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A novel approach of order diminution using time moment concept with Routh array and salp swarm algorithm

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Abstract: In control engineering, there may be two systems that have the same input-output characteristic with different degrees of complexity. This concept leads to order diminution(OD) of a large scale system. In this article, the authors propose a new hybrid order diminution technique based on the time moment matching method with the Routh array concept and a recently developed fast and accurate salp swarm optimization (SSO) technique. The proposed method combines the advantages of both the classical method of OD and the optimization technique. The unknown coefficient of the divisor of the reduced system is obtained by exploring the time moment matching methodology with the Routh array concept, whereas the unknown coefficients of dividend polynomial are obtained by the SSO technique. The time moment matching with the Routh array ensures the nature of the system in terms of stability, and better search capability of the SSO technique reduces the error between the original and the diminished system. The proposed technique is tested on different benchmark problems, including a time-delay system. An intensive comparative study in terms of different errors, time, and frequency domain provides better performance of the proposed method compared to the existing techniques. A good match to the parameters of transient specifications indicates the success of this proposed technique. Comparatively, the matching of rise time, settling time, and maximum overshoots are 99.6076%, 99.1611%, and 100%, respectively.

Key words: Error minimization, model order reduction, order diminution, Routh array, salp swarm optimization, time moment matching method

1. Introduction

In the system engineering and control systems, the order of the plant or designed controller could be very high, which adds a burden on simulation and computation. If a higher-order system or controller can be diminished in terms of its order, it saves computation time and cost. Order diminution (OD) or model order reduction (MOR) may be classified into three headings: classical methods, using optimization algorithms, and mixed methods.

The classical methods are proposed by different researchers such as Davison [1], Marshall [2], Fossard [3], Davidson and Lucas [4], and Krishnamurthy and Seshadri [5]. These techniques are based on some simple concepts such as dominant pole preservation, dividing state matrix into two parts, retention of predominated eigenvalues, continued fraction expansions, and Routh stability criterion. The disadvantages of these methods are the higher cost of computation and sometimes the problem of stability. These classical methods are also mixed by many researchers to get a simplified system [6, 7]. Recently, an improved balanced realization technique

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[8] was proposed in which the denominator was calculated based on the balanced realization, whereas the numerator was determined by the simple mathematical procedure.

In the second method, optimization techniques are used for order reduction. "They are based on minimizing the error between the source system and the reduced system" [9]. The error functions are taken as integral of absolute error (IAE) [10], integral of time multiplied by absolute error (ITAE) [11], integral of square error (ISE) [10], integral of time multiplied by square error (ITSE) [11], L1, L2, L-infinity norm [12], weighted error function [13], etc. Meta-heuristic algorithms inspired by nature are widely used in the optimization technique method [14]. First, in this category, comes a genetic algorithm (GA) [15] that works on the survival of the fittest. Some other models are particle swarm optimization (PSO) [16] based on flocking behavior of birds, harmony search algorithm (HSA) [17] based on musical improvisation, cuckoo search algorithm (CSA) [18, 19] based on a cuckoo bird, which lays their own eggs in the nest of other host bird, and Big Bang-Big Crunch [20] theory based on the evolution of the Universe. Generally, these methods do not retain dominant poles, which is a serious drawback.

In the third category, order reduction is performed by mixing classical methods and optimization algorithms. Such a method was proposed by Nadi [21] for single-input single-output (SISO) and multi-input multi-output (MIMO) systems in which dominant poles were chosen from the original system, while the other parameters were selected by PSO. Big Ban-Big Crunch optimization was mixed with Routh approximation by Desai et al. [22] and recently, it was also mixed with time moment matching [20]. In the above two methods, the numerator was calculated by Big Bang-Big Crunch optimization, and the denominator was estimated by Routh approximation and time moment matching, respectively. A unified hybrid metaheuristic algorithm [23] for the discrete-time system was proposed by Ganguli in which a gray wolf optimization and firefly algorithm were combined for order reduction.

The order diminution technique proposed in this article uses the time moment matching method with the Routh array concept and the recently developed fast and accurate salp swarm optimization (SSO) technique [20]. Time moment matching criterion ensures the dominant characteristic of the original system in the corresponding diminished system, and it is a very reliable technique for reducing the complexity of a given problem. This concept has been used by many researchers such as Pal [24], Shamash [6], and Biradar [20]. The denominator coefficients (in the frequency domain) of the diminished system are calculated with the help of the time moment matching method using the Routh array concept, which ensures the stable nature of the original system. On the other hand, the numerator of the diminished system is obtained by utilizing the fast and better search capability of the SSO technique using integral of square error (ISE) as an objective function. The SSO ensures the minimum error between the diminished and the original system. A variety of benchmark problems are considered here to show the superiority of this technique. The first problem is a SISO system of 8^{th} order, and the second one has a time delay. A very high order ((84^{th})) transfer function is taken as the third example, which is approximated to a 2^{nd} order system using the proposed technique. The obtained results of all problems are intensively compared with other classical as well as optimization methods provided in the literature; the method proposed in this paper provides the least ISE and the best approximated reduced system.

2. Problem background

2.1. General model order diminution

Let us define a system transfer function of order n as

$$G(s) : u \rightarrow y \tag{1}$$

The objective of order diminution is to find out a new transfer function $G_r(s)$ of diminished order n_r which is defined as

$$G_r(s) : u \rightarrow y_r \text{ with } n_r < n \tag{2}$$

provided input and output characteristics are the same for the above two systems, that is for the same input $u(t)$, output $y(t) \approx y_r(t)$.

2.2. Model order diminution in terms of transfer matrix/function for MIMO and SISO systems

Let a higher-order MIMO system of order n is denoted by the following transfer matrix

$$[G(s)] = \begin{bmatrix} G_{11}(s) & \cdots & G_{1j}(s) \\ \vdots & \ddots & \vdots \\ G_{i1}(s) & \cdots & G_{ij}(s) \end{bmatrix}; \qquad i, j > 0 \tag{3}$$

where, i = no of inputs, j = no of outputs and $G_{ij}(s)$ = transfer function for i^{th} input and j^{th} output. Equation (3) can be expressed in terms of numerator and denominator polynomials as

$$[G(s)] = \begin{bmatrix} \frac{N_{11}(s)}{D_{11}(s)} & \cdots & \frac{N_{1j}(s)}{D_{1j}(s)} \\ \vdots & \ddots & \vdots \\ \frac{N_{i1}(s)}{D_{i1}(s)} & \cdots & \frac{N_{ij}(s)}{D_{ij}(s)} \end{bmatrix} \tag{4}$$

$$= \frac{1}{D(s)} \begin{bmatrix} N_{11}(s) & \cdots & N_{1j}(s) \\ \vdots & \ddots & \vdots \\ N_{i1}(s) & \cdots & N_{ij}(s) \end{bmatrix} \tag{5}$$

where $N_{ij}(s)$ and $D_{ij}(s)$ are the numerator and denominator polynomials corresponding to i^{th} input and j^{th} output; $D(s)$ represents a common denominator. The transfer function $G_{ij}(s)$ can be expressed as

$$G_{ij}(s) = \frac{a_0 s^m + a_1 s^{m-1} + a_2 s^{m-2} + \cdots a_m s^0}{b_0 s^n + b_1 s^{n-1} + b_2 s^{n-2} + \cdots b_n s^0}, \ (m \leq n) \tag{6}$$

If we put $i = 1$ and $j = 1$ in Equation (6), the same equation represents the transfer function of the SISO system with $G_{ij}(s) = G_{11}(s) \ or \ G(s)$.

Then the reduced-order transfer function corresponding to $G_{ij}(s)$ is denoted by the following equation

$$G_{ij,r}(s) = \frac{c_0 s^{m_r-1} + c_1 s^{m_r-2} + c_2 s^{m_r-3} + \ldots c_{m_r-1} s^0}{d_0 s^{n_r} + d_1 s^{n_r-1} + d_2 s^{n_r-2} + \ldots d_{n_r} s^0}, m_r \leq n_r \tag{7}$$

In the case of the SISO system, $G_{ijr}(s)$ becomes $G_r(s)$. The Eqn. (7) may also be transformed into the state space form if required.

2.3. Order diminution approach applied in optimization technique

Let for input $u(t)$, the output of the original full-order system (Equation (6)) be $y(t)$ and for the same input, the output of the reduced system (Equation (7)) be $y_r(t)$. Then, the error between the two systems is estimated by the following equation

$$e(t) = y(t) - y_r(t) \tag{8}$$

The reduced-order system is obtained by minimizing the error criterion named as integral of square error [10]

$$ISE = \int_{0}^{T_{ss}} e(t)^2 dt \tag{9}$$

where T_{ss} is the steady-state time.

Remark 1: Only Linear time-invariant systems with poles on the left-hand side of s-planes are considered in this article.

3. Theoretical background of proposed approach

The denominator of the diminished system is calculated by using the time moment matching method with the Routh array concept, whereas the numerator is obtained with the help of the fast and more accurate salp swarm optimization algorithm. Integral of square error (ISE) between the original and diminished systems is selected as the objective function to get the numerator coefficient. The details of these two methods are given below.

3.1. Time moment matching

For time moment matching [20] consider a system with following transfer function

$$G(s) = \int_0^\infty y_\delta(t) e^{-st} dt \tag{10}$$

where $y_\delta(t)$ represents the impulse response of $G(s)$. By using the power series expansion of e^{-st}, Equation (10) can be written as

$$\begin{aligned} G(s) &= \int_0^\infty y_\delta(t) \left\{1 - st + \frac{s^2t^2}{2!} - \frac{s^3t^3}{3!} + \cdots \right\} dt \\ &= \int_0^\infty y_\delta(t)dt - s\int_0^\infty t y_\delta(t)dt + s^2 \int_0^\infty \frac{t^2}{2!} y_\delta(t)dt - \cdots \end{aligned} \tag{11}$$

In general, Equation (11) can be expressed as

$$G(s) = k_0 + k_1 s + k_2 s^2 + \cdots \tag{12}$$

where coefficients k_i

$$k_i = \frac{(-1)^i}{i!} \int_0^\infty t^i y_\delta(t)dt, \quad i = 0, 1, 2, ...$$

and $\int_0^\infty t^i y_\delta(t)dt$ denote the i^{th} moment of $y_\delta(t)$.

Thus, the time moments of the diminished system are proportional to time moments of the original system and

time moment matching method aims to match as many moments as possible.
Remark 2: Time moment matching criterion ensures the dominant characteristic of the original system in the corresponding diminished system [20].

3.2. Salp swarm optimization (SSO)

Salps, transparent, barrel-shaped fishes belong to the family Salpidae. They resemble jellyfishes. The movement of salp is very similar to jellyfish, in which the water is pumped through the body as propulsion to move forward. The swarming behavior is very important for these fishes. In the deep ocean, salps form a swarm known as a salp chain. As per the researchers, this salp chain is formed for achieving better locomotion using rapid coordinated changes and hunting.
The mathematical model of the swarming behavior of these salps is proposed by Mirjalili [25]. According to this model, salps are divided into two categories: leaders and followers. The salp at the front of the salp chain is known as a leader while the rest slaps are known as followers. The position of salps can be represented in the n-dimensions search space, where n is the number of variables in the problem. Therefore, the position of salps can be stored in two dimensions of space. The swarm's target can be assumed as a food source denoted by T. To update the position of the leader, following equation is used

$$x_j^1 = \begin{cases} T_j + c_1\left(\left(Ub_j - Lb_j\right)c_2 + Lb_j\right) c_3 \geq 0 \\ T_j - c_1\left(\left(Ub_j - Lb_j\right)c_2 + Lb_j\right) c_3 < 0 \end{cases} \tag{13}$$

where x_j^1 is the position of leader salp, T_j is the position of the food source, Ub_j and Lb_j are the upper and lower bounds in the j-th dimensions respectively, c_1, c_2, and c_3 are the random numbers. The coefficient c_1, which is responsible for exploration and exploitation, is given as

$$c_1 = 2e^{-\left(\frac{4l}{L}\right)^2} \tag{14}$$

where l and L are the current and total number of iterations, respectively. The coefficients c_1 and c_2 are randomly generated numbers in the interval of [0,1]. To update the position of followers, the following equation is used

$$x_j^i = \frac{1}{2}at^2 + v_0 t; \quad i \geq 2 \tag{15}$$

where x_j^i represents the position of i-th follower in j-th dimension, t is the time, v_0 is the initial velocity, and a is calculated as follows:

$$a = \frac{v_{final}}{v_0}, \ where \ v = \frac{x - x_0}{t} \tag{16}$$

In the optimization problem, time represents iterations; the difference between two iterations is equal to 1 and if the initial velocity v_0 is taken as zero, the above equation can be modified as

$$x_j^i = \frac{1}{2}\left(x_j^i + x_j^{i-1}\right), \ i \geq 2 \tag{17}$$

4. Schema of the proposed approach

4.1. Determine coefficients of diminished denominator polynomial $(d_0, d_1, d_2, ..., d_{n_r})$

Apply time moment matching technique with Routh array concept to find out denominator of diminished system

Step 1: Divide equation (6) by b_n to get the following equation

$$G_{ij}(s) = \frac{a_0/b_n\ s^m + a_1/b_n\ s^{m-1} + a_2/b_n\ s^{m-2} + ...a_{m-2}/b_n s^2 + a_{m-1}/b_n s^1 + a_m/b_n\ s^0}{b_0/b_n\ s^n + b_1/b_n\ s^{n-1} + b_2/b_n\ s^{n-2} + ...b_{m-2}/b_n s^2 + b_{n-1}/b_n s^1 + s^0} \tag{18}$$

The above equation can be simplified as:

$$G_{ij}(s) = \frac{q_{21}\ + q_{22}\ s^1 + q_{23}\ s^2 + q_{24}\ s^3 ... + q_{2m}\ s^m}{1\ + q_{12}\ s^1 + q_{13}\ s^2 + q_{14}\ s^3 ... + q_{1n}\ s^n} \tag{19}$$

In the similar manner, the transfer function of the reduced system Eqn (7) is written as:

$$G_{ij,r}(s) = \frac{\widetilde{q}_{21}\ + \widetilde{q}_{22}\ s^1 + \widetilde{q}_{23}\ s^2 + \widetilde{q}_{24}\ s^3 ... + \widetilde{q}_{2m_r}\ s^{m_r}}{1\ + \widetilde{q}_{12}\ s^1 + \widetilde{q}_{13}\ s^2 + \widetilde{q}_{14}\ s^3 ... + \widetilde{q}_{1n_r}\ s^{n_r}} \tag{20}$$

Step 2: Now, using Eqn (19) makes the following Routh array

$$\begin{array}{|cccccc} \hline 1 & q_{12} & q_{13} & q_{14} & \cdots & q_{1n} \\ q_{21} & q_{22} & q_{23} & q_{24} & \cdots & q_{2m} \\ q_{31} & q_{32} & q_{33} & q_{34} & \cdots & q_{3m} \\ q_{41} & q_{42} & q_{43} & q_{44} & \cdots & q_{4m} \\ q_{51} & q_{52} & q_{53} & q_{54} & \cdots & q_{5m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \end{array} \tag{21}$$

in general $q_{k,l} = q_{k-1,1} \times q_{1,l+1} - q_{k-1,l+1} \times q_{1,1}\ \forall k = 3, 4, 5, ...\ l = 1, 2, 3, ...,$ and $q_{11} = 1$. The first column of Routh array is used to get the following original system

$$G_{ij}(s) = q_{21} - q_{31}s + q_{41}s^2 - q_{51}s^3 + \cdots \tag{22}$$

Step 3: By using $k_i = (-1)^{k+2} \times q_{k,1}\ \ \forall\ k = 2, 3, 4, ...\ and\ \ i = 0, 1, 2, ...,$, form the following matrix

$$\begin{bmatrix} k_0 \\ k_1 \\ k_2 \\ \vdots \\ k_{m_r} \\ \hline k_{m_r+1} \\ k_{m_r+2} \\ \vdots \\ k_{m_r+n_r} \end{bmatrix} = \left[\begin{array}{cccccc|ccc} 0 & 0 & \dots & \dots & 0 & 0 & 0 & \dots & 0 \\ -k_0 & 0 & \dots & \dots & 0 & 0 & & & \\ -k_1 & -k_0 & \dots & \dots & 0 & 0 & \vdots & \ddots & \vdots \\ \vdots & \vdots & \dots & \dots & \vdots & 0 & & & \\ -k_{m_r-1} & -k_{m_r-2} & \dots & \dots & -k_0 & 0 & 0 & \dots & 0 \\ \hline -k_{m_r} & -k_{m_r-1} & \dots & -k_1 & -k_0 & 0 & 0 & \dots & 0 \\ -k_{m_r+1} & -k_{m_r} & \dots & -k_2 & -k_1 & 0 & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \dots & \dots & \vdots & \vdots & & \vdots \\ -k_{m_r+n_r+1} & -k_{m_r+n_r} & \dots & \dots & \dots & 0 & 0 & \dots & 0 \end{array}\right] \times \begin{bmatrix} \widetilde{q}_{12} \\ \widetilde{q}_{13} \\ \widetilde{q}_{14} \\ \vdots \\ \widetilde{q}_{1,n_r+1} \\ \hline 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} \widetilde{q}_{21} \\ \widetilde{q}_{22} \\ \widetilde{q}_{23} \\ \vdots \\ \widetilde{q}_{2,m_r+1} \\ \hline 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{23}$$

In compact form Eqn (23) can be written as:

$$\begin{bmatrix} \widetilde{K}_1 \\ \hline \widetilde{K}_2 \end{bmatrix} = \left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{21} & K_{22} \end{array}\right] \times \begin{bmatrix} \widetilde{Q}_1 \\ \hline 0 \end{bmatrix} + \begin{bmatrix} \widetilde{Q}_2 \\ \hline 0 \end{bmatrix} \tag{24}$$

Step 4: Calculate the value of vector $\widetilde{Q}_1 = {K_{21}}^{-1} \times \widetilde{K}_2 = \begin{bmatrix} \widetilde{q}_{12} & \widetilde{q}_{13} & \cdots & \widetilde{q}_{1n_r} \end{bmatrix}^T$.

Step 5: The denominator of a reduced 2^{nd} order system is given by $\widetilde{q}_{12}s^2 + \widetilde{q}_{13}s + 1$; for 3^{rd} order $\widetilde{q}_{12}s^3 + \widetilde{q}_{13}s^2 + \widetilde{q}_{14}s + 1$, etc. Hence, $[\widetilde{q}_{12}, \widetilde{q}_{13}, \ldots \widetilde{q}_{1n_r}]$ are the denominator coefficients $(d_0, d_1, d_2, ..., d_{n_r})$ of Eqn (7).

4.2. Determine coefficients of diminished numerator polynomial $(c_0, c_1, c_2, ..., c_{m_r})$

Step 1: Randomly initialize the salp population.

Step 2: Put the denominator calculated by the time moment method in Eqn (7) and find out the reduced system.

Step 3: Calculate the error, which is the difference between the original and the reduced system outputs for the step input, using the Eqn (8). Evaluate the fitness of each search agent (salp) using objective function as Eqn (9).

Step 4: Select T = salp with the lowest ISE value.

Step 5: For every salp, update the position of leader salp using Eqn (13) and follower salp by using Eqn (17).

Step 6: Amend the salps based on lower and upper bounds of variables.

Step 7: If maximum number of iterations is reached, return the best solution T, $(c_0, c_1, c_2, ..., c_{m_r})$, otherwise, go to step 3.

Remark 3: ISE is considered as an objective function because it eliminates errors in transient as well as steady-state response quickly as compared to other error criteria.

5. Numerical examples, results, and discussion

This section discusses three different examples showing the superiority of the proposed MOR method. The first example is the 8^{th} order SISO system, whereas the second example has a time delay which becomes a non-minimum system after Pade approximation. The third example has the 84^{th} order. The results of all reduced systems are compared chronically with other up-to-date order reduction techniques available in the literature. The parameters taken for the salp swarm optimization technique are shown in Table 1.

Remark 4: Several trials have been done to get the numerator coefficients using SSO and the best solutions are selected. The lower and upper limits for variables are selected using these trials.

Table 1. SSO parameters.

Parameters	values
Search agents (Number of salps, N)	30
Maximum iterations count (T)	1000
Number of variables (dim)	Problem dependent
Random number	[0,1]
Lower and upper bounds	–10000 and +10000

Example 1 *Consider the 8th order problem [26] with a transfer function given by the following equation*

$$G(s) = \frac{35s^7+1086s^6+13285s^5+82402s^4+278376s^3+511812s^2+482964s+194480}{s^8+33s^7+437s^6+3017s^5+11870s^4+27470s^3+37492s^2+28880s+9600} \quad (25)$$

First of all, apply the time moment method to find out the denominator and then apply the SSO technique for estimating the numerator as discussed in the proposed schema. The obtained 4^{th} *order reduced system has the following transfer function:*

$$G_r(s) = \frac{4.178s^3 + 22.48s^2 + 34.74s + 20.26}{0.1209s^4 + 0.8606s^3 + 1.98s^2 + 2.24s + 1} \quad (26)$$

The step and Bode graphs are shown in Figures 1 and 2 for the original and reduced systems. It is clear that the proposed system coincides with the original system throughout the entire time and frequency domain. Specifications in terms of transient response and the ISE *are shown in Table 2 for comparison purposes. The value of* ISE *is the least* (4.2241×10^{-05}) *and rise time, settling time, and overshoot are matched more closely with the original system as compared to other simplified systems. We can conclude that the proposed method depicts the original system more closely and accurately.*

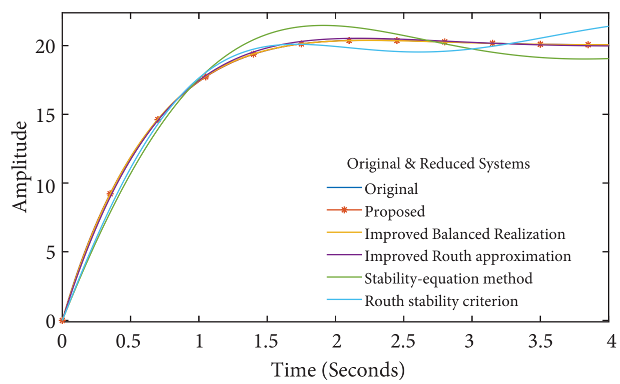


Figure 1. Time response of different systems of Example 1.

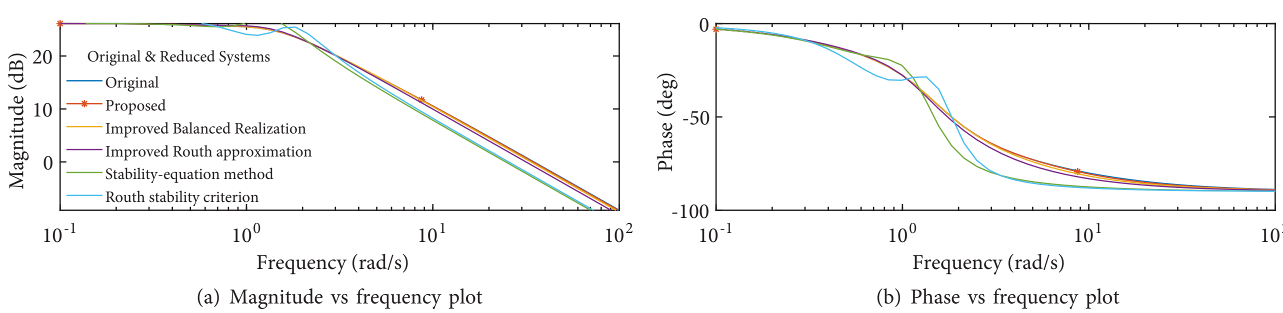


Figure 2. Frequency response of different systems of Example 1.

Example 2 *Consider the 7th order delayed system with following transfer function [20]*

$$G(s) = \frac{(4000s + 50000)e^{-0.3s}}{s^7 + 69s^6 + 1764s^5 + 20280s^4 + 102500s^3 + 221375s^2 + 187500s + 50000} \quad (27)$$

Table 2. ISE and transient response comparison for different systems of Example 1.

Systems	ISE	Rise time (Sec)	Settling time (Sec)	% Overshoot
Original 25	-	1.0725	1.5824	$0.6421 \times 10^{+00}$
Proposed (26)	4.2239×10^{-05}	1.0728	1.5831	$0.6425 \times 10^{+00}$
Prajapati(2019)[8]	0.0024×10^{-00}	1.0643	1.5777	$0.5909 \times 10^{+00}$
Prajapati(2018)[26]	0.0674×10^{-00}	1.0473	1.5058	$1.3692 \times 10^{+00}$
Sikander(2017)[29]	$1.6403 \times 10^{+08}$	0.0001	4.5413	$1.6403 \times 10^{+04}$
Singh(2016)[30]	$9.9101 \times 10^{+04}$	0.0847	9.2898	$1.4997 \times 10^{+01}$
Sikander(2015)[31]	$2.1448 \times 10^{+03}$	5.6520	1.3938	$1.4997 \times 10^{+01}$
Kranthi(2013)[32]	$9.7994 \times 10^{+11}$	0.0000	3.7559	$4.3290 \times 10^{+04}$
Vishwakarma(2013)[33]	$1.3550 \times 10^{+11}$	0.0000	5.6070	$1.2794 \times 10^{+04}$
Vishwakarma(2008)[34]	$3.2523 \times 10^{+11}$	0.0000	4.9212	$2.0647 \times 10^{+06}$
Nidhi(2006)[35]	$8.6722 \times 10^{+05}$	0.0060	8.7509	$1.3854 \times 10^{+01}$
Sinha(1990)[36]	$4.6388 \times 10^{+01}$	0.4506	2.7144	$5.9124 \times 10^{+01}$
Gutman(1982)[37]	2.6071×10^{-00}	0.9607	1.7605	$0.0000 \times 10^{+00}$
Moore(1981)[38]	1.0978×10^{-05}	1.0709	1.5780	$0.6944 \times 10^{+00}$
Shamash(1981)[39]	$1.2542 \times 10^{+02}$	2.4455	8.5958	$3.1687 \times 10^{+01}$
Chen(1980)[40]	$2.1463 \times 10^{+10}$	0.0001	10.654	$5.1208 \times 10^{+05}$
Chen(1979)[41]	3.0973×10^{-00}	0.9853	7.1210	$5.9752 \times 10^{+00}$
Krishnamurthy(1978)[5]	1.5067×10^{-00}	6.5973	3.2393	$8.3407 \times 10^{+00}$
Shamash(1975)[42]	$3.6452 \times 10^{+07}$	8.5353	1.3255	$2.5623 \times 10^{+04}$
Shamash(1974)[43]	$7.8388 \times 10^{+10}$	6.1306	1.3255	$9.8139 \times 10^{+05}$

In the above system, the time delay of 0.3 s can be approximated to the 3rd order by Pade approximation, so system given by Eqn. (27) converts to 10th order of the form $G(s) = N(s)/D(s)$, where $N(s)$ and $D(s)$ are given below:

$$N(s) = -4000s^4 + 110000s^3 - 666700s^2 - 15560000s + 222200000$$
$$D(s) = s^{10} + 109s^9 + 5191s^8 + 141300s^7 + 2396000s^6 + 25680000s^5 + 167500000s^4 + 610500000s^3 + 1111000000s^2 + 866700000s + 222200000 \quad (28)$$

The above 10^{th} order system is diminished to 2^{nd} order, and the simplified transfer function is given as:

$$G_r(s) = \frac{-0.6318s + 1.002}{2.927s^2 + 3.377s + 1} \quad (29)$$

The step and Bode graphs are shown in Figures 3 and 4, respectively. It can be noted from Bode plot that all methods are not reliable for higher frequencies, whereas they are performing well for a lower range of frequencies. The step response is matching more closely as compared to reduced models given by Biradar and Desai. A comparison, in terms of ISE and transient time specifications, is shown in Table 3, which depicts that the proposed method outperforms in terms of ISE and other parameters.

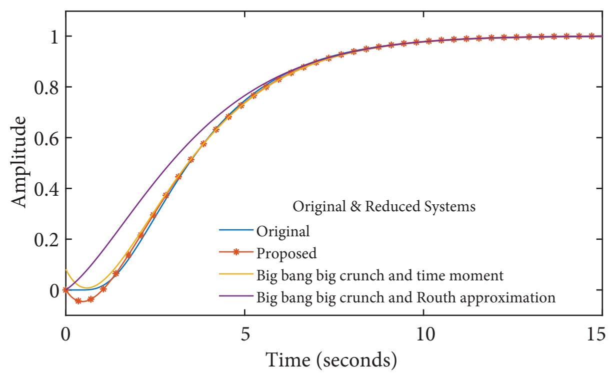
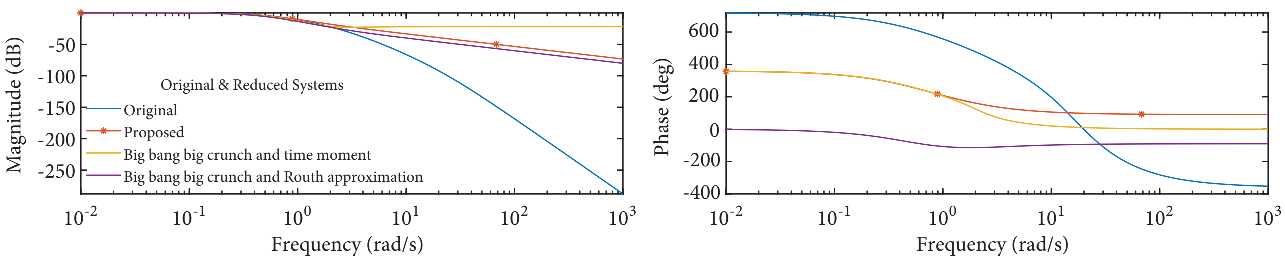


Figure 3. Time response of different systems of Example 2.



(a) Magnitude vs frequency plot

(b) Phase vs frequency plot

Figure 4. Frequency response of different systems of Example 2.

Table 3. ISE and transient response comparison for different systems of Example 2.

Systems	ISE	Rise Time(Sec)	Settling Time (Sec)	% Overshoot
Original 27	-	5.3179	10.2271	0.0000
Proposed (29)	0.0019	5.5231	10.1890	0.0000
Biradar(2016)[20]	0.0029	5.4573	10.3192	0.0000
Desai(2013)[22]	0.0593	6.2732	10.2156	0.0000

Example 3 *Consider another benchmark problem of* 84*th order* [27] *represented by following state-space model.*

$$A = \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 & 0 & 0 & a_{78} & 0 & 0 \\ 0 & A_2 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & A_3 & 0 & 0 & 0 & 0 & 0 & 0 & a_{154} \\ 0 & 0 & 0 & A_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_7 & 0 & 0 & 0 \\ a_1 & 0 & 0 & 0 & 0 & 0 & 0 & A_8 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & a_{77} & 0 & 0 & 0 & 0 & 0 & 0 & A_{42} \end{bmatrix}$$

where,

$$A_i = \begin{bmatrix} -734 & 171 \\ -9 & -734 \end{bmatrix}; \quad i = 1, 2 \ldots 42$$

and $a_j = 196; \quad j = 1, 2 \ldots 154$

$B = [B_{i,1}]_{84\times1}; \quad i = 1, 2 \ldots 84$

The values of $[B_i]$ *are given in Table 5 in the appendix,*

$$\begin{aligned} C &= [C_{1,j}]_{1\times84}; \quad j = 1, 2 \ldots 84 \\ &= B^T \\ D &= [0] \end{aligned}$$

By applying the technique proposed in this paper, the obtained reduced system has the following transfer function:

$$G_r(s) = \frac{0.01538s + 8.824}{5.490 \times 10^{-06}s^2 + 5.079 \times 10^{-03}s + 1} \tag{30}$$

The same 84*th order system can also be reduced by using particle swarm optimization (PSO) resulting in following* 2*nd order transfer function*

$$G_{rPSO}(s) = \frac{99.9s + 99.24}{0.03775s^2 + 11.36s + 11.33} \tag{31}$$

and by genetic algorithm(GA)

$$G_{rGA}(s) = \frac{22.24s + 7.154}{0.008615s^2 + 2.523s + 0.8108} \tag{32}$$

This higher-order system is also reduced to 2*nd order by Sikandar[28] with the following transfer function*

$$G_{rSikandar}(s) = \frac{41.64s + 1.08 \times 10^7}{s^2 + 2522s + 1.224 \times 10^6} \tag{33}$$

The step responses of the systems are shown in Figure 5 and Bode graphs in Figure 6. It is evident that the response in time and frequency domains is very well approximated by the proposed technique. This approach provides good approximation at lower as well as at higher frequencies as shown by the Bode plot. For this particular problem of 84*th order, PSO does not perform well* $(ISE = 3.8487 \times 10^{-2})$ *as compared to GA* $(ISE = 4.7727\times10^{-7})$*, and Sikander [28]* $(ISE = 2.1023\times10^{-3})$ *but proposed technique* $(ISE = 5.2610\times10^{-8})$ *provides the best results among all as shown in Table 4. This table shows the comparative analysis between the proposed and other optimization methods such as GA and PSO in terms of rise time, settling time, and overshoot. These values are almost equal to the original system by the proposed reduced* 2*nd order system. It concludes that the proposed technique also works for very high order systems.*

6. Conclusions

A rigorous study of the new hybrid method for order diminution has been carried out in this article. The technique used here is the conjugation of 2 strategies: time moment matching technique with the concept of the Routh array and ISE error- based salp swarm optimization (SSO) algorithm. The divisor of the reduced system is obtained by using the time moment matching method, whereas the dividend by SSO algorithm taking the integral of time multiplied by square error as the error minimization criterion. The Routh array of time moments matching technique guarantees the stability of the reduced system if the original system is stable and SSO ensures the minimum error between the diminished and the original system. A chronicle comparison of

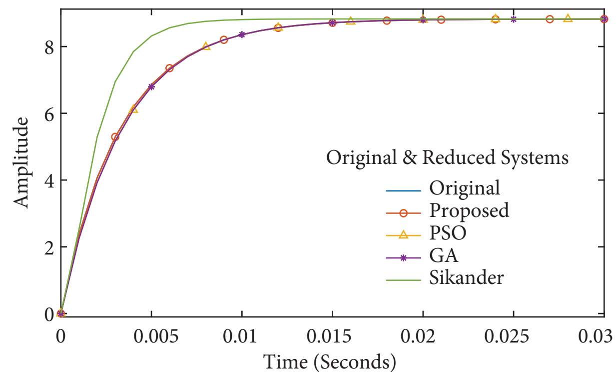
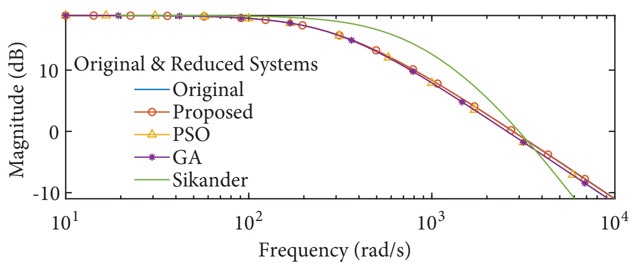
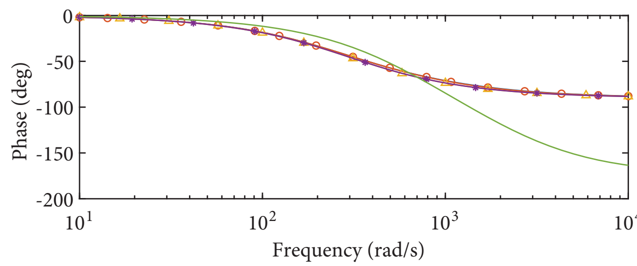


Figure 5. Time response of different systems of Example 3.



(a) Magnitude vs frequency plot



(b) Phase vs frequency plot

Figure 6. Frequency response of different systems of Example 3.

Table 4. ISE and transient response comparison for different systems of Example 3.

Systems	ISE	Rise time(Sec)	Settling time(Sec)	%Overshoot
Original[20]	-	7.4464×10^{-3}	1.3412×10^{-2}	0
Proposed (30)	5.2610×10^{-8}	7.4465×10^{-3}	1.3425×10^{-2}	0
PSO (31)	3.8487×10^{-2}	7.1125×10^{-3}	1.2007×10^{-2}	0
GA (32)	4.7727×10^{-7}	7.5106×10^{-3}	1.3374×10^{-2}	0
Sikander[28]	2.1023×10^{-3}	3.6748×10^{-3}	6.6207×10^{-3}	0

the proposed technique has been carried out with the variety of benchmark problems ranging from $8th$ to $84th$ order systems, and the comparative results prove the effectiveness of the proposed methodology. The following conclusions are drawn from this study.

- The range of problems (SISO and time-delayed) solved by the proposed method demonstrates the general nature of the technique.
- The proposed method also works very well on the $84th$ order system.
- The resulting ISE error is the lowest with the proposed technique.
- The obtained rise time, settling time, and percentage overshoot of the diminished system are very close to the original system.

Further, looking into the success of the proposed algorithm, a similar technique may be developed for a nonlinear system. Additionally, a reduced base controller can be designed to reduce the complexity of the design as well as to save simulation costs.

References

[1] Davison E. A method for simplifying linear dynamic systems. IEEE Transactions on Automatic Control 1966; 11(1):93-101.

[2] Marshall SA. An approximate method for reducing the order of a linear system. Control Engineering Practice 1966; 10:642-648.

[3] Fossard A. On a method for simplifying linear dynamic systems. IEEE Transactions on Automatic Control 1970; 15(2):261-262.

[4] Davidson AM, Lucas TN. Linear-system reduction by continued-fraction expansion about a general point. Electronics Letters 1974; 11;10(14):271-273.

[5] Krishnamurthy V, Seshadri V. Model reduction using the Routh stability criterion. IEEE Transactions on Automatic Control 1978; 23(4):729-731. doi: 10.1109/TAC.1978.1101805.

[6] Shamash Y. Model reduction using the Routh stability criterion and the Padé approximation technique. International Journal of Control 1975; 21(3):475-484.

[7] Jayanta P, Sarvesh B,Ghosh MK. A new mixed method for the reduction of high-order systems. In: NSC-94 ROC. of the National Systems Conference; India; 1994. pp. 667-670.

[8] Prajapati AK, Prasad R. Order reduction in linear dynamical systems by using improved balanced realization technique. Circuits, Systems, and Signal Processing 2019; 38:1-15. doi: 10.1007/s00034-019-01109-x

[9] Eitelberg E. Comments on model reduction by minimizing the equation error. IEEE Transactions on Automatic Control 1982; 27(4):1000-1002. doi: 10.1109/TAC.1982.1103048

[10] Bhaumik A, Kumar Y, Srivastava S, Islam SKM. Performance studies of a separately excited dc motor speed control fed by a buck converter using optimized piλdμ controller. In: IEEE 2016 ICCPCT International Conference on Circuit, Power and Computing Technologies; Nagercoil, Andaman and Nicobar Islands, India; 2016. pp. 1-6. doi: 10.1109/ICCPCT.2016.7530297

[11] White AS,Kelly C. Optimization of a control algorithm using a simulation package. Microprocessors and Microsystems 1994; 18(2):89-94. doi: 10.1016/0141-9331(94)90029-9

[12] El-Attar R. Vidyasagar M. Order reduction by l-1-and l-infinity-norm minimization. IEEE Transactions on Automatic Control 1978; 23(4):731-734. doi: 10.1109/TAC.1978.1101830

[13] Eitelberg E. Model reduction by minimizing the weighted equation error. International Journal of Control 1981; 34(6):1113-1123. doi: 10.1080/00207178108922585

[14] Philip B. Pal J. An evolutionary computation based approach for reduced order modelling of linear systems. In: IEEE 2010 International Conference on Computational Intelligence and Computing Research; Tirunelveli, India; 2010. pp. 1-8. doi: 10.1109/ICCIC.2010.5705729

[15] Golberg DE. Genetic algorithms in search, optimization, and machine learning. Boston, MA, USA: Addison-Wesley Longman Publishing Co., Inc. 1989.

[16] Kennedy J, Eberhart R. Particle swarm optimization. In: IEEE ICNN'95 International Conference on Neural Networks; Perth, WA, Australia; 1995 pp. 1942-1948. doi: 10.1109/ICNN.1995.488968

[17] Lee KS, Geem ZW. A new structural optimization method based on the harmony search algorithm. Computers & Structures 2004; 82(9-10):781-798. doi: 10.1016/j.compstruc.2004.01.002

[18] Sikander A, Prasad BR. A novel order reduction method using cuckoo search algorithm. IETE Journal of Research 2015; 61(2):83–90. doi:10.1080/03772063.2015.1009396

[19] Ahamad N, Sikander A, Singh G. Substructure preservation based approach for discrete time system approximation. Microsystem Technologies 2019; 25(2):641-649. doi:10.1007/s00542-018-3985-0

[20] Biradar S, Hote YV, Saxena S. Reduced-order modeling of linear time invariant systems using big bang big crunch optimization and time moment matching method. Applied Mathematical Modelling 2016; 40(15-16):7225-7244. doi:10.1016/j.apm.2016.03.006

[21] Abu-Al-Nadi D, Alsmadi OM, Abo-Hammour ZS. Reduced order modeling of linear mimo systems using particle swarm optimization. In: ICAS 2011 The 7th International Conference on Autonomic and Autonomous Systems: Venice/Mestre, Italy; 2011. pp. 3-7.

[22] Desai SR, Prasad R. A novel order diminution of lti systems using big bang big crunch optimization and routh approximation. Applied Mathematical Modelling 2013; 37(16-17):8016-8028. doi:10.1016/j.apm.2013.02.052

[23] Ganguli S, Kaur G, Sarkar P. A hybrid intelligent technique for model order reduction in the delta domain: a unified approach. Soft Computing 2018; 23 (13): 1-14. doi: 10.1007/s00500-018-3137-6

[24] Pal J. Improved Pade approximants using stability equation method. Electronics Letters 1983; 19(11):426-7.

[25] Mirjalili S, Gandomi AH, Mirjalili SZ, Saremi S, Faris H et al. Salp swarm algorithm: A bio-inspired optimizer for engineering design problems. Advances in Engineering Software 2017; 114:163-191. doi: 10.1016/j.advengsoft.2017.07.002

[26] Prajapati AK, Prasad R. Reduced order modelling of linear time invariant systems using the factor division method to allow retention of dominant modes. IETE Technical Review 2019; 36(5):449-462. doi: 10.1080/02564602.2018.1503567

[27] Chahlaoui Y, Van Dooren P. Benchmark examples for model reduction of linear time-invariant dynamical systems. In: Dimension Reduction of Large-Scale Systems; Berlin, Heidelberg; 2005. pp. 379-392.

[28] Sikander A. Reduced Order Modelling for Linear Systems and Controller Design. PhD, Indian Institute of Technology Roorkee, Roorkee, India, 2016.

[29] Sikander A, Prasad R. A new technique for reduced-order modelling of linear time-invariant system. IETE Journal of Research 2017; 63(3):316–324. doi: 10.1080/03772063.2016.1272436

[30] Singh J, Vishwakarma CB,Chattterjee K. Biased reduction method by combining improved modified pole clustering and improved Padé approximations. Applied Mathematical Modelling 2016; 40(2):1418–1426. doi: 10.1016/j.apm.2015.07.014

[31] Sikander A, Prasad R. Linear time-invariant system reduction using a mixed methods approach. Applied Mathematical Modelling 2015; 39(16):4848–4858. doi: 10.1016/j.apm.2015.04.014.

[32] Kumar DK, Nagar SK, Tiwari JP. A new algorithm for model order reduction of interval systems. Bonfring International Journal of Data Mining 2013; 3(1):06-11. doi: 10.9756/BIJDM.10131

[33] Vishwakarma CB, Prasad R. Order reduction using modified pole clustering and Padé approximations. World Academy of Science, Engineering and Technology 2011; 56:787-91. doi: 10.5281/zenodo.1059535

[34] Vishwakarma CB, Prasad R. Clustering method for reducing order of linear system using Padé approximation. IETE Journal of Research 2008; 54(5):326-30. doi: 10.4103/0377-2063.48531

[35] Singh N, Prasad R, Gupta HO. Reduction of linear dynamic systems using Routh Hurwitz array and factor division method. IETE Journal of Education 2006; 47(1):25-9. doi: 10.1080/09747338.2006.11415859

[36] Sinha AK, Pal J. Simulation based reduced order modelling using a clustering technique. Computers & Electrical Engineering 1990; 16(3):159-69. doi: 10.1016/0045-7906(90)90020-G

[37] Gutman P, Mannerfelt C, Molander P. Contributions to the model reduction problem. IEEE Transactions on Automatic Control 1982; 27(2):454-5. doi: 10.1109/TAC.1982.1102930

[38] Moore B. Principal component analysis in linear systems: Controllability, observability, and model reduction. IEEE Transactions on Automatic Control 1981; 26(1):17-32. doi: 10.1109/TAC.1981.1102568

[39] Smamash Y. Truncation method of reduction: a viable alternative. Electronics Letters 1981; 17(2):97-9. doi: 10.1049/el:19810070

[40] Chen TC, Chang CY, Han KW. Model reduction using the stability-equation method and the Padé approximation method. Journal of the Franklin Institute 1980; 309(6):473-90. doi: 10.1080/00207178008922845

[41] Chen TC, Chang CY, Han KW. Reduction of transfer functions by the stability-equation method. Journal of the Franklin Institute 1979; 308(4):389-404. doi: 10.1016/0016-0032(79)90066-8

[42] Shamash Y. Multivariable system reduction via modal methods and Padé approximation. IEEE Transactions on Automatic Control 1975; 20(6):815-7. doi: 10.1109/TAC.1975.1101090

[43] Shamash Y. Stable reduced-order models using Padé-type approximations. IEEE ransactions on Automatic Control 1974; 19(5):615-6. doi: 10.1109/TAC.1974.1100661

Appendix A

Table 5. Coefficients of $[B_{i,1}]$

i	$[B_{i,1}]$	i	$[B_{i,1}]$	i	$[B_{i,1}]$	i	$[B_{i,1}]$	i	$[B_{i,1}]$	i	$[B_{i,1}]$
1	7.624824	15	3.413381	29	3.468259	43	8.562598	57	2.056365	71	9.036562
2	2.354606	16	5.595597	30	3.058394	44	5.995426	58	2.93394	72	6.950758
3	7.570281	17	3.976737	31	9.01239	45	4.822466	59	2.561231	73	7.567942
4	2.321555	18	9.456895	32	1.065961	46	0.120089	60	3.126108	74	2.666435
5	4.608591	19	8.76504	33	2.963187	47	2.787891	61	5.486768	75	6.112229
6	0.353796	20	9.884494	34	8.515574	48	5.741295	62	8.308178	76	1.722831
7	5.242211	21	3.913699	35	7.294136	49	8.2792	63	2.974988	77	1.518785
8	3.992345	22	9.201152	36	6.427816	50	7.045744	64	8.77164	78	6.839423
9	8.994307	23	4.575071	37	2.681877	51	3.418584	65	7.548852	79	2.179521
10	1.71435	24	6.616429	38	7.281299	52	4.020264	66	3.25237	80	7.636844
11	0.247185	25	5.214498	39	8.92205	53	5.389039	67	2.581167	81	0.669466
12	5.921826	26	9.829012	40	5.099291	54	1.897445	68	7.341292	82	2.401817
13	5.702193	27	8.996692	41	5.530897	55	4.346914	69	4.374441	83	3.542152
14	5.824618	28	6.302044	42	8.808669	56	4.086833	70	3.124567	84	6.076599