left(\frac{\alpha_1}{\alpha_2} \right)^{p+1} \right) = \beta > 0. \tag{4.17}$$

Since by (4.6), $0 < \sigma q_i + \frac{q_i+1}{p+1} \leq 1, i = 1, 2$, then using the algebraic inequality

$$z^\nu \leq z + 1 \leq \left(1 + \frac{1}{a} \right) (z + a), \forall z \geq 0, 0 < \nu \leq 1, a > 0, \tag{4.18}$$

we obtain

$$\begin{aligned}
 &\left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right)^{\sigma q_i + \frac{q_i+1}{p+1}} \leq \\
 &\leq \left(1 + \frac{1}{H(0)} \right) \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx + H(0) \right),
 \end{aligned}$$

or denoting $1 + \frac{1}{H(0)} = \gamma$, by (4.3) we get

$$\begin{aligned}
 &\left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right)^{\sigma q_i + \frac{q_i+1}{p+1}} \leq \\
 &\leq \gamma \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx + H(t) \right), i = 1, 2.
 \end{aligned} \tag{4.19}$$

Using (4.17) and (4.19) in (4.16), we have

$$\begin{aligned} \Theta'(t) &\geq (1 - \sigma - \varepsilon P) H^{-\sigma}(t) H'(t) + 2\varepsilon H(t) + 2\varepsilon \int_{\Omega_1} u_t^2 dx + \\ &\quad + 2\varepsilon \int_{\Omega_2} v_t^2 dx - \varepsilon \int_{\Gamma_2} \delta u_t d\Gamma_2 + \\ &\quad + \varepsilon \beta \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right) + \varepsilon \left\| \sqrt{M} \delta_t \right\|_{\Gamma_2}^2 + \varepsilon \left\| \sqrt{K} \delta \right\|_{\Gamma_2}^2 - \\ &\quad - \varepsilon \gamma \left(\frac{k_1}{P_1^{q_1} (p+1)^{\sigma q_1}} + \frac{k_2}{P_2^{q_2} (p+1)^{\sigma q_2}} \right) \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx + H(t) \right) \end{aligned}$$

or since $-\varepsilon \int_{\Gamma_2} \delta u_t d\Gamma_2 \geq -\frac{\varepsilon}{2} \int_{\Gamma_2} \delta^2 d\Gamma_2 - \frac{\varepsilon}{2} \int_{\Gamma_2} u_t^2 d\Gamma_2$ and $K \geq 1$, then

$$\begin{aligned} \Theta'(t) &\geq (1 - \sigma - \varepsilon P) H^{-\sigma}(t) H'(t) + \\ &\quad + \frac{3\varepsilon}{2} \int_{\Omega_1} u_t^2 dx + 2\varepsilon \int_{\Omega_2} v_t^2 dx + \frac{\varepsilon}{2} \int_{\Gamma_2} \delta^2 d\Gamma_2 + \\ &\quad + \varepsilon \left[2 - \gamma \left(\frac{k_1}{P_1^{q_1} (p+1)^{\sigma q_1}} + \frac{k_2}{P_2^{q_2} (p+1)^{\sigma q_2}} \right) \right] H(t) + \varepsilon \left\| \sqrt{M} \delta_t \right\|_{\Gamma_2}^2 + \\ &\quad + \varepsilon \left[\beta - \gamma \left(\frac{k_1}{P_1^{q_1} (p+1)^{\sigma q_1}} + \frac{k_2}{P_2^{q_2} (p+1)^{\sigma q_2}} \right) \right] \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right). \end{aligned} \tag{4.20}$$

We choose P_1, P_2 large enough such that

$$\begin{aligned} \theta_1 &= 2 - \gamma \left(\frac{k_1}{P_1^{q_1} (p+1)^{\sigma q_1}} + \frac{k_2}{P_2^{q_2} (p+1)^{\sigma q_2}} \right) > 0, \\ \theta_2 &= \beta - \gamma \left(\frac{k_1}{P_1^{q_1} (p+1)^{\sigma q_1}} + \frac{k_2}{P_2^{q_2} (p+1)^{\sigma q_2}} \right) > 0. \end{aligned}$$

Then we choose ε small enough such that $1 - \sigma - \varepsilon P > 0$ and

$$\Theta(0) = H^{1-\sigma}(0) + \varepsilon \left(\int_{\Omega_1} u_0 u_1 dx + \int_{\Omega_2} v_0 v_1 dx - \int_{\Gamma_2} u_0 \delta_0 d\Gamma_2 \right) > 0 \tag{4.21}$$

or

$$\varepsilon < \min \left\{ \frac{1 - \sigma}{P}, \frac{H^{1-\sigma}(0)}{\left| \int_{\Omega_1} u_0 u_1 dx + \int_{\Omega_2} v_0 v_1 dx - \int_{\Gamma_2} u_0 \delta_0 d\Gamma_2 \right|} \right\}.$$

Then from (4.20) we have

$$\begin{aligned} \Theta'(t) &\geq \varepsilon \left(\int_{\Omega_1} u_t^2 dx + \int_{\Omega_2} v_t^2 dx \right) + \frac{\varepsilon}{2} \int_{\Gamma_2} \delta^2 d\Gamma_2 + \varepsilon \theta_1 H(t) + \\ &\quad + \varepsilon \theta_2 \left(\int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx \right). \end{aligned} \tag{4.22}$$

Therefore, $\Theta(t)$ is a nondecreasing function for $t \geq 0$. Using (4.21), we obtain that

$$\Theta(t) \geq \Theta(0) > 0, t \geq 0.$$

Since $0 < \sigma < 1$, it is obvious that $\frac{1}{1-\sigma} > 1$. Then using the inequality

$$(a + b)^r \leq C (a^r + b^r), a, b \geq 0, r > 0, C > 0,$$

from (4.5) we get

$$\Theta^{\frac{1}{1-\sigma}}(t) \leq C_0 \left[H(t) + \left(\int_{\Omega_1} u u_t dx + \int_{\Omega_2} v v_t dx - \int_{\Gamma_2} u \delta d\Gamma_2 \right)^{\frac{1}{1-\sigma}} \right], \tag{4.23}$$

where C_0 is a positive constant.

On the other hand, from Hölder and Young inequalities we obtain that

$$\begin{aligned} & \left(\int_{\Omega_1} u u_t dx + \int_{\Omega_2} v v_t dx - \int_{\Gamma_2} u \delta d\Gamma_2 \right)^{\frac{1}{1-\sigma}} \leq \\ & \leq \left[|\Omega_1|^{\frac{p-1}{2(p+1)}} \left(\int_{\Omega_1} |u|^{p+1} dx \right)^{\frac{1}{p+1}} \left(\int_{\Omega_1} u_t^2 dx \right)^{\frac{1}{2}} + \right. \\ & + |\Omega_2|^{\frac{p-1}{2(p+1)}} \left(\int_{\Omega_2} |v|^{p+1} dx \right)^{\frac{1}{p+1}} \left(\int_{\Omega_2} v_t^2 dx \right)^{\frac{1}{2}} + \\ & \left. + |\Omega_1|^{\frac{p-1}{2(p+1)}} \left(\int_{\Omega_1} |u|^{p+1} dx \right)^{\frac{1}{p+1}} \left(\int_{\Gamma_2} \delta^2 d\Gamma_2 \right)^{\frac{1}{2}} \right]^{\frac{1}{1-\sigma}} \leq \\ & \leq |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left(\int_{\Omega_1} |u|^{p+1} dx \right)^{\frac{1}{(p+1)(1-\sigma)}} \left(\int_{\Omega_1} u_t^2 dx \right)^{\frac{1}{2(1-\sigma)}} + \\ & + |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left(\int_{\Omega_2} |v|^{p+1} dx \right)^{\frac{1}{(p+1)(1-\sigma)}} \left(\int_{\Omega_2} v_t^2 dx \right)^{\frac{1}{2(1-\sigma)}} + \\ & + |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left(\int_{\Omega_1} |u|^{p+1} dx \right)^{\frac{1}{(p+1)(1-\sigma)}} \left(\int_{\Gamma_2} \delta^2 d\Gamma_2 \right)^{\frac{1}{2(1-\sigma)}} \leq \\ & \leq |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\frac{1-2\sigma}{2(1-\sigma)} \left(\int_{\Omega_1} |u|^{p+1} dx \right)^{\frac{2}{(p+1)(1-2\sigma)}} + \frac{1}{2(1-\sigma)} \int_{\Omega_1} u_t^2 dx \right] + \\ & + |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\frac{1-2\sigma}{2(1-\sigma)} \left(\int_{\Omega_2} |v|^{p+1} dx \right)^{\frac{2}{(p+1)(1-2\sigma)}} + \frac{1}{2(1-\sigma)} \int_{\Omega_2} v_t^2 dx \right] + \\ & + |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\frac{1-2\sigma}{2(1-\sigma)} \left(\int_{\Omega_1} |u|^{p+1} dx \right)^{\frac{2}{(p+1)(1-2\sigma)}} + \frac{1}{2(1-\sigma)} \int_{\Gamma_2} \delta^2 d\Gamma_2 \right]. \end{aligned} \tag{4.24}$$

Since by (4.6), $0 < \frac{2}{(p+1)(1-2\sigma)} \leq 1$, then using (4.18) we obtain

$$\begin{aligned} & \left(\int_{\Omega_1} |u|^{p+1} dx \right)^{\frac{2}{(p+1)(1-2\sigma)}} \leq \left(1 + \frac{1}{H(0)} \right) \left(\int_{\Omega_1} |u|^{p+1} dx + H(0) \right) \leq \\ & \leq \left(1 + \frac{1}{H(0)} \right) \left(\int_{\Omega_1} |u|^{p+1} dx + H(t) \right) = \gamma \left(\int_{\Omega_1} |u|^{p+1} dx + H(t) \right). \end{aligned} \tag{4.25}$$

Similarly, we have

$$\left(\int_{\Omega_2} |v|^{p+1} dx \right)^{\frac{2}{(p+1)(1-2\sigma)}} \leq \gamma \left(\int_{\Omega_2} |v|^{p+1} dx + H(t) \right). \tag{4.26}$$

From (4.24)–(4.26) we get

$$\begin{aligned} & \left(\int_{\Omega_1} u u_t dx + \int_{\Omega_2} v v_t dx - \int_{\Gamma_2} u \delta d\Gamma_2 \right)^{\frac{1}{1-\sigma}} \leq \\ & \leq |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\frac{1-2\sigma}{2(1-\sigma)} \gamma \left(\int_{\Omega_1} |u|^{p+1} dx + H(t) \right) + \frac{1}{2(1-\sigma)} \int_{\Omega_1} u_t^2 dx \right] + \\ & + |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\frac{1-2\sigma}{2(1-\sigma)} \gamma \left(\int_{\Omega_2} |v|^{p+1} dx + H(t) \right) + \frac{1}{2(1-\sigma)} \int_{\Omega_2} v_t^2 dx \right] + \\ & + |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\frac{1-2\sigma}{2(1-\sigma)} \gamma \left(\int_{\Omega_1} |u|^{p+1} dx + H(t) \right) + \frac{1}{2(1-\sigma)} \int_{\Gamma_2} \delta^2 d\Gamma_2 \right]. \end{aligned}$$

Using this in (4.23), we obtain

$$\begin{aligned} \Theta^{\frac{1}{1-\sigma}}(t) & \leq C_0 \left[1 + 2|\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)} + |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)} \right] H(t) + \\ & + C_0 |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{1-\sigma} \int_{\Omega_1} |u|^{p+1} dx + C_0 |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)} \int_{\Omega_2} |v|^{p+1} dx + \\ & + C_0 \frac{|\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}}}{2(1-\sigma)} \int_{\Omega_1} u_t^2 dx + C_0 \frac{|\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}}}{2(1-\sigma)} \int_{\Omega_2} v_t^2 dx + C_0 \frac{|\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}}}{2(1-\sigma)} \int_{\Gamma_2} \delta^2 d\Gamma_2 \end{aligned}$$

or

$$\begin{aligned} \Theta^{\frac{1}{1-\sigma}}(t) & \leq C_1 \left(H(t) + \int_{\Omega_1} |u|^{p+1} dx + \right. \\ & \left. + \int_{\Omega_2} |v|^{p+1} dx + \int_{\Omega_1} u_t^2 dx + \int_{\Omega_2} v_t^2 dx + \int_{\Gamma_2} \delta^2 d\Gamma_2 \right), \end{aligned} \tag{4.27}$$

where

$$\begin{aligned} C_1 = C_0 \max \left\{ 1 + 2|\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)} + \right. \\ \left. + |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)}, |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{1-\sigma}, \right. \\ \left. |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)}, \frac{|\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}}}{2(1-\sigma)}, \frac{|\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}}}{2(1-\sigma)} \right\}. \end{aligned}$$

Putting $C_2 = \varepsilon \min \{ 1; \theta_1; \theta_2 \}$, from (4.22) we get

$$\Theta'(t) \geq C_2 \left(H(t) + \int_{\Omega_1} |u|^{p+1} dx + \int_{\Omega_2} |v|^{p+1} dx + \int_{\Omega_1} u_t^2 dx + \int_{\Omega_2} v_t^2 dx \right). \tag{4.28}$$

It follows from (4.27) and (4.28) that

$$\Theta'(t) \geq C_3 \Theta^{\frac{1}{1-\sigma}}(t),$$

where $C_3 = \frac{C_2}{C_1}$. Integrating both sides of this inequality over $[0, t]$ yields

$$\Theta(t) \geq \left(\Theta^{-\frac{\sigma}{1-\sigma}}(0) - \frac{C_3 \sigma}{1-\sigma} t \right)^{-\frac{1-\sigma}{\sigma}}.$$

Noting that $\Theta(0) > 0$, there exists $T_1 = \frac{(1-\sigma)\Theta^{-\frac{\sigma}{1-\sigma}}(0)}{C_3\sigma}$ such that $\Theta(t) \rightarrow +\infty$ as $t \rightarrow T_1$. In other words, the solution of problem (1.1)–(1.8) blows up in finite time.

Theorem 2.5 is proved.

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References

- [1] Aliev AB, Mammadhasanov EH. Well-posedness of initial boundary value problem on longitudinal impact on a composite linear viscoelastic bar. *Math Method Appl Sci* 2017; 40: 5380-5390.
- [2] Bae JJ. Nonlinear transmission problem for wave equation with boundary condition of memory type. *Acta Appl Math* 2010; 110: 907-919.
- [3] Ball J. Remarks on blow up and nonexistence theorems for nonlinear evolution equations. *Quart J Math Oxford* 1977; 28: 473-486.
- [4] Bastos WD, Raposo CA. Transmission problem for waves with frictional damping. *Electron J Differ Eq* 2007; 60: 1-10.
- [5] Beale JT. Spectral properties of an acoustic boundary condition. *Indiana Univ Math J* 1976; 25: 895-917.
- [6] Beale JT. Acoustic scattering from locally reacting surfaces. *Indiana Univ Math J* 1977; 26: 199-222.
- [7] Beale JT, Rosencrans SI. Acoustic boundary conditions. *B Am Math Soc* 1974; 80: 1276-1278.
- [8] Bilgin BA, Kalantarov VK. Blow up of solutions to the initial boundary value problem for quasilinear strongly damped wave equations. *J Math Anal Appl* 2013; 403: 89-94.
- [9] Boukhatem Y, Benabderrahmane B. Existence and decay of solutions for a viscoelastic wave equation with acoustic boundary conditions. *Nonlinear Anal* 2014; 97: 191-209.
- [10] Boukhatem Y, Benabderrahmane B. Polynomial decay and blow up of solutions for variable coefficients viscoelastic wave equation with acoustic boundary conditions. *Acta Math Sin* 2016; 32: 153-174.
- [11] Bucci F, Lasiecka I. Exponential decay rates for structural acoustic model with an over damping on the interface and boundary layer dissipation. *Appl Anal* 2002; 81: 977-999.
- [12] Cousin AT, Frota CL, Larkin NA. On a system of Klein-Gordon type equations with acoustic boundary conditions. *J Math Appl* 2004; 293: 293-309.
- [13] Dautray R, Lions JL. *Analyse et Calcul Numerique pour les Sciences et les Techniques*. Paris, France: Masson, 1984 (in French).
- [14] Frota CL, Cousin AT, Larkin NA. Global solvability and asymptotic behaviour of a hyperbolic problem with acoustic boundary conditions. *Funkcial Ekvac* 2001; 44: 471-485.
- [15] Frota CL, Goldstein JA. Some wave equations with acoustic boundary conditions. *J Differ Equations* 2000; 164: 92-109.
- [16] Frota CI, Larkin NA. Uniform stabilization for a hyperbolic equation with acoustic boundary conditions in simple connected domains. *Prog Nonlin* 2005; 66: 297-312.
- [17] Frota CI, Medeyros LA, Vicente A. A mixed problem for semilinear wave equations with acoustic boundary conditions in domains with non-locally reacting boundary. *Electron J Differ Eq* 2014; 2014: 1-14.
- [18] Frota CL, Vicente A. A hyperbolic system of Klein-Gordon type with acoustic boundary conditions. *Int J Pure Appl Math*. 2008; 47: 185-198.

- [19] Gal CG, Goldstein GR, Goldstein JA. Oscillatory boundary conditions for acoustic wave equations. *J Evol Equ* 2003; 3: 623-635.
- [20] Gao Y, Liang J, Xiao TJ. Observability inequality and decay rate for wave equations with nonlinear boundary conditions. *Electron J Differ Eq* 2017; 2017: 1-12.
- [21] Georgiev V, Todorova G. Existence of a global solution of the wave equation with nonlinear damping and source term. *J Differ Equations* 1994; 109: 295-308.
- [22] Graber PJ. Wave equation with porous nonlinear acoustic boundary conditions generates a well-posed dynamical system. *Nonlinear Anal* 2010; 73: 3058-3068.
- [23] Graber PJ. Strong stability and uniform decay of solutions to a wave equation with semilinear porous acoustic boundary conditions. *Nonlinear Anal* 2011; 74: 3137-3148.
- [24] Graber PJ, Said-Houari B. On the wave equation with semilinear porous acoustic boundary conditions. *J Differ Equations* 2012; 252: 4898-4941.
- [25] Haraux A, Zuazua E. Decay estimates for some semilinear damped hyperbolic problems. *Arch Ration Mech Anal* 1988; 150: 191-206.
- [26] Isayeva SE. Existence of local solutions for nonlinear wave equations with transmission acoustic conditions. *News of Baku University Series of Physico-mathematical Sciences* (in press).
- [27] Jeong JM, Park JY, Kang YH. Global nonexistence of solutions for a quasilinear wave equation with acoustic boundary conditions. *Boundary Value Problems* 2017; 2017: 1-10.
- [28] Kalantarov VK, Ladyzhenskaya OA. The occurrence of collapse for quasilinear equations of parabolic and hyperbolic types. *J Soviet Math* 1978; 10: 53-70.
- [29] Kopackova M. Remarks on bounded solutions of a semilinear dissipative hyperbolic equation. *Comment Math Univ Cardin* 1989; 30: 713-719.
- [30] Lasiecka I. Boundary stabilization of a 3-dimensional structural acoustic model. *J Math Pure Appl* 1999; 78: 203-232.
- [31] Levine HA. Instability and nonexistence of global solutions to nonlinear wave equations of the form $Pu_{tt} = Au + F(u)$. *T Am Math Soc* 1974; 192: 1-21.
- [32] Levine HA. Some additional remarks on the nonexistence of global solutions to nonlinear wave equations. *SIAM J Math Anal* 1974; 5: 138-146.
- [33] Levine HA, Park SR. Global existence and global nonexistence of solutions of the Cauchy problem for a non-linearly damped wave equation. *J Math Anal Appl* 1998; 228: 181-205.
- [34] Levine HA, Serrin J. Global nonexistence theorems for quasilinear evolution equations with dissipation. *Arch Ration Mech Anal* 1997; 137: 341-361.
- [35] Lions JL. *Quelques méthodes de résolution des problèmes aux limites non linéaires*. Paris, France: Dunod Gauthier-Villars, 1969 (in French).
- [36] Lions JL, Magenes E. *Non-Homogeneous Boundary Value Problems and Applications. I*. Berlin, Germany: Springer Verlag, 1972.
- [37] Liu W, Williams G. The exponential stability of the problem of transmission of the wave equation. *Bull Austral Math Soc* 1998; 57: 305-327.
- [38] Morse PM, Ingard KU. *Theoretical Acoustics*. New York, NY, USA: McGraw-Hill, 1968.
- [39] Mugnolo D. Abstract wave equations with acoustic boundary conditions. *Math Nachr* 2006; 279: 299-318.
- [40] Muñoz Rivera JE, Portillo Oquendo H. The transmission problem of viscoelastic waves. *Acta Applicandae Mathematicae* 2000; 60: 1-21.

- [41] Park JY, Ha TG. Well-posedness and uniform decay rates for the Klein-Gordon equation with damping term and acoustic boundary conditions. *J Math Phys* 2009; 50: 1-18.
- [42] Park JY, Park SH. Decay rate estimates for wave equations of memory type with acoustic boundary conditions. *Nonlinear Anal* 2011; 74: 993-998.
- [43] Vicente A. Wave equation with acoustic/memory boundary conditions. *Bol Soc Parana Mat* 2009; 27: 29-39.
- [44] Vitillaro E. Global existence theorems for a class of evolution equations with dissipation. *Arch Ration Mech Anal* 1999; 149: 155-182.