Turkish Journal of Physics

Volume 32 | Number 6 Article 7

1-1-2008

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Turk J Phys
32 (2008) , 357 – 361.

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Received 10.07.2008

Abstract

Field equations in the presence of a perfect fluid distribution for static plane symmetric metric are obtained in the scale invariant theory of gravitation proposed by Wesson [1]. A static Zeldovich fluid model corresponding to perfect fluid is presented. Physical and kinematical properties of the model are discussed.

Key Words: Plane Symmetric, Gauge Function, Zeldovich Fluid, Space-time.
PACS Nos.: 04.50.+h; 04.80.C; 04.20.H; 95.30; 95.30.S

1. Introduction

In order to modify Einstein's theory of gravitation in certain aspects, several theories of gravitation have been proposed from time to time. Prominent among them are scalar-tensor theories of gravitation formulated by Brans and Dicke [2], Nordtvedt [3], Wagoner [4], Ross [5], Dunn [6] and Saez and Ballester [7]. In the Brans-Dicke theory, there exists a variable gravitation parameter. Wesson [1] formulated a scale invariant theory of gravitation, which also admits a variable G as a viable alternative to scale covariant theory of gravitation (Canuto et al.; [8, 9]). In the scale invariant theory of gravitation, Einstein equations have been written in a scale-independent way by performing the conformal or scale transformation as

$$\bar{g}_{ij} = \beta^2 \left(x^k\right) g_{ij}, \tag{1.1}$$

where the gauge function β $(0 < \beta < \infty)$, in its most general formulation, is a function of all space-time coordinates. Thus using the conformal transformation of the type given by equation (1.1), Wesson [1] transforms the usual Einstein field equations into

$$G_{ij} + 2\frac{\beta_{;ij}}{\beta} - 4\frac{\beta_{,i}\beta_{,j}}{\beta^2} + \left(g^{ab}\frac{\beta_{,a}\beta_{,b}}{\beta^2} - 2g^{ab}\frac{\beta_{;ab}}{\beta}\right) g_{ij} + \Lambda_0\beta^2 g_{ij} = -\kappa T_{ij} \tag{1.2}$$

$$G_{ij} \equiv R_{ij} - \frac{1}{2}R g_{ij}. \tag{1.3}$$

In these equations, G_{ij} is the conventional Einstein tensor involving g_{ij}. Semicolon and comma, respectively, denote covariant differentiation with respect to g_{ij} and partial differentiation with respect to coordinates. R_{ij} is the Ricci tensor, and R is the Ricci scalar. The cosmological term Λg_{ij} of Einstein theory is now transformed to $\Lambda_0\beta^2 g_{ij}$ in scale invariant theory with a dimensionless cosmological constant Λ_0. G and κ are, respectively, the Newtonian and Wesson's gravitational parameter. T_{ij} is the energy momentum tensor of the matter field and $\kappa = \frac{8\pi G}{c^4}$. A particular feature of this theory is that no independent equation for β exists.

Beesham [10, 11, 12], Reddy and Venkateswaralu [13], Mohanty and Mishra [14, 15] have investigated several aspects of this theory of gravitation. Recently Mishra [16] have investigated the problem of non-static plane symmetric perfect fluid distribution in scale invariant theory of gravitation with a time dependent gauge function.

2. Field Equations

The non-static plane symmetric metric with a gauge function $\beta = \beta\,(ct)$ can be given as

$$ds_W^2 = \beta^2 ds_E^2 \tag{2.1}$$

with

$$ds_E^2 = e^{2A}\left(c^2dt^2 - dx^2\right) - e^{2B}\left(dy^2 + dz^2\right), \tag{2.2}$$

where $A = A\,(t)$, $B = B\,(t)$, and c is the velocity of light. ds_W and ds_E represent the intervals in Wesson and Einstein theory of gravitation, respectively.

The energy momentum tensor for a perfect fluid can be given as

$$T_{ij}^m = \left(p_m + \rho_m c^2\right) U_i U_j - p_m g_{ij} \tag{2.3}$$

together with

$$g^{ij}U_iU_j = 1, \tag{2.4}$$

where $U^i$is the four velocity vector of the fluid. $p_m$and $\rho_m$are proper isotropic pressure and energy density of the matter, respectively.

The non-vanishing components of conventional Einstein's tensor (1.3) for the metric (2.2) can be obtained as

$$G_{11} \equiv \frac{1}{c^2}\left[2B_{44} + 3B_4^2 - 2A_4B_4\right] \tag{2.5}$$

$$G_{22} = G_{33} \equiv \frac{e^{2B}}{c^2e^{2A}}\left[A_{44} + B_{44} + B_4^2\right] \tag{2.6}$$

$$G_{44} \equiv -\left[B_4^2 + 2A_4B_4\right]. \tag{2.7}$$

From here on, the suffix 4 after a field variable denotes exact differentiation with respect to time t only.

Using the comoving coordinate $\left(0,0,0,ce^A\right)$, the non-vanishing components of the field equation (1.2) for the metric (2.1) can be written in the explicit form

$$G_{11} = -\kappa p_m e^{2A} - \frac{1}{c^2}\left[2\frac{\beta_{44}}{\beta} - \frac{\beta_4^2}{\beta^2} + \left(-2A_4 + 4B_4\right)\frac{\beta_4}{\beta} - \Lambda_0\beta^2c^2e^{2A}\right] \tag{2.8}$$

$$G_{22} = G_{33} = -\kappa p_m e^{2B} - \frac{e^{2B}}{c^2e^{2A}}\left[2\frac{\beta_{44}}{\beta} - \frac{\beta_4^2}{\beta^2} + \left(2B_4\right)\frac{\beta_4}{\beta} - \Lambda_0\beta^2c^2e^{2A}\right] \tag{2.9}$$

$$G_{44} = -\kappa\rho_m c^4e^{2A} + \left[3\frac{\beta_4^2}{\beta^2} + \left(2A_4 + 4B_4\right)\frac{\beta_4}{\beta} - \Lambda_0\beta^2c^2e^{2A}\right]. \tag{2.10}$$

Now, equation (1.2) and equations (2.8)–(2.10) suggest the definition of quantities p_v(vacuum pressure) and ρ_v(vacuum density), which involve neither the Einstein tensor of conventional theory nor the properties of conventional matter [1]. These two quantities can be obtained as

$$2\frac{\beta_{44}}{\beta} - \frac{\beta_4^2}{\beta^2} + \left(-2A_4 + 4B_4\right)\frac{\beta_4}{\beta} - \Lambda_0\beta^2c^2e^{2A} = \kappa p_v c^2 e^{2A} \tag{2.11}$$

$$2\frac{\beta_{44}}{\beta} - \frac{\beta_4^2}{\beta^2} + (2B_4)\frac{\beta_4}{\beta} - \Lambda_0\beta^2c^2e^{2A} = \kappa p_v c^2 e^{2A} \tag{2.12}$$

$$3\frac{\beta_4^2}{\beta^2} + (2A_4 + 4B_4)\frac{\beta_4}{\beta} - \Lambda_0\beta^2c^2e^{2A} = -\kappa\rho_v c^4 e^{2A}. \tag{2.13}$$

When there is no matter and the gauge function β is a constant, one can recover the relation

$$c^2\rho_v = -c^4\frac{\lambda_{GR}}{8\pi G} = -p_v, \tag{2.14}$$

which is the equation of state for vacuum. p_v is dependent on constants λ_{GR}, G and c is uniform in all directions and hence isotropic in nature. The cosmological model with the equation of state is rare in the literature and is known as ρ-vacuum universe or false vacuum or degenerate vacuum model [17–20]. The corresponding model in static case is the well-known De Sitter model. Hence p_v being isotropic is consistent only when

$$A = B + k_1, \tag{2.15}$$

where $k_1$is an integrating constant, since $\beta_4 \neq 0$.

Using the consistency condition (2.15), pressure and energy density for the vacuum case reduce to

$$p_v = \frac{1}{\kappa c^2 e^{2A}}\left[2\frac{\beta_{44}}{\beta} - \frac{\beta_4^2}{\beta^2} + (2A_4)\frac{\beta_4}{\beta} - \Lambda_0\beta^2c^2e^{2A}\right] \tag{2.16}$$

$$\rho_v = \frac{1}{\kappa c^4 e^{2A}}\left[3\frac{\beta_4^2}{\beta^2} + (6A_4)\frac{\beta_4}{\beta} - \Lambda_0\beta^2c^2e^{2A}\right]. \tag{2.17}$$

Here, p_v and ρ_v relate to the properties of vacuum only in conventional physics. The definition of p_v and ρ_v are natural as regards to the scale invariant properties of the vacuum (see Wesson, [1]). The total pressure and energy density can be defined as

$$p_t \equiv p_m + p_v \tag{2.18}$$

$$\rho_t \equiv \rho_m + \rho_v. \tag{2.19}$$

Using the aforementioned definition of p_t and ρ_t, the field equations in scale invariant theory of gravity, i.e. equations (2.8)–(2.10), can now be written by using the components of Einstein tensor (2.5)–(2.7) and the results obtained in equations (2.15)–(2.17) as:

$$2A_{44} + A_4^2 = -\kappa p_t c^2 e^{2A} \tag{2.20}$$

$$3A_4^2 = \kappa\rho_t c^4 e^{2A}. \tag{2.21}$$

3. Solutions of the Field Equations

Equations (2.20)–(2.21) are two equations with three unknowns, viz. p_t, ρ_t and A. For complete determinacy one extra condition is required. So, the equation of state $p_t = \rho_t c^2$, i.e. Zeldovich fluid model, is considered. Also, $\beta = \frac{1}{ct}$.

Using the equation of state $p_t = \rho_t c^2$, equations (2.20) and (2.21) yield

$$A = \log{(2t + k_3)}^{1/2} + k_2, \tag{3.1}$$

where k_2 and k_3 are constants of integration.

Without loss of generality, $k_1 = 0$ is assumed in equation (2.15). Subsequently

$$A = B = \log{(2t + k_3)}^{1/2} + k_2. \tag{3.2}$$

The total pressure p_t and total energy density ρ_t can be calculated as

$$p_t = \rho_t c^2 = \frac{3}{\kappa c^2 e^{2k_2} \left(2t + k_3\right)^3}. \tag{3.3}$$

The vacuum pressure p_v and vacuum energy density $\rho_v$can be calculated as

$$p_v = \frac{1}{\kappa c^2 e^{2k_2}} \left[\frac{1}{t^2 \left(2t + k_3\right)} - \frac{2}{t \left(2t + k_3\right)^2} - \frac{\Lambda_0 e^{2k_2}}{t^2} \right] \tag{3.4}$$

$$\rho_v = \frac{1}{\kappa c^4 e^{2k_2}} \left[\frac{3}{t^2 \left(2t + k_3\right)} - \frac{6}{t \left(2t + k_3\right)^2} - \frac{\Lambda_0 e^{2k_2}}{t^2} \right]. \tag{3.5}$$

The matter pressure p_m and matter energy density ρ_m can be calculated as

$$p_m = \frac{1}{\kappa c^2 e^{2k_2}} \left[\frac{3}{\left(2t + k_3\right)^3} - \frac{1}{t^2 \left(2t + k_3\right)} - \frac{2}{t \left(2t + k_3\right)^2} - \frac{\Lambda_0 e^{2k_2}}{t^2} \right] \tag{3.6}$$

$$\rho_m = \frac{1}{\kappa c^4 e^{2k_2}} \left[\frac{3}{\left(2t + k_3\right)^3} - \frac{3}{t^2 \left(2t + k_3\right)} + \frac{6}{t \left(2t + k_3\right)^2} + \frac{\Lambda_0 e^{2k_2}}{t^2} \right]. \tag{3.7}$$

So, the Zeldovich static plane symmetric model in scale invariant theory can be given by equations (2.15), (3.2) and (3.3). The metric for this case can be written as

$$ds_W^2 = \frac{1}{c^2 t^2} \left[e^{2k_2} \left(2t + k_3\right) \left\{ c^2 dt^2 - dx^2 - dy^2 - dz^2 \right\} \right]. \tag{3.8}$$

4. Some Physical and Kinematical Properties of the Model

In this section, the physical and kinematical properties of the cosmological model given by equation (3.8) have been studied.

The scalar expansion of the model can be calculated as

$$\Theta = U^i_{;i} = \frac{3}{c e^{2k_2} \left(2t + k_3\right)^2}, \tag{4.1}$$

from which it is evident that $\Theta \to 0$ as $t \to \infty$ and $\Theta \to \frac{3}{c e^{2k_2} k_3^2}$ (constant) as $t \to 0$. So, the universe is expanding with increase of time and the rate of expansion is slow with increase in time.

It has also been observed that,

$$\frac{\rho_m}{\Theta^2} \to \infty \text{ as } t \to 0 \text{ and } \frac{\rho_m}{\Theta^2} \to \text{constant as } t \to \infty, \tag{4.2}$$

which confirms the homogeneity nature of the space-time during evolution. Further,

$$\rho_m \to \infty \text{ as } t \to 0 \text{ and } \rho_m \to 0 \text{ as } t \to \infty, \tag{4.3}$$

which indicates that there is a big bang like singularity at initial epoch.

The shear scalar $\sigma = 0$ indicates that the shape of the universe remains unchanged during evolution. Moreover, since $\frac{\sigma^2}{\Theta^2} = 0$, the space-time is isotropized during evolution in scale invariant theory. As the acceleration is found to be zero, the matter particle follows geodesic path in this theory. The vorticity W of the model vanishes, which indicates that U^i is hypersurface orthogonal.

The spatial volume of the model is found to be

$$V = e^{3k_2} \left(2t + k_3\right)^{3/2}. \tag{4.4}$$

So, $V = 0$ at $t_0 = -k_3/2$, which shows that the universe starts evolution with zero volume at $t = t_0$and expands with t.

The deceleration parameter q for the model (3.8) can be obtained as

$$q = -3\theta^2 \left[\theta_{;\alpha} u^\alpha + \frac{1}{3}\theta^2\right] = \frac{54}{e^{4k_2}\left(2t + k_3\right)^6}. \tag{4.5}$$

The positive value of the deceleration parameter shows that the model decelerates in the standard way.

5. Conclusion

It is well known that plane symmetric solutions are important in the study of relativistic cosmology and Astrophysics. Here, plane symmetric static Zeldovich fluid model in the presence of perfect fluid distribution, in scale invariant theory of gravitation is obtained. As far as matter is concerned, the model admits big bang singularity at initial epoch. It is also observed that the model decelerates in a standard way. Moreover, the universe starts evolution with zero volume and expands with time t.

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