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Positivity of continuous time irregular linear descriptor systems

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Abstract: The positivity of continuous-time irregular linear descriptor systems is investigated in this study. The present method is introduced based on Weierstrass decomposition. We obtain necessary and sufficient conditions for the positivity of the linear irregular descriptor systems. The conditions are investigated by two examples with Simulink/MATLAB.

Key words: Descriptor systems, positivity, Weierstrass decomposition, classical solution

1. Introduction

In recent years a lot of work has been done on systems having the form $E\dot{x}(t) = Ax(t) + Bu(t)$, where E is a singular matrix, $x(t)$ is a state vector, and $u(t)$ is an input vector. These systems are called differential algebraic equations (DAEs) or descriptor systems. The analysis of DAEs provides substantial research in different fields such as engineering, biology, economics, and social science. DAEs could model the phenomena of these fields. In some models like chemical reactors, storage systems, and air pollution, the nonnegativity of the variables must be ensured. Therefore, positive DAEs or positive descriptor systems in practical applications are important and their study is valuable.

During the two last decades, we have witnessed the emergence of studies on positive descriptor systems. Nonetheless, the developed theories for these systems are very limited compared to descriptor systems and positive systems.

In the present paper the irregular linear descriptor systems in continuous time are considered as follows:

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \end{cases} \tag{1.1}$$

where $E, A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{m\times r}$, and $C \in \mathbb{R}^{l\times n}$ are constant coefficient matrices and $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$, and $y(t) \in \mathbb{R}^l$ are respectively the state, input, and output vectors.

One of the first studies on positive descriptor systems was that of Trzaska [11]. He considered a positive descriptor system as follows:

$$\begin{cases} E\dot{x}(t) = Ax(t) + f(t), \\ y(t) = Cx(t), \end{cases}$$

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and he examined the asymptotic stability problem of the above system in a homogeneous mode (without the presence of $f(t)$).

Dai [4] studied controllability and observability of irregular systems. Bru et al. [2] discussed controllability of positive descriptor systems. Chou [3] proposed a new approach to explore the robust controllability problems of linear descriptor systems with both structured (elemental) and unstructured (normbounded) parameter uncertainties in the system matrix and the input matrix. Zhang [14] studied rectangular descriptor systems by dynamic compensation. Virnik [13] studied stability properties of positive descriptor systems in the continuous and discrete time cases. Herrero [8] proposed some conditions on nonnegativity and stability of discrete-time descriptor systems based on rearranging the information involved in the original matrices of a descriptor system. Rami [10] addressed the positivity and stability of autonomous descriptor systems. Kaczorek [9] analyzed the positivity and asymptotic stability of discrete and continuous-time nonlinear systems. However, few studies have been carried out on irregular systems. The present study aims to attain some conditions on the positivity of continuous-time irregular systems.

The study is organized as follows: Section 2 is allocated to the definitions and theorems on the solutions of the irregular systems. In Section 3, an important theorem about positivity is proved. In Section 4, two examples verify the validity of theorem 3.1. Finally, the conclusions are presented in Section 5.

2. Preliminaries

A matrix $A \in \mathbb{R}^{n\times n}$ is called nonnegative $(A \geq 0)$ if all of its elements are nonnegative, and it is called exponentially nonnegative if $e^{At} \geq 0$. The matrix A is Metzler if its off-diagonal elements are nonnegative $(a_{ij} \geq 0, \forall (i,j); i \neq j)$ [1].

System (1.1) is called a differential algebraic equation or descriptor system or generalized state-space system. If $m = n$ and $det(\lambda E - A) \neq 0$ for some $\lambda \in \mathbb{C}$, system (1.1) is called regular; otherwise, it is called irregular.

A matrix P is called nilpotent if there exists $n \in \mathbb{N}$ such that $P^n = 0$ and the smallest n that satisfies this condition is named the index of nilpotency. A monomial matrix is a square matrix over an associative ring with identity, in each row and column of which there is exactly one nonzero element. If the nonzero entries of a monomial matrix are equal to 1, then the matrix is called a permutation matrix. Every monomial matrix is the product of a permutation matrix and a diagonal matrix.

The following lemma states the relation between an exponentially nonnegative matrix and a Metzler matrix.

Lemma 2.1 *[12] Let $A \in \mathbb{R}^{n\times n}$. Then A is a Metzler matrix if and only if $e^{At} \geq 0$ for all $t \geq 0$.*

We now states the lemma that is famous for Weierstrass decomposition.

Lemma 2.2 *[7] The state solution of system (1.1), if it exists, is unique if and only if two nonsingular matrices W and T exist such that*

$$WET = diag(I_{n_f}, N, J) \quad , \quad WAT = diag(A_1, I_{n_\infty}, L), \tag{2.1}$$

where $A_1 \in \mathbb{R}^{n_f\times n_f}$ is a constant matrix, $N \in \mathbb{R}^{n_\infty\times n_\infty}$ is a nilpotent matrix in Jordan canonical form, and

$$J = diag(J_i), \quad L = diag(L_i), \quad i = 1, 2, ..., p,$$

where

$$J_i = \begin{pmatrix} 1 & & & & & & \\ 0 & 1 & & & & & \\ & 0 & 1 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & 1 \\ & & & & & & 0 \end{pmatrix} \in \mathbb{R}^{(p_i+1)\times p_i}, \qquad L_i = \begin{pmatrix} 0 & & & & & & \\ 1 & 0 & & & & & \\ & 1 & 0 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & 0 \\ & & & & & & 1 \end{pmatrix} \in \mathbb{R}^{(p_i+1)\times p_i}.$$

Let $n_3 = \Sigma p_i$. *Then* $n_f + n_\infty + n_3 = n$ *and* $n + p = m$.

Without loss of generality, we assume $p = 1$, and then $n + 1 = m$, $n_3 = p_1$, $J = J_1$, and $L = L_1$. Let

$$x(t) = T \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}, \quad x_i(t) \in \mathbb{R}^{n_i}, \quad i = 1, 2, 3,$$

$$WB = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix}, \quad CT = \begin{pmatrix} C_1 & C_2 & C_3 \end{pmatrix},$$

where B_i and C_i are constant matrices of appropriate dimensions. Equations (2.1) show that system (1.1) is equivalent to

$$\dot{x}_1(t) = A_1 x_1(t) + B_1 u(t), \tag{2.2}$$

$$N\dot{x}_2(t) = x_2(t) + B_2 u(t), \tag{2.3}$$

$$J\dot{x}_3(t) = L x_3(t) + B_3 u(t), \tag{2.4}$$

$$y(t) = C_1 x_1(t) + C_2 x_2(t) + C_3 x_3(t). \tag{2.5}$$

Equations (2.2) and (2.3) form the regular subsystems. For any sufficiently piecewise continuously differentiable $u(t)$, they have a unique solution in the form below:

$$x_1(t) = e^{A_1 t} x_1(0) + \int_0^t e^{A_1(t-\tau)} \mathrm{B}_1 u(\tau) d\tau, \tag{2.6}$$

$$x_2(t) = -\sum_{i=1}^{k-1} N^i \delta^{(i-1)}(t) x_2(0) - \sum_{i=1}^{k-1} N^i B_2 u^{(i)}(t), \tag{2.7}$$

where k is the nilpotency index of N and $\delta(t)$ is the delta function, which has an infinite value at $t = 0$ and a zero value at any other point.

If we rewrite (2.4) as

$$\dot{x}_3(t) = K x_3(t) + B_{31} u(t), \tag{2.8}$$

$$0 = e_n x_3(t) + B_{32} u(t), \tag{2.9}$$

where

$$e_n = \begin{pmatrix} 0 & 0 & \ldots & 1 \end{pmatrix}, K = \begin{pmatrix} 0 & & & & & & \\ 1 & 0 & & & & & \\ & 1 & & & & & \\ & & . & & & & \\ & & & . & & & \\ & & & & . & & \\ & & & & & 1 & 0 \end{pmatrix}, B_3 = \begin{pmatrix} B_{31} \\ B_{32} \end{pmatrix}. \tag{2.10}$$

From (2.4), (2.8), and (2.9) we have a unique solution, $x_3(t)$,

$$x_3(t) = e^{Kt}x_3(0) + \int_0^t e^{K(t-\tau)}\mathrm{B}_{31}u(\tau)d\tau \tag{2.11}$$

if and only if the input vector $u(t)$ satisfies

$$e_n[e^{Kt}x_3(0) + \int_0^t e^{K(t-\tau)}\mathrm{B}_{31}u(\tau)d\tau] + B_{32}u(t) \equiv 0, \tag{2.12}$$

where any $u(t)$ satisfying (2.12) is called admissible.

We define the index of (E, A) as the nilpotency index of the matrix N in the Weierstrass decomposition and it is denoted by $ind(E, A)$.

Definition 2.3 *The set of consistent initial conditions is defined by*

$$H_0(u) = \{\eta \mid \begin{pmatrix} 0 & I \end{pmatrix} T^{-1}\eta = -\sum_{i=0}^{k-1} N^i B_2 u^{(i)}(0)\}, \tag{2.13}$$

where T is given in Lemma 2.2.

A linear standard system, i.e $det(E) \neq 0$, is named positive if for every $u(t) \geq 0$ and $x(0) \geq 0$, $x(t)$ and $y(t)$ are nonnegative [6]. Now we define the positivity of system (1.1) [6].

Definition 2.4 *The continuous-time system (1.1) is said to be positive if for every consistent nonnegative initial condition $x(0)$ and every input function $u(t)$ such that $u^{(i)}(t) \geq 0$ for $i = 0, 1, ..., k-1$, where $k = ind(E, A)$, the state vector $x(t)$ and the output $y(t)$ are nonnegative for all $t \geq 0$.*

Theorem 2.5 *[4] System (1.1) is solvable with a unique state solution $x(t)$ for any admissible $u(t)$ if and only if*

$$\{s \in \mathbb{C} | rank(sE - A) \neq n\}$$

is a finite set.

Solutions (2.6) and (2.7) are called distributional solutions for regular subsystems (2.2) and (2.3). By imposing the restriction on the initial value $x(0)$, another solution called the classical solution is obtained [5].

In this regard, we state the following theorem.

Theorem 2.6 *[5] Assume that the index of regular subsystems (2.2) and (2.3) is k and their input function $u(t)$ is k times piecewise continuously differentiable. For any $x(0) \in H_0(u)$ subsystems (2.2) and (2.3) have the following classical solutions:*

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} e^{A_1 t}x_1(0) + \int_0^t e^{A_1(t-\tau)}\mathrm{B}_1 u(\tau)d\tau \\ -\sum_{i=0}^{k-1} N^i B_2 u^{(i)}(t) \end{pmatrix}. \tag{2.14}$$

3. Positivity of irregular descriptor systems

In this section, we study the concept of positivity of an irregular system in the continuous-time state. With the Weierstrass decomposition and special case $p = 1$, and with the same W, T, $x_1(t)$, $x_2(t)$, $x_3(t)$, B_1, B_2, B_{31}, B_{32}, A_1, C_1, C_2, and C_3 that are used in Lemma 2.2, we characterize the positivity of system (1.1) in the following theorem.

Theorem 3.1 *Consider system (1.1). Assume that T is a positive monomial matrix. Then system (1.1) is positive if and only if the following conditions are satisfied:*

1. *A_1 is Metzler.*
2. *$B_1 \geq 0, B_2 \leq 0, B_{31} \geq 0, B_{32} \leq 0$.*
3. *$C_1, C_2, C_3 \geq 0$.*

Proof Assume that system (1.1) is positive. Then for every $x(0) \geq 0$ and every $u(t) \geq 0$, we have $x(t) \geq 0$ and $y(t) \geq 0$. Since T is positive monomial, we conclude for every $x_1(0) \geq 0$, $x_2(0) \geq 0$, $x_3(0) \geq 0$ and for every $u(t) \geq 0$, and then $x_1(t) \geq 0$, $x_2(t) \geq 0$, $x_3(t) \geq 0$ and $y(t) \geq 0$. Let $u(t) \equiv 0$. Then from (2.8), (2.11), (2.12), and (2.14) we have

$$x_1(t) = e^{A_1 t}x_1(0), \tag{3.1}$$

$$x_2(t) \equiv 0, \tag{3.2}$$

$$x_3(t) \equiv 0. \tag{3.3}$$

Since the system is positive, we have $x_1(t) \geq 0$ and thus $e^{A_1 t} \geq 0$, and from Lemma 2.1, we conclude that A_1 must be a Metzler matrix. Now let $x_1(0) = x_2(0) = x_3(0) = 0$, and $u(t) = \eta \geq 0$, where η is a constant vector. We thus have the following:

$$x_1(t) = \int_0^t e^{A_1(t-\tau)}\mathrm{B}_1 \eta d\tau \geq 0, \tag{3.4}$$

$$x_2(t) = -B_2 u(t) = -B_2\eta \geq 0, \tag{3.5}$$

$$x_3(t) = \int_0^t e^{K(t-\tau)}\mathrm{B}_{31}\eta d\tau \geq 0, \tag{3.6}$$

$$e_n x_3(t) + B_{32}\eta = 0, \tag{3.7}$$

because the integration is monotone, and, as (3.4) shows, then $B_1 \geq 0$, and as (3.5) shows, we conclude $B_2 \leq 0$, because matrix K is Metzler and since integration is monotone and, as (3.6) shows, $B_{31} \geq 0$ and we have $B_{32} \leq 0$ as (3.7) indicates. We suppose the system is positive, so

$$y(t) = C_1 x_1(t) + C_2 x_2(t) + C_3 x_3(t) \geq 0, \tag{3.8}$$

and since (3.8) holds for every nonnegative $x_1(t), x_2(t)$, and $x_3(t)$, thus C_1, C_2, and C_3 must be nonnegative.

Conversely, since A_1 is Metzler, from Lemma 2.1, $e^{A_1 t} x_1(0)$ is nonnegative, and from $B_1 \geq 0$, we have $x_1(t) \geq 0$. Considering $B_2 \leq 0$, we have $x_2(t) \geq 0$. Because K is Metzler, $e^{Kt} x_3(0)$ is nonnegative, and since $B_{31} \geq 0$, $x_3(t) \geq 0$ is concluded. Since $x_1(t), x_2(t)$ and $x_3(t)$ are nonnegative, from the third condition of the theorem, $y(t)$ is nonnegative. To complete the proof it must be emphasized that, since T is a positive monomial matrix, $x(t)$ is nonnegative and therefore system (1.1) is positive. □

4. Numerical examples

In this section, two examples are presented. They are performed in the Simulink environment of MATLAB. In the first example the conditions of the theorem are satisfied and in the next one the conditions are violated.

Example 4.1 *We consider system (1.1) with the following E, A, B, and C:*

$$E = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{9} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, A = \begin{pmatrix} 0 & -\frac{3}{2} & 4 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 2 & 2 \\ \frac{1}{2} & 1 \\ 0 & -1 \\ 0 & \frac{1}{2} \\ -1 & -3 \\ -\frac{2}{3} & -\frac{1}{3} \\ 2 & 0 \\ 0 & -1 \end{pmatrix},$$

$$C = \begin{pmatrix} 2 & 1 & 3 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

We thus have the following DAEs:

$$\begin{cases} \frac{1}{2}\dot{x}_2 = -\frac{3}{2}x_2 + 4x_3 + 2u_1 + 2u_2, \\ \frac{1}{2}\dot{x}_3 = \frac{1}{4}x_2 - x_3 + \frac{1}{2}u_1 + u_2, \\ 0 = \frac{1}{3}x_4 - u_2, \\ \frac{1}{2}\dot{x}_7 = \frac{1}{2}u_2, \\ \frac{1}{2}\dot{x}_6 = x_5 - u_1 - 3u_2, \\ \frac{1}{9}\dot{x}_4 = \frac{1}{6}x_6 - \frac{2}{3}u_1 - \frac{1}{3}u_2, \\ \dot{x}_1 = x_7 + 2u_1, \\ 0 = \frac{1}{2}x_1 - u_2. \end{cases} \tag{4.1}$$

The matrices W and T are as follows:

$$W = \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

.

Based on Lemma 2.2 and the special case $p = 1$, we have

$$A_1 = \begin{pmatrix} -2 & 1 \\ 4 & -3 \end{pmatrix}, K = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0 & -1 \\ -2 & -1 \\ -1 & -3 \end{pmatrix}, B_{31} = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}, B_{32} = \begin{pmatrix} 0 & -2 \end{pmatrix}.$$

As we see, the conditions of Theorem 3.1 are satisfied, so the states and the output of the system must be positive. By Simulink of MATLAB the diagram of the states is illustrated in Figure 1.

In this example we have chosen

$$u_1(t) = u_2(t) = \begin{cases} 0 & 0 \le t < 1, \\ 1 & 1 \le t \end{cases}$$

and $x_1(0) = 1$, $x_2(0) = 2$, $x_3(0) = 4$, $x_4(0) = \frac{3}{2}$, $x_5(0) = 0$, $x_6(0) = 5$, *and* $x_7(0) = 5$.

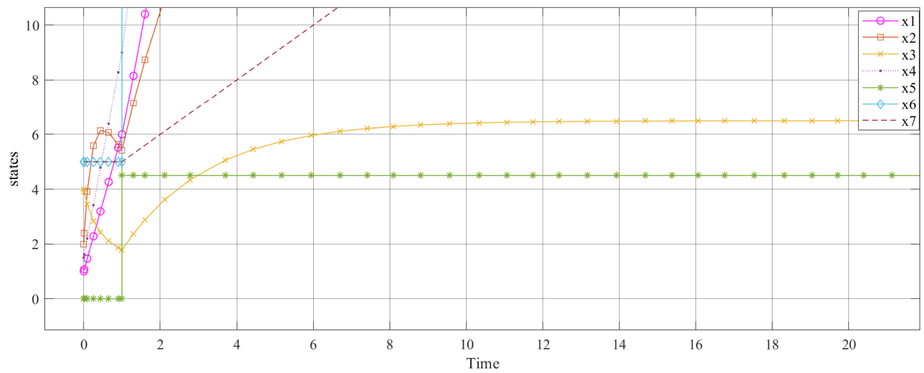

Figure 1. Positive states of the system of example 4.1.

Example 4.2 *If the conditions of the theorem 3.1 are violated, e.g., if we choose*

$$B = \begin{pmatrix} 2 & 2 \\ \frac{1}{2} & 1 \\ 0 & -1 \\ 0 & \frac{1}{2} \\ -1 & -3 \\ \frac{2}{3} & -\frac{1}{3} \\ 2 & 0 \\ 0 & -1 \end{pmatrix},$$

then with the same E, A, and C as in example 4.1 we have

$$B_2 = \begin{pmatrix} 0 & -1 \\ 2 & -1 \\ -1 & -3 \end{pmatrix},$$

so, based on the conditions of Theorem 3.1, because $B_2 \nleq 0$, this system is not positive and it means that there exist $x(0) \geq 0$ and $u(t) \geq 0$, such that $x(t)$ or $y(t)$ are not positive. By Simulink of MATLAB the diagram of the states is illustrated in Figure 2.

In this example we have chosen $u_1(t) = e^t$, $u_2(t) = t^2$ and $x_1(0) = 6$, $x_2(0) = 12$, $x_3(0) = 14$, $x_4(0) = 9$, $x_5(0) = 1$, $x_6(0) = 6$, and $x_7(0) = 8$.

5. Conclusion

The positivity of continuous-time irregular linear descriptor systems is investigated in this work. A new method is introduced based on Weierstrass decomposition. We obtain necessary and sufficient conditions for the positivity of the linear irregular descriptor systems in Theorem 3.1. In this method, we confine our results to a particular case in which matrix T is positive monomial, because we want to relate mutual positivity between $x(t)$ and $x_1(t), x_2(t)$, and $x_3(t)$. Two examples are presented for the validity of the conditions with Simulink/MATLAB. As we see in example 4.2, when the conditions are violated, the positivity is not satisfied.

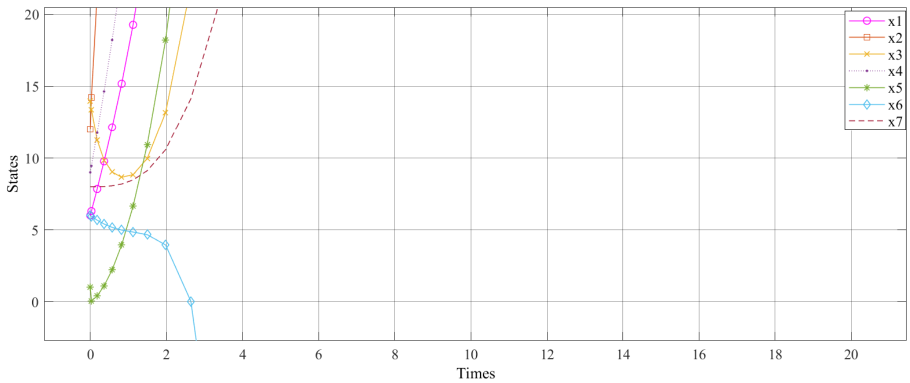

Figure 2. Nonpositive states of the system of example 4.2.

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