Turkish Journal of Physics

Volume 35 | Number 2 Article 3

1-1-2011

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Turk J Phys
35 (2011) , 115 – 120.

doi:10.3906/fiz-1012-67

Analytical expression for mode-coherence coefficient of a uniformly-distributed wave propagating in different homogeneous media

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Received: 20.12.2010

Abstract

Presented is a derivation of an analytical expression for the mode-coherence coefficients of uniform-distributed wave propagating within different homogeneous media—as in the case of hyperbolic Gaussian beams—and a simple method involving the superposition of two such beams is proposed. The results obtained from this work are very applicable to study and analysis of Hermite-Gaussian beam propagation, especially in the problems of radiation-matter interaction, and laser beam propagation such as employed in free-space and fiber optical communications.

Key Words: Wave propagation, mode coherence coefficient, Gaussian beams, uniformly-distributed waves

1. Introduction

In 1978, Collett and Wolf predicted that Gaussian Schell-model (GSM) beams may have the same directionality as a fully coherent laser beam in free space [1, 2], which was later confirmed experimentally [3, 4]. It implies that full spatial coherence is not a necessary condition for highly directional light beams. Many works have been carried out concerning the spreading of laser beams in atmospheric turbulence [5–8].

With decreasing availability of RF spectrum and the increasing demand for higher communications bandwidths, the terahertz laser communications bandwidths are seen as a viable augmentation of RF communications capability. Yet, cloud cover effects can impact link availability. Among the key strategies to increase availability and mitigate cloud cover effects is the global deployment of ground stations in atmospherically independent cells. Yet with such a deployment, one needs to address the impact of the uplink laser beams on the flying public and on space assets sensitive to laser radiation. Near damage threshold of human eye, the power densities on the communications downlink are usually eye safe. Although the power densities of the uplink beacon required for Earth orbiters to track the ground station can, depending on mission, be within eye-safe laser levels, this will not be so when operations call for transmitting a beacon or commands to deep-space probes [9].

The discovery of propagation-invariant beams naturally led to the idea of similar pulses or wave packets. Solitons are, of course, well-known for waves propagating in nonlinear media where the nonlinearity serves to counterbalance the effect of diffraction. Similarly (radial) changes in the index of refraction can be used to form a waveguide that supports localized waves. In free space or in a linear medium, no such equities are available. Periodically propagating waves are not strictly propagation-invariant although they avoid diffractive spreading by returning to their initial pattern after a certain propagation distance or time. They are further allowed to rotate in-between.

A systematic approach has been introduced for all periodically evolving pulsed waves for velocities $0 < v < \infty$. Their spectral characteristics vary according to whether this velocity of propagation equals, exceeds, or is below the speed of light.

Recently, we have found that, besides the equivalent Gaussian-Schell model beams, there also exist other equivalent partially and fully coherent beams which may have the same directionality as a fully coherent laser beam in free space and also in atmospheric turbulence, such as the equivalent partially and fully coherent Hermite-Gaussian beams [10], and the equivalent partially and fully coherent Hermite-cosh-Gaussian beams and cosh-Gaussian [11, 12].

Casperson et al. has presented a novel type of beam, Hermite–sinusiodal-Gaussian (HSG) beam [13-15]. Among the family of Hermite–sinusiodal-Gaussian beams, the cosh-Gaussian beams are of much interest, because their beam profiles are suitable for practical applications [13, 15]. The second order irradiance moments definition has been used to investigate the beam parameters such as M^2-factor, the power in bucket (PIB), beam width, curvature radius and kurtosis [16].

On the other hand, it was shown by Siegman [17, 18], Weber [19] and Du et al. [20, 21] that the beam-propagation factor (M^2 factor) and the mode coherence coefficients are very useful beam parameters for characterizing various laser beams and their mode structures. In this letter, the beam-propagation factor and the mode coherence coefficients of cosh-Gaussian beams were studied to propose a simple method for producing cosh-Gaussian beams experimentally.

2. Model

The field distribution $E(x, z)$ of two-dimensional cosh-Gaussian beams at the plane $z = 0$ is characterized by [15]

$$E(x,0) = \exp\left(-\frac{x^2}{\omega_0^2}\right)\cosh\left(\Omega_0 x\right), \tag{1}$$

where ω_0 is the waist width of the Gaussian amplitude distribution, Ω_0 is the normalized parameter of cosh-Gaussian beams, and cosh denotes the hyperbolic cosine function, which can be written as

$$\cosh\left(\theta\right) = \frac{e^{\theta} + e^{-\theta}}{2}. \tag{2}$$

Substituting equation (2) into equation (1) yields

$$E(x,0) = \frac{\exp\left(\frac{\omega_0^2\Omega_0^2}{4}\right)}{2}\left(e^{-a} + e^{-b}\right), \tag{3}$$

where

$$a = \frac{x - \frac{\omega_0^2 \Omega_0^2}{2}}{\omega_0^2}; \quad b = \frac{x + \frac{\omega_0^2 \Omega_0^2}{2}}{\omega_0^2}.$$

An alternative interpretation of equation (3) is that a cosh-Gaussian beam can be regarded as a superposition of two Gaussian beams with the same waist width and in phase, whose centers are located at ($\omega_0^2\Omega_0/2$,0) and (-$\omega_0^2\Omega_0/2$,0) in the xz plane. Thus, cosh-Gaussian beams can be simply realized experimentally by superposition of two decentered Gaussian beams. Furthermore, the most-general complex form of HSG mode can be obtained by superposition of two of the generalized Hermite–Gaussian beams [13]. The intensity distribution of cosh-Gaussian beams at the z=0 plane reads as

$$I(x,0) = E(x,0)E^*(x,0), \tag{4}$$

with the asterisk (*) denoting the complex conjugate.

Keeping in mind the definition of the second-moments of the variance σ_x^2 in the spatial domain, and the variance σ_k^2 in the spatial-frequency domain [15], after performing the standard integral procedures (see, e.g., references [4] and [6]) with equations (3) and (4) taken into account, we have

$$\sigma_x^2 = \frac{\omega_0^2}{4}\left[1 + \frac{\delta}{1+\exp\left(-\frac{\delta}{2}\right)}\right] \tag{5}$$

$$\sigma_k^2 = \frac{1}{4\pi^2\omega_0^2}\left[1 + \frac{\delta\exp\left(-\frac{\delta}{2}\right)}{1+\exp\left(-\frac{\delta}{2}\right)}\right], \tag{6}$$

where

$$\delta = \omega_0^2\Omega_0^2. \tag{7}$$

Therefore the M^2 factor of the cosh-Gaussian beams is obtained readily from equations (5) and (6) and is given by

$$M^2 \;=\; 4\pi\sigma_x\sigma_k\ , \tag{8a}$$

$$M^2 = \frac{\sqrt{(1-\delta)\,e^{-\delta} + (2-\delta^2)\,e^{-\frac{\delta}{2}} + \delta + 1}}{1+e^{-\frac{\delta}{2}}}. \tag{8b}$$

3. Results and discussion

Equation (8) indicates that the M^2 factor of the cosh-Gaussian beams depends only on $\delta = \omega_0^2\Omega_0^2$. Figure 1 gives the variation of the M^2 factor of a cosh-Gaussian beam versus δ, from which it turns out that the M^2 factor of the cosh-Gaussian beam decreases monotonically with δ ($\delta \geq 0$). In addition, $M^2 \geq 1$ and reaches the minimum value 1 if $\delta = 0$ (i.e., $\Omega_0 = 0$) in the limiting case of the Gaussian beam.

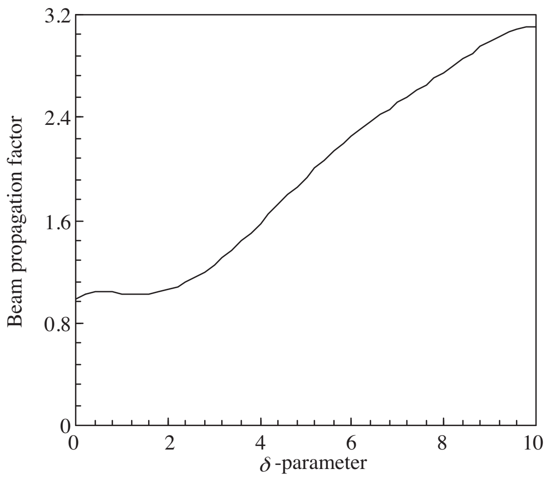

Figure 1. Beam propagation factor M^2 of a hyperbolic Gaussian beam (e.g., laser beam) as a function of δ u.

It is well known that the field distribution $E(x,\ z)$ of light at the plane of $z\ =0$ can be expanded into a series of orthogonal basis modes $\varphi_{\ m}(x)$ [22], i.e.

$$E(x,0)=\sum_{m}c_{m}\varphi_{m}(x), \tag{9}$$

where c_m denotes the mode coefficients, m is the mode indices; and $\varphi_{\ m}(x)$ is the series of orthogonal basis modes, for example, the Hermite-Gaussian modes of the form

$$\varphi_{m}(x)=U_{m}\exp\left(-\frac{\alpha^{2}x^{2}}{2}\right)H_{m}(\alpha x), \tag{10}$$

where U_m is the normalized factor, α is related to the waist width $\omega_{\ 0h}$ of the basis Gaussian mode by [23]

$$\alpha=\frac{\sqrt{2}}{\omega_{0h}}. \tag{11}$$

The unimportant phase factor is omitted in equation (10) for the sake of convenience.

Substituting equation (1) into equation (9), and using the orthogonality of the Hermite-Gaussian series, we have

$$c_{m}=\int_{-\infty}^{+\infty}\varphi_{m}^{*}(x)e^{-\frac{x^{2}}{\omega_{0}^{2}}}\cosh\left(\Omega_{0}x\right)dx. \tag{12}$$

The direct combination of equations (10) and (12) leads to indivisible integral relation as

$$c_{m}=\sqrt{\frac{\sqrt{\pi}v}{\alpha(2+v)}}\exp\left(\frac{\delta}{2(1+v)}\right)\left(\frac{2-v}{4+2v}\right)^{\frac{m}{2}}\frac{1}{\sqrt{m!}}H_{m}\left(\frac{v\delta}{\sqrt{4-v^{2}}}\right),\quad m=\text{even}. \tag{13}$$

Otherwise, c_m=0 when m is odd, where

$$v\ =\ \omega_{0}^{2}\alpha_{0}^{2}. \tag{14}$$

Equation (13) implies that cosh-Gaussian beams contain only even Hermite-Gaussian modes. Figure 2 shows the analytical relation of the mode coefficient c_m to the mode index m within the examined values of beam propagation factor.

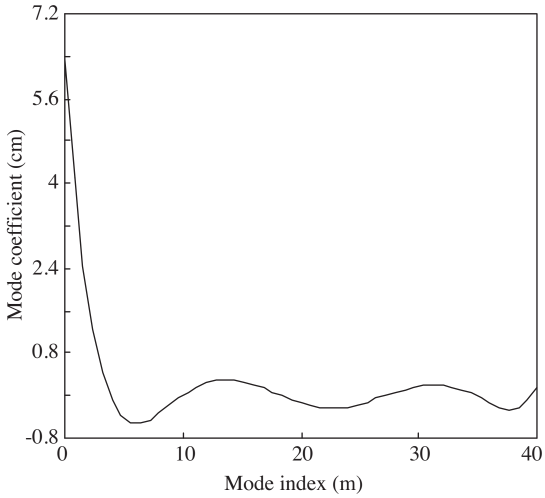

Figure 2. The analytical relation of the mode coefficient c_m to the mode index m within the examined values of beam propagation factor $M^2 = 1$ to 3.

4. Conclusion

A simple method has been proposed by which hyperbolic Gaussian beams can be realized experimentally without the use of a sophisticated aperture. Both the M^2 factor and the mode coherence coefficients of hyperbolic Gaussian beams have been expressed in the closed form, which is suitable for use in applications and provides a comprehensive characterization of hyperbolic Gaussian beam qualities such as beam invariance, beam quality, mode structure, and correlation. Finally, it should be stressed that here the hyperbolic Gaussian beams have been taken only as an illustrative example. The above approach and results have more generally applicable advantages and can be used to study three-dimensional hyperbolic Gaussian beams those can be obtained experimentally by superposition of two decentered Gaussian beams with the same width but dephased by π.

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