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## Perturbative approach to the spin-flavor precession of Majorana-type solar neutrinos

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**Abstract:** Spin-flavor precession (SFP) of Majorana-type solar neutrinos is investigated in the case of 2 generations. The evolution equation in the SFP framework is solved by using the perturbative method, in which  $\mu_B$  is accepted to be small. The approximate analytical formula including SFP effect is provided.

**Key words:** Perturbative method, spin-flavor precession, solar neutrinos, Majorana

### 1. Introduction

After neutrino oscillation was confirmed by serious solar, atmospheric, and reactor neutrino experiments [1–10] in the last decades, neutrino oscillation became one of the implications of the physics beyond the Standard Model (SM). Since neutrinos have a mass in a minimal extension of the SM, they also have magnetic moment:

$$\mu_\nu = \frac{3eG_f m_\nu}{8\pi^2 \sqrt{2}} = \frac{3eG_f m_e m_\nu}{4\pi^2 \sqrt{2}} \mu_B, \quad (1)$$

where  $G_f$  is the Fermi constant;  $m_e$  and  $m_\nu$  are the masses of electrons and neutrinos, respectively;  $e$  is the proton charge; and  $\mu_B$  is the Bohr magneton. The limits on the neutrino magnetic moment have been obtained by astrophysical arguments [11], solar neutrino experiments combined with KamLAND data [12], and reactor neutrino experiments [13,14]. The new limit recently obtained by the GEMMA experiment on the neutrino magnetic moment is  $\mu_\nu < 2.9 \times 10^{-11} \mu_B$  at 90% CL [15]. Detailed discussion on neutrino magnetic moment was given by Balantekin [16].

Neutrinos can be affected by the large magnetic fields throughout the sun due to their magnetic moments. Their spin can flip and a left-handed neutrino becomes a right-handed neutrino when they are passing through the magnetic region of the sun [17–20]. Even though information about the solar magnetic field is not well known, some bounds are given in the literature:  $\sim 10^7 G$  at the core and a maximum magnitude of  $\sim 10^5 G$  at the bottom of the convective zone [21,22]. The combined effect of the matter and the magnetic field, called spin-flavor precession (SFP), can change a left-handed electron neutrino into the right-handed type of neutrino. In the Majorana case, a right-handed neutrino is called an antineutrino. This can also be responsible for the solar electron neutrino deficit. The SFP effect has been investigated by several studies in different aspects [23–30].

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In this paper, neutrinos are assumed to be of the Majorana type. The SFP effect in the case of 2 neutrino generations is studied. The approximate analytical formula is provided by using the perturbative method, in which  $\mu B$  is taken to be small.

## 2. Formalism and analysis

The evolution equation for Majorana neutrinos passing through the matter and the magnetic field in the case of 2 generations is given by:

$$i \frac{d}{dt} \begin{pmatrix} \psi_e \\ \psi_\mu \\ \bar{\psi}_e \\ \bar{\psi}_\mu \end{pmatrix} = \mathcal{H} \begin{pmatrix} \psi_e \\ \psi_\mu \\ \bar{\psi}_e \\ \bar{\psi}_\mu \end{pmatrix}, \quad (2)$$

where  $\mathcal{H}$  is described as follows [23]:

$$\mathcal{H} = \begin{pmatrix} H & B_{mag} \\ -B_{mag} & \bar{H} \end{pmatrix}. \quad (3)$$

The  $2 \times 2$  submatrices,  $H$  and  $\bar{H}$ , are

$$H = \begin{pmatrix} \frac{1}{2}(V_c - c2_{12}\Delta_{21}) & \frac{s2_{12}\Delta_{21}}{2} \\ \frac{s2_{12}\Delta_{21}}{2} & \frac{1}{2}(-V_c + c2_{12}\Delta_{21}) \end{pmatrix}, \quad (4)$$

$$\bar{H} = \begin{pmatrix} \frac{1}{2}(-3V_c - 4V_n - c2_{12}\Delta_{21}) & \frac{s2_{12}\Delta_{21}}{2} \\ \frac{s2_{12}\Delta_{21}}{2} & \frac{1}{2}(-V_c - 4V_n + c2_{12}\Delta_{21}) \end{pmatrix}, \quad (5)$$

where  $s2_{12}$ ,  $c2_{12}$ , and  $\Delta_{12}$  are defined as

$$\begin{aligned} s2_{12} &= \sin(2\theta_{12}), \\ c2_{12} &= \cos(2\theta_{12}), \\ \Delta_{12} &= \frac{\delta m_{12}^2}{2E}. \end{aligned} \quad (6)$$

The matter potentials used in these equations are

$$V_c = \sqrt{2}G_F N_e \quad (7)$$

and

$$V_n = -\frac{G_F}{\sqrt{2}} N_n, \quad (8)$$

where  $N_e$  and  $N_n$  are electron and neutron density, respectively, and  $G_F$  is the Fermi constant. The magnetic part of Eq. (3) is

$$B_{mag} = \begin{pmatrix} 0 & \mu B \\ -\mu B & 0 \end{pmatrix}, \quad (9)$$

where  $\mu$  and  $B$  are the transition magnetic moment and the magnetic field, respectively. Unlike the Dirac neutrinos that have diagonal and off-diagonal magnetic moments, Majorana neutrinos have only off-diagonal magnetic moments.

By taking into account the definitions given above, we can split the  $\mathcal{H}$  into 2 parts:

$$\mathcal{H} = \mathcal{H}_M + \mathcal{H}_B. \quad (10)$$

The matter part,  $\mathcal{H}_M$ , and the magnetic part,  $\mathcal{H}_B$ , are given by

$$\mathcal{H}_M = \begin{pmatrix} H & 0 \\ 0 & \bar{H} \end{pmatrix}, \quad (11)$$

$$\mathcal{H}_B = \begin{pmatrix} 0 & B_{mag} \\ -B_{mag} & 0 \end{pmatrix}. \quad (12)$$

The evolution equation for the neutrinos, which is associated with the upper diagonal part of  $\mathcal{H}_M$ , is

$$i \frac{d}{dt} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = H \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}, \quad (13)$$

which is the standard MSW equation for 2 neutrino cases. Thus, the solution for  $H$  can be chosen as

$$U_H = \begin{pmatrix} \psi_1(t) & -\psi_2^*(t) \\ \psi_2(t) & \psi_1^*(t) \end{pmatrix}, \quad (14)$$

where  $\psi_1(t)$  and  $\psi_2(t)$  are solutions of Eq. (13) with the initial conditions  $\psi_1(t=0) = 1$  and  $\psi_2(t=0) = 0$  [31].

Now we can similarly solve the antineutrino part. The evolution equation for the antineutrinos, which is associated with the lower diagonal part of  $\mathcal{H}_M$ , is

$$i \frac{d}{dt} \begin{pmatrix} \bar{\psi}_e \\ \bar{\psi}_\mu \end{pmatrix} = \bar{H} \begin{pmatrix} \bar{\psi}_e \\ \bar{\psi}_\mu \end{pmatrix}. \quad (15)$$

This equation can be separated into 2 parts:

$$i \frac{d}{dt} \begin{pmatrix} \bar{\psi}_e \\ \bar{\psi}_\mu \end{pmatrix} = [\bar{H}^\alpha + \bar{H}^\beta] \begin{pmatrix} \bar{\psi}_e \\ \bar{\psi}_\mu \end{pmatrix}, \quad (16)$$

where

$$\bar{H}^\beta = \begin{pmatrix} -V_c - \frac{\Delta_{21}}{2} c_{212} & \frac{s_{212} \Delta_{21}}{2} \\ \frac{s_{212} \Delta_{21}}{2} & -V_c - \frac{\Delta_{21}}{2} c_{212} \end{pmatrix}.$$

$\bar{H}^\alpha$  is the phase part, which can be found by subtracting  $\bar{H}^\beta$  from  $\bar{H}$ . The solution including the phases can then be written for  $\bar{H}$ :

$$\bar{U}_{\bar{H}} = \begin{pmatrix} \bar{\psi}_1^\alpha(t) & -\bar{\psi}_2^{\alpha*}(t) \\ \bar{\psi}_2^\alpha(t) & \bar{\psi}_1^{\alpha*}(t) \end{pmatrix}, \quad (17)$$

where

$$\begin{aligned} \alpha &= -\frac{V_c}{2} - 2V_n, \\ \bar{\psi}^\alpha &= e^{i \int \alpha dt} \bar{\psi}. \end{aligned} \quad (18)$$

Thus, the solution matrix for the  $\mathcal{H}_M$  is given by

$$U_M = \begin{pmatrix} U_H & 0 \\ 0 & \bar{U}_H \end{pmatrix}. \quad (19)$$

The evolution operator of  $\mathcal{H}$  satisfies

$$i \frac{\partial U}{\partial t} = \mathcal{H}U, \quad (20)$$

where  $U = U_M U_B$  and  $\mathcal{H} = \mathcal{H}_M + \mathcal{H}_B$ . Now the complete solution of all of  $\mathcal{H}$  can be found by looking at  $\mathcal{H}_B$ :

$$i \frac{\partial U_B}{\partial t} = (U_M^\dagger \mathcal{H}_B U_M) U_B = h_b(t) U_B. \quad (21)$$

Because  $\mu B$  is small, we will approximate the solution to this equation to the second order in  $\mu B$ :

$$U_B = \left[ 1 - i \int_0^t h_b(t') dt' - \int_0^t dt' \int_0^{t'} (h_b(t') h_b(t'')) dt'' - \mathcal{O} \left( \int_0^t dt' \int_0^{t'} dt'' \int_0^{t''} dt''' \dots \right) \right]. \quad (22)$$

Thus, the total evolution is characterized by

$$U = U_M U_B = \begin{pmatrix} A & C \\ D & B \end{pmatrix}, \quad (23)$$

where  $A$ ,  $B$ ,  $C$ , and  $D$  are  $2 \times 2$  matrices given by

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \quad (24)$$

and

$$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}, \quad (25)$$

and so on. The state of the system evolves with the unitary operator of  $U$  from the initial state

$$\Psi(t = T) = U \Psi(t = 0), \quad (26)$$

where

$$\Psi(t = T) = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \bar{\psi}_e \\ \bar{\psi}_\mu \end{pmatrix}, \quad \Psi(t = 0) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (27)$$

Therefore, the electron neutrino amplitude,  $\psi_e$ , is found as

$$\psi_e = A_{11}. \quad (28)$$

The highly oscillating integrations that came out in  $A_{11}$  are ignored. The other integrals, such as

$$I = \int_0^\tau dt' e^{iQ} g(t) \quad Q = \int_0^{t'} dt'' A(t''), \quad (29)$$

can be calculated by using the stationary phase approximation method given by Balantekin et al. [32]. The stationary point,  $t_R$ , is where

$$A(t_R) = 0. \quad (30)$$

Since  $t_R$  appears as the SFP resonance point, which takes place before the MSW resonance point,  $\psi_2$  and  $\bar{\psi}_2$  are rather small at  $t_R$ . Therefore, we can ignore them at  $t_R$  [28]. Thus,  $A_{11}$  is found as

$$A_{11} = \psi_1(T) \left( 1 - \frac{1}{2} \frac{2\pi(\mu B)^2}{|d(\chi-2\kappa)/dt|_{(\chi-2\kappa)=0}} |\psi_1(t_R)|^2 \right), \quad (31)$$

where

$$\kappa = \frac{\Delta_{21}}{2} c_{212}, \quad \chi = \frac{G_f}{\sqrt{2}} (2N_e - 2N_n), \quad (32)$$

and  $\psi_e$  can be obtained as

$$\psi_e = \psi_1(T) \left( 1 - \frac{1}{2} \frac{2\pi(\mu B)^2}{|d(\chi-2\kappa)/dt|_{(\chi-2\kappa)=0}} |\psi_1(t_R)|^2 \right). \quad (33)$$

One can finally get the survival probability in the SFP framework by ignoring the terms that have higher order than  $(\mu B)^2$ .

$$P_{2 \times 2}(\nu_e \rightarrow \nu_e, \mu B \neq 0) = P_{2 \times 2}(\nu_e \rightarrow \nu_e, \mu B = 0) \left( 1 - \frac{2\pi(\mu B)^2}{|d(\chi-2\kappa)/dt|_{(\chi-2\kappa)=0}} |\psi_1(t_R)|^2 \right) \quad (34)$$

### 3. Conclusion

In summary, neutrinos are exposed to SFP resonances due to their magnetic moments and MSW resonance when they are passing through the sun. In this paper, the spin-flavor precession of Majorana-type solar neutrinos was investigated by using the perturbative method, in which  $\mu B$  was accepted to be small (in order for SFP to be responsible for the solar neutrino deficit,  $\mu B$  must be at least  $\sim 10^{-7} \mu_B G$ ). Two neutrino cases were considered. An analytical formula involving the SFP effect was obtained. Since  $\mu B$  was accepted to be small, the terms that have higher order than  $(\mu B)^2$  were ignored. The final result, Eq. (34), is consistent with the results of Balantekin and Volpe [28]. When  $|\psi_1(t_R)|^2$  is taken to be 1, it corresponds to the first 2 terms in the expansion of the exponential factor in Eq. (A31) in the work of Balantekin and Volpe [28]. In the 3 neutrino cases, the situation gets more complex and hard to solve analytically. To solve such complex situations, we may need the perturbative method. One can generalize this study to the 3 neutrino cases.

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