

1-1-2013

## Best possible tilt angle for optimal solar radiation on an inclined surface for photovoltaic and solar thermal application in Yenagoa, Nigeria

KENNETH PRIYE AINAH

JOHN TARILANYO AFA

Follow this and additional works at: <https://journals.tubitak.gov.tr/physics>



Part of the [Physics Commons](#)

---

### Recommended Citation

AINAH, KENNETH PRIYE and AFA, JOHN TARILANYO (2013) "Best possible tilt angle for optimal solar radiation on an inclined surface for photovoltaic and solar thermal application in Yenagoa, Nigeria," *Turkish Journal of Physics*: Vol. 37: No. 2, Article 12. <https://doi.org/10.3906/fiz-1206-16>  
Available at: <https://journals.tubitak.gov.tr/physics/vol37/iss2/12>

This Article is brought to you for free and open access by TÜBİTAK Academic Journals. It has been accepted for inclusion in Turkish Journal of Physics by an authorized editor of TÜBİTAK Academic Journals. For more information, please contact [academic.publications@tubitak.gov.tr](mailto:academic.publications@tubitak.gov.tr).

## Best possible tilt angle for optimal solar radiation on an inclined surface for photovoltaic and solar thermal application in Yenagoa, Nigeria

Kenneth Priye AINAH, John Tarilanyo AFA\*

Department of Electrical and Electronic Engineering, Niger Delta University, Wilberforce Island,  
P.M.B. 071 Yenagoa, Bayelsa State, Nigeria

Received: 03.07.2012 • Accepted: 20.11.2012 • Published Online: 19.06.2013 • Printed: 12.07.2013

**Abstract:** Due to the increased use of solar energy resources globally, the study of the available solar resources is essential for systems that use solar energy. This paper uses the Duffie and Beckhman method for estimating the best possible tilt angle for solar radiation using some climatic data such as declination, latitude, and clear index for photovoltaic and solar thermal systems application in the city of Yenagoa.

This research was conducted with different tilt angle ranging from  $15^\circ$  to  $65^\circ$  tilt and it was shown that the best tilt angle for solar energy system application for Yenagoa falls in the range of  $15^\circ$  to  $45^\circ$  tilt.

**Key words:** Photovoltaic cell, solar thermal system, solar radiation, Duffie and Beckhman method, tilt angle

### 1. Introduction

Yenagoa is the capital city of the Nigerian state of Bayelsa with a population of over 200,000 people. The rapid influx of people into the city has resulted in an increase in the demand for energy. With this development, solar energy seems to be a solution. Solar energy technology is growing rapidly and it is becoming significant in our world today and in the future to come. Global solar radiation data are essential for the study and design of the economic viability of systems that use solar energy [1]. It is the main source of energy that drives soil, vegetation, and atmospheric processes. It also determines the electricity and hot water produced by photovoltaic (PV) and solar thermal systems, respectively. Solar energy reaching the earth in tropical zones is about  $1 \text{ kWh/m}^2$ , giving approximately 5 to  $10 \text{ kWh/m}^2$  per day. In countries within 3200 km of the equator, use of such energy can be economically significant [2]. According to [3], Yenagoa has annual daily global horizontal radiation of  $4.61 \text{ kWh/m}^2$ , as averaged over 22 years. Monthly data reveal that radiation peaks at  $5.69 \text{ kWh/m}^2$  daily in February and is lowest in July at  $3.49 \text{ kWh/m}^2$  daily. This corresponds with the region's dry and rainy seasons. PV system application is one of the most important areas in the field of renewable energy and has attracted much research [3–6]. It is anticipated that use of PV systems will experience an enormous increase in decades to come. However, a successful integration of solar energy technologies into the existing energy structure depends on a detailed knowledge of the solar resources [7]. This has made researchers look at different ways and methods of estimating solar radiation in the extraterrestrial region and on the surface of the earth using some climatic data. In [8], a self-calibrating method for estimating solar radiation from an air temperature-based equation was used for predicting daily and monthly radiation, while in [9], a regression coefficient of Angstrom-type

\*Correspondence: priyeainah@yahoo.com

correlation was calculated for selected sites and used for estimating solar radiation on the sites. The tilt angle of the solar energy applications determines the amount of solar radiation that is generated by the systems. This is illustrated in Figure 1.

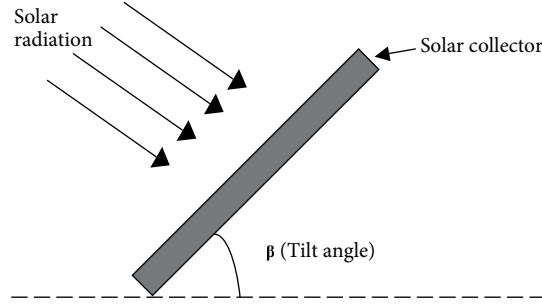


Figure 1. Angle of inclination for photovoltaic cell.

## 2. Methodology

The estimation of solar irradiation varies with climatic data and methods. However, in this paper, the Duffie and Beckman method in [10] was used to calculate the solar radiation on a tilted surface using the following parameters: global solar radiation  $G_o$ , declination  $\delta$ , clear index  $K_t$ , zenith angle  $\theta_z$ , sunset hour angle  $W_s$ , beam radiation  $R_b$ , incident angle  $\theta$ , diffuse radiation  $H_d$ , and ground reflectivity  $\rho$ .

### 2.1. Declination, $\delta$

$$\delta = 23.45 \sin 360(284 + n/365) \quad (1)$$

### 2.2. Calculation of sunset hour angle ( $W_s$ ) and daylight hours (N)

Sunset hour angle ( $W_s$ ) is the solar hour angle corresponding to the time when the sun sets.

$\cos W_s = -\tan \varphi \tan \delta$ , the day length is  $2 W_s$ , and therefore the number of daylight hours (N) is:

$$N = 2/15 \times \cos^{-1}(-\tan \varphi \tan \delta) \quad (2)$$

Then sunrise hour = 1200 (noon) - N / 2 and sunset hour = 1200 (noon) + N / 2.

The solar hour angle is now given by  $\omega = 15 (12 - t)^\circ$ , where t is solar time (24-h format) from sunrise to sunset.

### 2.3. Incident angle ( $\theta$ ) and solar zenith angle ( $\theta_z$ ) starting from sunrise hour to sunset hour

$$\begin{aligned} \cos \theta = \sin \delta \sin \varphi \cos \beta - \sin \delta \cos \varphi \sin \beta \cos \gamma + \cos \delta \cos \varphi \cos \beta \cos \omega \\ + \cos \delta \sin \varphi \sin \beta \cos \omega + \cos \delta \sin \beta \sin \gamma \sin \omega \end{aligned} \quad (3)$$

$$\cos \theta_z = \sin \delta \sin \varphi + \cos \delta \cos \varphi \cos \omega \quad (4)$$

Here,  $\varphi$  = latitude angle,  $\gamma$  = surface azimuth, and  $\beta$  = tilt angle.

### 2.4. Calculation of the beam radiation ( $R_b$ ) on the tilted plane and horizontal plane

$$R_b = \cos \theta / \cos \theta_z \quad (5)$$

**2.5. Calculation of the global solar radiation ( $G_o$ )**

$$G_o = G_{sc}[1 + 0.033 \cos(360/365n)] \cos \theta_z \tag{6}$$

$$G_o = 1367Wh/m^2$$

The total daily extraterrestrial radiation on a horizontal plane  $H_o$  is given by integrating the above equation from sunrise to sunset. Hence,  $H_o = G_o \times \text{time}$ , for time in every 1 h [4].

**2.6. Calculation of solar radiation at the earth’s surface with use of clear index  $K_t$**

$$\text{Clear index } K_t = H/H_o \tag{7}$$

Here, H = monthly average daily solar radiation.

**2.7. Calculation of diffuse and beam radiation**

When the solar hour angle is less than  $81.4^\circ$  and  $K_t$  lies between 0.3 and 0.8, the equation to calculate the diffuse radiation is given by  $H_d/H = 1.391 - 3.56 K_t + 4.189 K_t^2 - 2.137 K_t^3$ , and when the solar hour angle is greater than  $81.4$  and  $K_t$  lies between 0.3 and 0.8, we use the equation  $H_d/H = 1.311 - 3.022 K_t + 3.427 K_t^2 - 1.821 K_t^3$ .

Therefore, the average beam radiation,  $H_b$ , is as follows.

$$H_b = H - H_d \tag{8}$$

The solar radiation on an inclined surface is given as follows.

$$H_T = H_b R_b + H_d(1 + \cos \beta/2) + H\rho(1 - \cos \beta/2) \tag{9}$$

Here,  $\rho$  is given as ground reflectivity, which varies from 0.3 to 0.8.

**Table 1.** Solar radiation of inclined surface (tilt angle at  $\beta = 15^\circ$ ).

Assumed monthly average						
Date	Beam radiation ratio, $R_b$	Global solar radiation, $G_o$	Daily solar radiation, H	Diffuse radiation, $H_d$	Beam radiation, $H_b$	Tilt angle radiation, $H_T$ (Wh/m <sup>2</sup> )
12 Jan	13.07	9498.64	6649.05	1663.77	4985.28	10,290.38
15 Feb	15.61	10,034.05	7023.83	1757.55	5266.28	11,768.68
10 Mar	11.28	10,349.96	7244.97	1812.88	5432.08	10,476.48
20 Apr	9.85	10,313.32	7219.32	1806.47	5412.85	9820.49
20 May	7.76	9936.26	6955.38	1740.42	5214.96	9128.47
15 Jun	7.56	9734.29	6814	1705.05	5108.96	8814.29
12 Jul	8.05	9712.16	6798.51	1701.17	5097.34	9089.39
5 Aug	9.36	10,026.28	7018.39	1756.19	5262.21	9334.52
10 Sep	10.08	10,257.81	7180.46	1796.75	5383.72	9870.33
19 Oct	11.89	10,037.88	7026.52	1758.22	5268.29	10,410.02
15 Nov	12.44	9802.68	6861.87	1717.03	5144.85	10,376.59
20 Dec	13.29	9326.49	6528.54	1633.62	4894.93	10,182.37

**3. Estimated results**

Using Eqs. (1) through (9) with different tilt angles, the following results from Tables 1–9 were achieved. The results were estimated for different days of each month and were assumed to be the monthly averages.

4. Discussion of results

For better application for photovoltaic modules and solar thermal systems in Yenagoa, the best possible tilt angle is the key to optimal performance. Tables 1–9 show the solar radiation at different tilt angles ranging from 15° to 65° in different months of the year from January to December 2011.

The result, as demonstrated in Figure 2, gives us insight into the importance of tilt angle. In Figure 2, it can be observed that the lower the tilt angle, the better the radiation. It can be seen that 15° tilt was better than 25°, while 25° was better than 30° and so on. It is also observed that tilt angles from 15° tilt to 45° tilt tend to produce solar radiation of 11,768 Wh/m<sup>2</sup> as the maximum and 6625.93 Wh/m<sup>2</sup> as the minimum, while 55° to 65° tilts produce 11,012 Wh/m<sup>2</sup> as the maximum and 4995 Wh/m<sup>2</sup> as the minimum.

**Table 2.** Solar radiation of inclined surface (tilt angle at  $\beta = 25^\circ$ ).

Assumed monthly average						
Date	Beam radiation ratio, $R_b$	Global solar radiation, $G_o$	Daily solar radiation, $H$	Diffuse radiation, $H_d$	Beam radiation, $H_b$	Tilt angle radiation, $H_T$ (Wh/m <sup>2</sup> )
12 Jan	13.96	9498.65	6649.05	1663.77	4985.28	11,071.02
15 Feb	14.81	10,034.05	7023.83	1757.55	5266.28	11,486.59
10 Mar	11.04	10,349.96	7244.97	1812.88	5432.08	10,425.11
20 Apr	8.71	10,313.32	7219.32	1806.47	5412.85	9392.89
20 May	6.63	9936.26	6955.38	1740.42	5214.95	8524.17
15 Jun	6.32	9734.29	6814	1705.05	5108.96	8150.46
12 Jul	6.77	9712.16	6798.51	1701.17	5097.34	8410.11
5 Aug	7.9	10,026.28	7018.39	1756.19	5262.21	8797.15
10 Sep	9.08	10,257.81	7180.46	1796.75	5383.72	9502.87
19 Oct	12.05	10,037.88	7026.52	1758.22	5268.29	10,533.77
15 Nov	12.93	9802.678	6861.81	1717.03	5144.85	10,642.33
20 Dec	14.34	9326.49	6528.54	1633.62	4894.92	10,651.83

**Table 3.** Solar radiation of inclined surface (tilt angle at  $\beta = 30^\circ$ ).

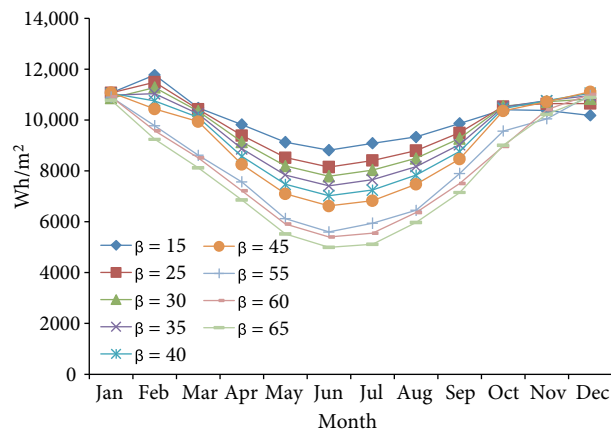
Assumed monthly average						
Date	Beam radiation ratio, $R_b$	Global solar radiation, $G_o$	Daily solar radiation, $H$	Diffuse radiation, $H_d$	Beam radiation, $H_b$	Tilt angle radiation, $H_T$ (Wh/m <sup>2</sup> )
12 Jan	14.25	9498.65	6649.05	1663.77	4985.28	10,863.79
15 Feb	14.23	10,034.65	7023.83	1757.55	5266.28	11,281.96
10 Mar	10.79	10,349.96	7244.97	1812.88	5432.08	10,350.45
20 Apr	8.03	10,313.32	7219.32	1806.47	5412.85	9140.79
20 May	5.98	9936.26	6955.38	1740.42	5214.96	8190.57
15 Jun	5.62	9734.29	6814	1705.05	5108.96	7789.78
12 Jul	6.06	9712.16	6798.51	1701.17	5097.34	8038.59
5 Aug	7.08	10,026.28	7018.39	1756.19	5262.21	8494.68
10 Sep	8.48	10,257.81	7180.46	1796.75	5383	9279.39
19 Oct	11.98	10,037.88	7026.52	1758.22	5268.29	10,543.47
15 Nov	13.02	9802.68	6861.87	1717.03	5144.85	10,720.36
20 Dec	14.69	9326.49	6528.54	1633.62	4894.93	10,828.56

**Table 4.** Solar radiation of inclined surface (tilt angle at  $\beta = 35^\circ$ ).

Assumed monthly average						
Date	Beam radiation ratio, $R_b$	Global solar radiation, $G_o$	Daily solar radiation, $H$	Diffuse radiation, $H_d$	Beam radiation, $H_b$	Tilt angle radiation, $H_T$ ( $Wh/m^2$ )
12 Jan	14.43	9498.65	6649.05	1663.77	4985.28	10,976.56
15 Feb	13.55	10,034.05	7023.83	1757.55	5266.28	11,036.93
10 Mar	10.46	10,349.96	7244.97	1812.88	5432.08	10,243.83
20 Apr	7.29	10,313.32	7219.32	1806.47	5412.85	8865.66
20 May	5.3	9936.26	6955.38	1740.42	5214.96	7839.33
15 Jun	4.88	9734.29	6814.01	1705.05	5108.96	7413.54
12 Jul	5.29	9712.16	6798.51	1701.17	5097.34	7649.53
5 Aug	6.21	10,026.28	7018.39	1756.19	5262.21	8172.71
10 Sep	7.81	10,257.81	7180.46	1796.75	5383.72	9031.62
19 Oct	11.83	10,037.88	7026.52	1758.22	5268.29	10,518.22
15 Nov	13.02	9802.68	6861.87	1717.03	5144.85	10,760.93
20 Dec	14.94	9326.49	6528.54	1633.62	4894.93	10,964.72

**Table 5.** Solar radiation of inclined surface (tilt angle at  $\beta = 40^\circ$ ).

Assumed monthly average						
Date	Beam radiation ratio, $R_b$	Global solar radiation, $G_o$	Daily solar radiation, $H$	Diffuse radiation, $H_d$	Beam radiation, $H_b$	Tilt angle radiation, $H_T$ ( $Wh/m^2$ )
12 Jan	14.5	9498.65	6649.05	1663.77	4985.28	11,048.46
15 Feb	12.77	10,034.05	7023.83.83	1757.55	5266.28	10,753.35
10 Mar	10.05	10,349.05	7244.97	1812.88	5432.08	10,106.04
20 Apr	6.5	10,313.32	7219.32	1806.47	5412.85	8569.58
20 May	4.57	9936.26	6955.38	1740.43	5214.96	7473.13
15 Jun	4.1	9734.29	6814	1705.05	5108.96	7024.6
12 Jul	4.49	9712.16	6798.51	1701.17	5097.34	7245.89
5 Aug	5.28	10,026.28	7018.39	1756.19	5262.21	7833.68
10 Sep	7.08	10,257.81	7180.46	1796.75	5383.72	8761.45
19 Oct	11.59	10,037.88	7026.52	1758.22	5268.29	10,458.2
15 Nov	12.92	9802.68	6861.87	1717.03	5144.85	10,763.74
20 Dec	15.07	9326.49	6528.54	1633.62	4894.93	11,059.29



**Figure 2.** Results at different tilt angles ( $\beta$ ).

**Table 6.** Solar radiation of inclined surface (tilt angle at  $\beta = 45^\circ$ ).

Assumed monthly average						
Date	Beam radiation ratio, $R_b$	Global solar radiation, $G_o$	Daily solar radiation, $H$	Diffuse radiation, $H_d$	Beam radiation, $H_b$	Tilt angle radiation, $H_T$ (Wh/m <sup>2</sup> )
12 Jan	14.46	9498.64	6649.05	1663.77	4985.28	11,078.94
15 Feb	11.88	10,034.05	7023.83	1757.55	5266.23	10,433.38
10 Mar	9.57	10,349.96	7244.97	1812.88	5432.08	9938.14
20 Apr	5.66	10,313.32	7219.32	1806.46	5412.85	8254.81
20 May	3.81	9936.26	6955.38	1740.42	5214.96	7094.75
15 Jun	3.29	9734.29	6814	1705.05	5108.96	6625.93
12 Jul	3.65	9712.16	6798.51	1701.17	5097.34	6830.75
5 Aug	4.32	10,026.28	7018.39	1756.19	5662.2	7480.19
10 Sep	6.3	10,257.18	7180.46	1796.75	5383	8470.92
19 Oct	11.26	10,037.88	7026.52	1758.22	5268.29	10,363.86
15 Nov	12.72	9802.07	6861.87	1717.03	5144.85	10,728.77
20 Dec	15.08	9326.49	6528.54	1633.62	4894.93	11,111.54

**Table 7.** Solar radiation of inclined surface (tilt angle at  $\beta = 55^\circ$ ).

Assumed monthly average						
Date	Beam radiation ratio, $R_b$	Global solar radiation, $G_o$	Daily solar radiation, $H$	Diffuse radiation, $H_d$	Beam radiation, $H_b$	Tilt angle radiation, $H_T$ (Wh/m <sup>2</sup> )
12 Jan	14.05	9498.65	6649.05	1663.77	4985.28	10,901.65
15 Feb	9.86	10,034.05	7023.83	1757.55	5266.28	9780.74
10 Mar	8.39	10,349.96	7244.97	1812.88	5432.08	8612.45
20 Apr	3.86	10,313.32	7219.32	1806.47	5412.85	7569.41
20 May	2.21	9936.26	6955.38	1740.42	5214.96	6125.49
15 Jun	1.6	9734.29	6814	1705.05	5108.96	5602.19
12 Jul	1.89	9712.16	6798.51	1701.17	5097.34	5942.28
5 Aug	2.31	10,026.28	7018.39	1756.19	5262.21	6446.27
10 Sep	4.59	10,257.81	7180.46	1796.75	5383.72	7901.04
19 Oct	10.34	10,037.88	7026.52	1758.22	5268.29	9567.50
15 Nov	12.04	9802.68	6861.87	1717.03	5144.85	10,051.09
20 Dec	14.77	9326.49	6528.54	1633.62	4894.92	10,975.11

**Table 8.** Solar radiation of inclined surface (tilt angle at  $\beta = 60^\circ$ ).

Assumed monthly average						
Date	Beam radiation ratio, $R_b$	Global solar radiation, $G_o$	Daily solar radiation, $H$	Diffuse radiation, $H_d$	Beam radiation, $H_b$	Tilt angle radiation, $H_T$ (Wh/m <sup>2</sup> )
12 Jan	13.68	9498.65	6649.05	1663.77	4985.28	10,921.17
15 Feb	8.73	10,034.05	7023.83	1757.55	5266.81	9580.74
10 Mar	7.69	10,349.96	7244.97	1812.88	5432.08	8512.65
20 Apr	2.91	10,313.32	7219.32	1806.47	5412.85	7222.92
20 May	1.37	9936.26	6955.38	1740.43	5214.96	5915.69
15 Jun	0.73	9734.29	6814.01	1705.05	5108.96	5402.04
12 Jul	0.99	9712.16	6798.51	1701.17	5097.34	5548.19
5 Aug	1.26	10,026.28	7018.39	1756.19	5262.21	6360.16
10 Sep	3.69	10,257.81	7180.46	1796.75	5383	7500.21
19 Oct	9.76	10,037.88	7026.52	1758.22	5268.29	8960.44
15 Nov	11.56	9802.68	6861.87	1717.03	5144.85	10,401.27
20 Dec	14.45	9326.49	6528.54	1633.62	4894.92	11,012.03

**Table 9.** Solar radiation of inclined surface (tilt angle at  $\beta = 65^\circ$ ).

Assumed monthly average						
Date	Beam radiation ratio, $R_b$	Global solar radiation, $G_o$	Daily solar radiation, $H$	Diffuse radiation, $H_d$	Beam radiation, $H_b$	Tilt angle radiation, $H_T$ (Wh/m <sup>2</sup> )
12 Jan	13.21	9498.65	6649.05	1663.77	4985.28	10,786.84
15 Feb	7.53	10,034.05	7023.83	1757.55	5266.28	9242.04
10 Mar	6.94	10,349.96	7244.97	1812.88	5432.08	8125.22
20 Apr	1.94	10,313.32	7219.32	1806.47	5412.85	6858.49
20 May	0.53	9936.26	6955.38	1740.42	5214.96	5517.99
15 Jun	0.43	9734.29	6814	1705.05	5108.96	4995.14
12 Jul	0.08	9712.16	6798.51	1701.17	5097.34	5119.13
5 Aug	0.22	10,026.28	7018.39	1756.19	5262.21	5976.46
10 Sep	2.76	10,257.81	7180.46	1796.75	5383.72	7151.49
19 Oct	9.12	10,037.88	7026.52	1758.22	5268.29	9011.55
15 Nov	10.99	9802.68	6861.87	1717.03	5144.85	10,220.67
20 Dec	14.01	9326.49	6528.54	1633.62	4894.92	10,894.28

## 5. Conclusion

It is concluded that the best possible tilt angle is within the range of  $15^\circ$  to  $45^\circ$  tilt. When the solar cell is horizontally placed, it receives the maximum solar resources but the cooling process is necessary to have a balance, and the angles from  $15^\circ$  to  $45^\circ$  are recommended, with greater emphasis on the average. The results of the estimated solar radiation on different tilt angles provide a template for solar energy installers and consumers of the best possible tilt angle to achieve the optimal radiation from the sun. It also reveals the huge potential of solar radiation in the state of Bayelsa and the benefits of tapping such resources for investors and the government to address the energy challenges of the city. The government can also take advantage of this new trend and make steps to provide incentives for users of renewable energy so as to create a friendlier environment.



### References

- [1] L. E. Akpabio, S. O. Udo and S. E. Etuk, *Turk. J. Phys.*, **28**, (2004), 205.
- [2] J. B. Gupta, A Course in Power Systems, 10th ed. (S.K. Kataria and Sons, New Delhi, 2004).
- [3] A. Mellit, M. Benhanem and S. A. Kalogirou, *Ren. Energ.*, **32**, (2007), 285.
- [4] T. Ikegami, T. Maezono, F. Nakarushi, Y. Yamagata and K. Ebihara, *Sol. Energ. Mat. Sol. C.*, **67**, (2001), 389.
- [5] K. H. Chao, S. H. Ho and M. Huiwang, *Electric Pow. Sys. Research*, **78**, (2008), 97.
- [6] G. Petrone, G. Spagnudo and M. Vitelli, *Sol. Energ. Mat. Sol. C.*, **91**, (2007), 1652.
- [7] S. Ganguli and J. Singh, *Int. J. App. Eng. Research*, **1**, (2010), 253.
- [8] R. G. Allen, *J. Hydrologic Eng.*, **57**, (1997), 56.
- [9] S. A. Khalil and A. M. Fathy, *Acta Polytechnica*, **48**, (2008), 48.
- [10] J. A. Duffie and W. A. Beckham, *Solar Engineering of Thermal Processes*, (John Wiley and Sons, New York, 1980).