

1-1-2021

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### Recommended Citation

ACAR, YUSUF (2021) "Novel OFDM transmission scheme using generalized prefix with subcarrierindex modulation," *Turkish Journal of Electrical Engineering and Computer Sciences*: Vol. 29: No. 3, Article 25. <https://doi.org/10.3906/elk-2006-10>

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## Novel OFDM transmission scheme using generalized prefix with subcarrier index modulation

Yusuf ACAR\* 

Group of Electronic Warfare Systems, STM Defense Technologies Engineering, Inc., Ankara, Turkey

Received: 02.06.2020

Accepted/Published Online: 31.12.2020

Final Version: 31.05.2021

### Abstract:

The cyclic prefix (CP) is a prefix technique widely used in orthogonal frequency division multiplexing (OFDM) systems in order to eliminate the intersymbol interference (ISI) caused by the dispersion of wireless channels. However, CP reduces the number of symbols that can be transmitted in one OFDM symbol. Therefore, CP is one of the bottlenecks of OFDM systems limiting their spectral efficiency (SE). This limitation on the SE of the classical CP-based OFDM system is the main motivation for this work to introduce a novel method. In this paper, the design of a new CP structure, which is based on the generalized prefix (GP), is presented. In the proposed structure, extra bits are assigned to the GP structure to increase the SE of OFDM systems. Simulation results show that the proposed method has the same performance as conventional CP-based OFDM but higher spectral efficiency. Specifically, it is shown that the proposed method has 5.5% more spectral efficiency than the classical OFDM system. A simple algorithm is also proposed to detect the additional information bits conveyed by GP. As a result, the proposed method provides both high spectral efficiency and low computational complexity.

**Key words:** Orthogonal frequency division multiplexing, cyclic prefix, generalized prefix, index modulation, spectral efficiency, cross-correlation method

### 1. Introduction

In wireless communication systems, intersymbol interference (ISI) is a significant channel impairment due to frequency-selective fading channels caused by the multipath effect [1]. Orthogonal frequency division multiplexing (OFDM) has attracted a lot of attention for broadband wireless communication systems due to its robustness against frequency-selective fading channels and its simple implementation [2, 3]. Cyclic prefix (CP) is one of the widely used transmission techniques in the OFDM method to eliminate ISI due to frequency-selective fading channels. As CP retransmits data symbols that have already been transmitted when no new information can be transmitted, it reduces the spectral efficiency (SE). Hence, CP is one of the bottlenecks of OFDM systems limiting their SE. Consequently, the loss in SE of the classical CP-based OFDM system is the main motivation for this work to introduce a novel prefix method.

Several prefix approaches such as known symbol padding OFDM, pseudo-random postfix OFDM, zero-padding OFDM, unique word OFDM, without guard intervals (NG-OFDM) have been proposed [4–8]. While the zero-padding method is used for frequency-domain channel estimation, the channel matrix of the zero-padding method is not circulant and cannot be diagonalized by the discrete Fourier transform (DFT) [9]. Pseudo-random

\*Correspondence: [yusuf.acar@stm.com.tr](mailto:yusuf.acar@stm.com.tr)

postfix scheme is used to enable semiblind channel estimation; known symbol padding and unique word are used for channel, timing, and carrier offset estimation [10, 11]. However, the aforementioned methods decrease the spectral efficiency of the classical OFDM system.

One of the ways to increase the SE is to use a CP shorter than the length of the channel impulse response [12], but shorter CP degrades the performance of an OFDM system due to ISI. Moreover, these methods either decrease the SE or have a worse bit error rate (BER) performance and also have a high computational complexity. Please note that while CP converts the linear convolution between the channel impulse response and the transmitted signal into a circular convolution, i.e. it diagonalizes the channel matrix, aforementioned prefixes are not able to convert linear convolution to circular convolution, which provides a very simple equalizer structure (single-tap equalization) for practical OFDM systems. As a result, enhancement of spectral efficiency without degradation in BER performance with low complexity (simple equalizer structure) is one of the key objectives of OFDM-based next generation wireless communication technologies.

Recently, the generalized prefix (GP) has been proposed instead of CP for especially deep fading channels [13–15]. It not only reduces the effect of the deep fading channels but also effectively reduces the spatial correlation of the MIMO channel [16]. It was demonstrated that GP has a better BER performance than the other proposed prefix techniques and it also diagonalizes the channel matrix (i.e. it was shown that CP is not the only prefix method with diagonalization property [13]). Hence, GP transforms linear convolution into a symbol-by-symbol circular convolution. GP-based OFDM structure shifts the frequency of the multipath channel and effectively transforms it into a less fading channel. GP method uses classical CP weighted by a complex coefficient  $\beta = e^{j\alpha}$ , where  $\alpha$  stands for the amount of shift in the frequency domain of the channel. By means of this shifting, the channel that has spectral nulls or deep fades experienced by the OFDM system is converted into a wireless channel with fades that are less deep or without nulls in the frequency spectrum [13–15].

Lately, index modulation (IM) concepts have received increasing attention. Recent works show that it is a promising technique for future generation wireless communication technologies. IM method allows transmitting additional bits with respect to conventional modulation schemes by mapping data bits to the indexes of different media. For example, in IM-based works, additional bits are carried by subcarriers [17], transmit antenna indexes [18], orthogonal IQ components [19], reconfigurable antennas [20], different distinguishable constellations [21], pilot positions schemes [22–24], indices of activated time slots [25], molecular communication [26], massive MIMO [27], millimeter-wave (mmWave) transmission [28], etc. For more details on this important field, the reader is referred to a survey of IM schemes in [29]. However, to the best of the author's knowledge, in the literature there does not exist any work to carry extra bits by prefix structure using the IM technique.

The undeniable potential of both GP and IM has been another motivation for this study. Inspired by IM and GP methods, a novel technique is proposed to increase the SE of OFDM systems. Hence, for the first time in the literature, the IM method allows transmitting additional bits by mapping data bits to the prefix structure. Therefore, the SE problem caused by CP can be solved by the proposed method.

In the proposed method, the additional bits are assigned to the shifted amount of frequency of GP. For example, the information bits are assigned to each  $\alpha_n$  value as given in Table 1. Assume that we want to transmit additional  $b = 2$  bits over the proposed GP scheme, (i.e. total number of  $\alpha_n$  is  $2^b = 4$  and each frequency shift will be  $\alpha_n = n\pi/4$  radians for  $n = 1, 2, 3, 4$ ). If the incoming bits for the proposed GP structure are [1, 1], then the amount of shift in the frequency domain is selected as  $\alpha_4$  and each CP symbol is multiplied by

the complex number  $\beta_4 = e^{j\alpha_4}$ . Compared to conventional CP-based OFDM, the proposed method is capable of enhancing the spectral efficiency without extra energy consumption since the amplitudes of  $\beta_n = 1 \times e^{j\alpha_n}$  are 1. As a result, the information bits are conveyed not only by the modulated symbols but also by the prefix structure.

**Table 1.** A look-up table for a total of 4 frequency shift values where  $\beta_n = e^{j\alpha_n}$ .

Bits	$\alpha_n$	$\beta_n$
[0, 0]	$\alpha_1$	$e^{j\frac{\pi}{4}}$
[0, 1]	$\alpha_2$	$e^{j\frac{2\pi}{4}}$
[1, 0]	$\alpha_3$	$e^{j\frac{3\pi}{4}}$
[1, 1]	$\alpha_4$	$e^{j\frac{4\pi}{4}}$

The rest of the paper is organized as follows: Sections 2 and 3 provide essential information on CP and GP structures, respectively. Section 4 presents the proposed method. Section 5 explains the proposed receiver structure. Spectral efficiency and computational complexity calculations are given in Sections 6 and 7, respectively. Finally, computer simulation results are provided in Section 8 while conclusions are given in Section 9.

## 2. Cyclic prefix-based OFDM system model

Assume that the  $N$ -point frequency-domain OFDM symbol  $\mathbf{z}$  is written as

$$\mathbf{z} = [z_1, z_2, \dots, z_N]^T, \tag{1}$$

where  $z_n, n = 1, 2, \dots, N$  are data symbols. The equivalent time-domain signal can be calculated by using the IDFT as follows:

$$\mathbf{s} = \mathbf{F}^* \mathbf{z} = [s_1, s_2, \dots, s_N]^T, \tag{2}$$

where  $\mathbf{F} \triangleq [\exp(-2\pi in/N)]_{i,n=1,\dots,N}$  is the DFT matrix and  $(.)^*$  is the Hermitian operator. Then, cyclic prefix of length  $N_{CP}$  is inserted as  $\tilde{\mathbf{s}} = \mathbf{M}_{\text{CP}} \mathbf{s}$ , where  $\mathbf{M}_{\text{CP}}$  has  $N + N_{CP}$  rows and  $N$  columns:

$$\mathbf{M}_{\text{CP}} = \begin{bmatrix} \mathbf{O}_{N_{CP} \times (N - N_{CP})} & \mathbf{I}_{N_{CP}} \\ & \mathbf{I}_N \end{bmatrix}, \tag{3}$$

where  $\mathbf{I}_X$  denotes the  $X \times X$  identity matrix and  $\mathbf{O}_{X \times Y}$  denotes the  $X \times Y$  zero matrix. The transmitted signal becomes

$$\tilde{\mathbf{s}} = \left[ \underbrace{s_{N-N_{CP}+1} \ s_{N-N_{CP}} \ \dots \ s_N}_{N_{CP}} \middle| \underbrace{s_1, s_2, \dots, s_N}_N \right]^T. \tag{4}$$

After signal  $\tilde{\mathbf{s}}$  is transmitted through the channel, the OFDM symbol is obtained in matrix form as

$$\mathbf{y} = \mathbf{H}\tilde{\mathbf{s}} + \mathbf{w}. \tag{5}$$

The CP-based circulant channel matrix can be given in matrix form as [13]:

$$\mathbf{H}_{CP} = \begin{bmatrix} h[1] & 0 & \cdots & 0 & h[L-1] & h[L-2] & \cdots & h[2] \\ h[2] & h[1] & 0 & \ddots & 0 & h[L-1] & \ddots & h[3] \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h[L-1] & h[L-2] & \cdots & h[2] & h[1] \end{bmatrix}_{(N+N_{CP}+L-1) \times (N_{CP}+N)} \quad (6)$$

where  $h$  is the channel impulse response with length  $L$ . Then, at the receiver, the cyclic prefix is removed and the DFT algorithm is applied as follows:

$$\tilde{\mathbf{y}} = \mathbf{FR}_{CP}\mathbf{y} = \mathbf{FR}_{CP}\mathbf{H}\mathbf{M}_{CP}\mathbf{F}^*\mathbf{z} + \mathbf{n}, \quad (7)$$

where  $\mathbf{n} = \mathbf{FR}_{CP}\mathbf{w}$  and  $\mathbf{R}_{CP} = \begin{bmatrix} \mathbf{O}_{N \times N_{CP}} & \mathbf{I}_N & \mathbf{O}_{N \times (L-1)} \end{bmatrix}$ . Equation (7) can be given in simplified form as follows:

$$\tilde{\mathbf{y}} = \bar{\mathbf{H}}\mathbf{z} + \mathbf{n} \quad (8)$$

where  $\bar{\mathbf{H}} = \mathbf{FR}_{CP}\mathbf{H}\mathbf{M}_{CP}\mathbf{F}^*$  is a diagonal matrix. Hence, the frequency response of the CP-based channel can be given as follows [13]:

$$\bar{\mathbf{H}} = \text{diag}[H(0), H(1), \dots, H(N-1)], \quad (9)$$

where

$$H(e^{jw}) = \sum_{n=0}^{N-1} h[n] e^{-jwn}. \quad (10)$$

### 3. Generalized prefix-based OFDM system model

GP method uses classical CP weighted by a complex coefficient. GP procedure can be implemented by adding the following two steps to the CP procedure:

**Step 1:** Apply  $\mathbf{B}^{-1}$  matrix to (2) from the left side as:

$$\mathbf{S}_\beta = \mathbf{B}^{-1}\mathbf{s}, \quad (11)$$

where

$$\mathbf{B} = \begin{bmatrix} 1 & & & & \\ & \beta_n & & & \\ & & \beta_n^2 & & \\ & & & \ddots & \\ & & & & \beta_n^{N-1} \end{bmatrix}, \quad (12)$$

and  $\beta_n = e^{j\alpha_n}$ , where  $\alpha_n$  denotes the amount of shift in the frequency domain of the channel.

**Step 2:** Apply generalized prefix addition matrix  $\mathbf{G}_\beta$  to (11) from the left side as

$$\mathbf{s}_\beta = \mathbf{G}_\beta\mathbf{S}_\beta, \quad (13)$$

where

$$\mathbf{G}_\beta = \begin{bmatrix} \mathbf{O}_{N_{\text{CP}} \times (N - N_{\text{CP}})} & \beta^N \mathbf{I}_{N_{\text{CP}}} \\ & \mathbf{I}_N \end{bmatrix}. \quad (14)$$

After signal  $\mathbf{s}_\beta$  is transmitted through the channel, the GP-based OFDM symbol is obtained in matrix form as

$$\mathbf{y}_\beta = \mathbf{H}\mathbf{s}_\beta + \mathbf{w}, \quad (15)$$

The equivalent circulant channel matrix for GP structure can be represented as follows [13]:

$$\mathbf{H}_{\text{GP}} = \begin{bmatrix} h[1] & 0 & \cdots & 0 & \beta_n h[L-1] & \beta_n h[L-2] & \cdots & \beta_n h[2] \\ h[2] & h[1] & 0 & \ddots & 0 & \beta_n h[L-1] & \ddots & \beta_n h[3] \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h[L-1] & h[L-2] & \cdots & h[2] & h[1] \end{bmatrix}_{(N+N_{\text{CP}}+L-1) \times (N_{\text{CP}}+N)} \quad (16)$$

Generalized prefix is removed by multiplying (15) by  $\mathbf{R}_{\text{CP}}$  matrix at the receiver. Then, after applying  $\mathbf{B}$  and FFT matrices, we have

$$\tilde{\mathbf{y}}_\beta = \mathbf{F}\mathbf{B}\mathbf{R}_{\text{CP}}\mathbf{H}\mathbf{G}_\beta\mathbf{B}^{-1}\mathbf{F}^*\mathbf{z} + \tilde{\mathbf{w}}, \quad (17)$$

where  $\tilde{\mathbf{w}} = \mathbf{F}\mathbf{B}\mathbf{R}_{\text{CP}}\mathbf{w}$  is the noise vector. Let us express (17) in a simple form as

$$\tilde{\mathbf{y}}_\beta = \bar{\mathbf{H}}_\beta\mathbf{z} + \tilde{\mathbf{w}}, \quad (18)$$

where  $\bar{\mathbf{H}}_\beta = \mathbf{F}\mathbf{B}\mathbf{R}_{\text{CP}}\mathbf{H}\mathbf{B}^{-1}\mathbf{G}_\beta\mathbf{F}^*$  is the frequency response of the time-domain wireless channel and in [13] it is shown that it has the following form:

$$\bar{\mathbf{H}}_\beta = \text{diag}[H_\beta(0), H_\beta(1), \dots, H_\beta(N-1)], \quad (19)$$

where

$$H_\beta(e^{jw}) = \sum_{n=0}^{N-1} \beta^n h[n] e^{-jwn}. \quad (20)$$

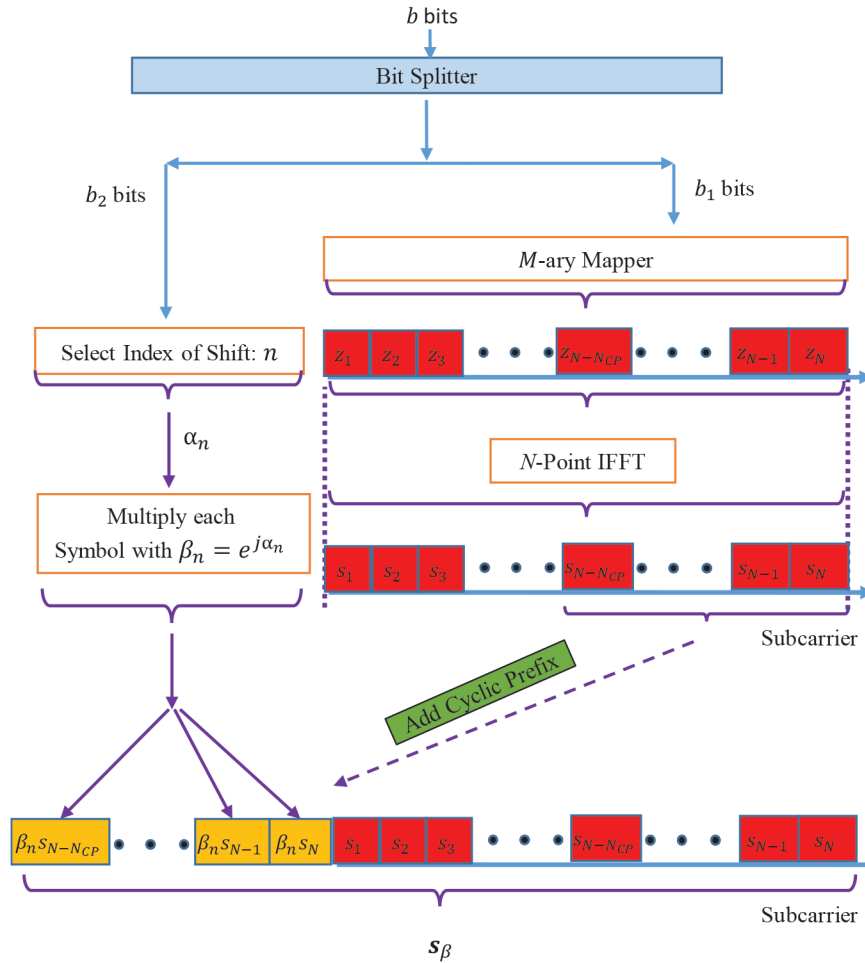
By applying the above procedure, the CP-based channel (10) has been transformed into a GP-based channel (20) as  $H_\beta(e^{jw}) = H(e^{j(w-\alpha)})$ . For more details on the mathematical model of GP-OFDM, the reader is referred to [13].

#### 4. Proposed method

Figure 1 shows a block diagram of the proposed scheme. In the proposed scheme, incoming bits are split into two parts as  $b = b_1 + b_2$  where each OFDM symbol contains  $b_1$  bits for the  $M$ -ary modulation scheme as usual and  $b_2$  (additional) bits for the amount of frequency shift (i.e.  $\alpha_n$ ). The total equivalent frequency-domain shift for the incoming  $b_2$  bits are as follows:

$$f_n = n\Delta, \quad n = 1, 2, \dots, N_\alpha \quad (21)$$

where  $N_\alpha$  is the total number of frequency shifts and  $\Delta = N/N_\alpha$  (i.e.  $\alpha_n = n\pi/N_\alpha$ ,  $n = 1, 2, \dots, N_\alpha$ ). Hence,  $b_2 = \log_2(N_\alpha)$  bits can be transmitted over the proposed GP structure. For example, assume that the incoming



**Figure 1.** Frame structure of the proposed method.

bits for transmission are  $\underbrace{[0\ 0\ 0\ 1\ 0\ \dots\ 1\ 1\ 0]}_{b_2}$  and  $N_\alpha = 8$ . The first three bits  $[0\ 0\ 0]$  show the index of the frequency shift, (i.e.  $\alpha_1$  will be used according to Table 2) and the remaining bits  $[1\ 0\ \dots\ 1\ 1\ 0]$  are used for the  $M$ -ary modulation scheme. Therefore, in the proposed scheme, in contrast to classical OFDM, the prefix is used to transfer additional data bits through the index domain. Hence, the proposed method has more spectral efficiency than the classical OFDM method.

**5. Receiver design**

The proposed structure requires an algorithm to estimate the transmitted bits. In the literature, there are several delay estimation methods that are generally based on cross-correlation method (CCM), least-squares criterion, adaptive filters, interpolation methods, higher-order statistics methods, etc. [30–35]. Firstly, CCM-based adaptive algorithm (Algorithm-1) is used as a benchmark for estimating the transmitted additional  $b_2$  bits. However, the main drawback of this method is the computational load due to heavy signal processing. Therefore, a simple algorithm is proposed as shown in Algorithm-2.

**Table 2.** A look-up table for  $N_\alpha = 8$  where  $\beta_n = e^{j\alpha_n}$ .

Bits	$\alpha_n$	$\beta_n$
[0, 0, 0]	$\alpha_1$	$e^{j\frac{\pi}{8}}$
[0, 0, 1]	$\alpha_2$	$e^{j\frac{2\pi}{8}}$
[0, 1, 0]	$\alpha_3$	$e^{j\frac{3\pi}{8}}$
[0, 1, 1]	$\alpha_4$	$e^{j\frac{4\pi}{8}}$
[1, 0, 0]	$\alpha_5$	$e^{j\frac{5\pi}{8}}$
[1, 0, 1]	$\alpha_6$	$e^{j\frac{6\pi}{8}}$
[1, 1, 0]	$\alpha_7$	$e^{j\frac{7\pi}{8}}$
[1, 1, 1]	$\alpha_8$	$e^{j\frac{8\pi}{8}}$

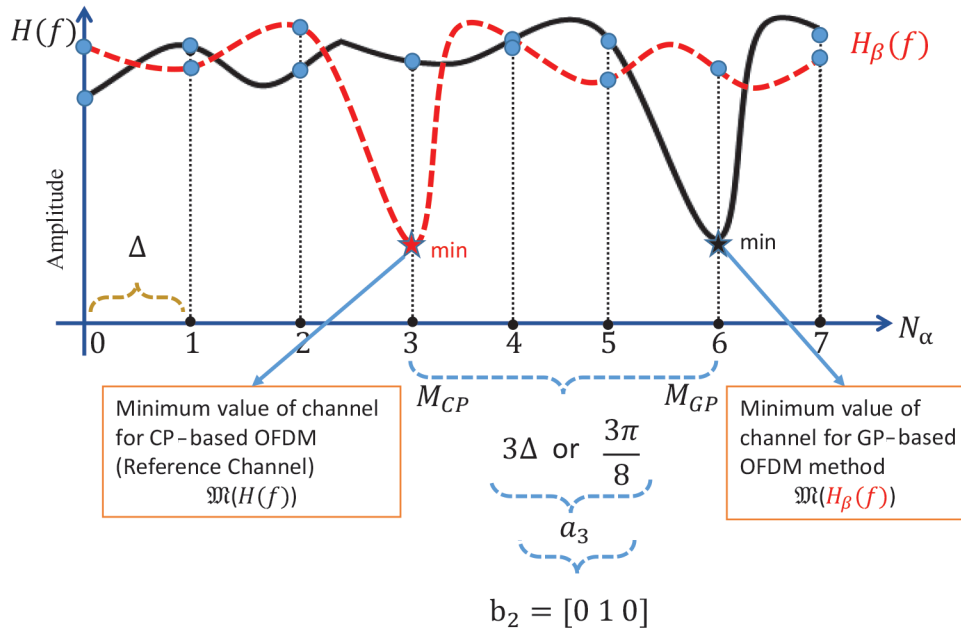
Algorithm-1: Cross-Correlation method (CCM)-based adaptive algorithm

1. Step: Calculate cross-correlation of CP and GP based channel,  $C(l) = \text{crosscorr}(H(k), H_\beta(k))$
2. Step: Find peaks of cross-correlation,  $\max(C(l))$
3. Step: Initialize search window,
4. Step: Predict location of maximum,
5. Step: Shift signal to predict location,
6. Step: Cross-correlate signals,
7. Step: Update narrow search window,
8. Step: if window width > threshold go to Step 4,
9. Step: else (i.e. window width  $\leq$  threshold), finish,
10. Step: Estimate the information bits  $\hat{b}_2$ : Convert index value of the total delay  $n\Delta$  to binary number,  $\hat{b}_2 = \log_2 n$

The proposed algorithm detects the additional information bits without cross-correlation and adaptive structure. The proposed algorithm first determines the index of the minimum (or maximum) value of the reference channel (CP-based channel) and GP-based channel (the shifted version of CP-based channel) as shown in Figure 2. This process is represented as  $\mathfrak{M}(F(x))$  in Algorithm-2 where  $\mathfrak{M}(\cdot)$  finds the minimum or maximum value of its argument. Afterward, the relative distance of these two indexes is calculated in steps 3 and 4 of the proposed algorithm. Finally, the corresponding bits  $\hat{b}_2$  that are used in frequency shift selection are estimated by using a decimal to binary conversion. It is clear that steps 3 and 4 require only simple arithmetic operations which make the complexity of the proposed algorithm very low. Therefore, the proposed algorithm reduces the complexity of the system significantly. To obtain the transmitted  $b_1$  information bits on each OFDM symbol, the receiver uses hard decision decoding for  $M$ -ary symbols. Moreover, the proposed method does not need any optimization and feedback processes as in GP-based OFDM systems [13–15].

In the simulation section, it is shown that the BER performance of the well-known Algorithm-1 is slightly





**Figure 2.** Frequency responses of CP- and GP-based channels with  $N_\alpha = 8$  subcarriers.

better than the proposed Algorithm-2, especially under channel estimation error with more complexity. There is a trade-off between complexity and BER performance of Algorithm-1 and the proposed Algorithm-2. As a result, the proposed method with Algorithm-2 is a good candidate for practical systems due to its simple implementation.

### 6. Spectral efficiency

The spectral efficiency of the classical OFDM, by taking into account the CP overhead, is given as follows:

$$\eta_{\text{CP-OFDM}} = \frac{N \log_2 M}{N + N_{\text{CP}}}, \tag{22}$$

where  $N, N_{\text{CP}}$ , and  $M$  denote the total number of subcarriers, CP size, and the degree of the modulation scheme, respectively. Spectral efficiency of the proposed scheme is given as follows:

$$\eta_{\text{GP-OFDM}} = \frac{N \log_2 M + \log_2 N_\alpha}{N + N_{\text{CP}}}. \tag{23}$$

As seen from (22) and (23), CP size  $N_{\text{CP}}$  at the denominator causes loss of spectral efficiency. It is clear that spectral efficiency of the proposed method is better than the classical method owing to the term  $\log_2 N_\alpha$  at the nominator of (23).

Table 3 shows the spectral efficiency of the proposed method for different OFDM sizes. As seen from the table, the proposed GP-based method has 10% more spectral efficiency than the classical CP-based OFDM system with  $N = N_\alpha = 64$  and BPSK modulation scheme (similar to the Wi-Fi standard).

Algorithm-2: Proposed algorithm

Use CP for the first OFDM symbol and find the index of the maximum

1. Step: or minimum value of the CP-based channel as:  $\mathfrak{M}(\mathbf{H}_{CP})$ , then set this value to nearest  $M_{CP} = n\Delta$ ,  $n = 1, 2, \dots, N_\alpha$

Use GP for the remaining OFDM symbols and find the index of the

2. Step: maximum or minimum value of GP-based channel as:  $\mathfrak{M}(\mathbf{H}_{GP})$ , then set this value to nearest  $M_{GP} = n\Delta$ ,  $n = 1, 2, \dots, N_\alpha$

3. Step: **if**  $M_{CP} \leq M_{GP}$ :  
Compute the amount of shift:  $\hat{M} = M_{GP} - M_{CP}$

4. Step: **else** (i.e.  $M_{CP} > M_{GP}$ ):  
Compute the amount of shift:  $\hat{M} = N_\alpha - M_{CP} + M_{GP}$

5. Step: Estimate the index bits  $\hat{b}_2$ : Convert calculated decimal number  $\hat{M}$  to binary number,  $\hat{b}_2 = \log_2(\hat{M}) = \log_2(n)$

**Table 3.** Spectral efficiencies for BPSK scheme and different  $N$  and  $N_\alpha$  values.

Cases	CP-OFDM	GP-OFDM	Gain of the proposed method (%)
$N = 64, N_\alpha = 256$	0.80 bpcu	0.90 bpcu	12.5%
$N = 64, N_\alpha = 128$	0.80 bpcu	0.89 bpcu	11.25%
$N = 64, N_\alpha = 64$	0.80 bpcu	0.88 bpcu	10%
$N = 128, N_\alpha = 512$	0.89 bpcu	0.95 bpcu	5.67%
$N = 128, N_\alpha = 128$	0.89 bpcu	0.94 bpcu	5.62%
$N = 256, N_\alpha = 256$	0.94 bpcu	0.97 bpcu	3.19%

## 7. Computational complexity

CP-based OFDM has low computational complexity since CP diagonalizes the channel matrix; hence, all practical OFDM systems use CP. Recently, it has been shown that CP is not the only prefix method that diagonalizes the channel matrix. In [13], it is shown that GP also diagonalizes the channel matrix and that the computational complexity of the GP-based OFDM system is more than that of the CP-based OFDM system. This is due to the fact that the optimization process in [13] entails more computational complexity than CP-based OFDM systems. In the proposed method, computational complexity is reduced by removing the optimization steps that require heavy numerical computations. Unlike CP-based OFDM, the proposed method uses  $\mathbf{B}^{-1}$  and  $\mathbf{B}$  matrices at the transmitter and receiver, respectively. These matrices require  $3N + L$  complex number multiplications [13]. The computational complexity of cross-correlation is clearly  $\mathcal{O}(N)$  [36]. Therefore, the computational complexity of Algorithm-1 for main steps 1, 2, and 6 are  $\mathcal{O}(N_\alpha \times N_\alpha)$ ,  $\mathcal{O}(N_\alpha)$ , and  $\mathcal{O}(N_\alpha \times N_\alpha)$ , respectively. Decimal to binary conversion in step 10 has a complexity of  $\mathcal{O}(\log_2(N_\alpha)) \approx \mathcal{O}(1)$ . After  $i^{th}$  iteration (i.e. steps 8 and 9), the overall computational complexity of Algorithm-1 is about  $\sim i \times (2\mathcal{O}(N_\alpha \times N_\alpha) + \mathcal{O}(N_\alpha)) + \mathcal{O}(1)$ .

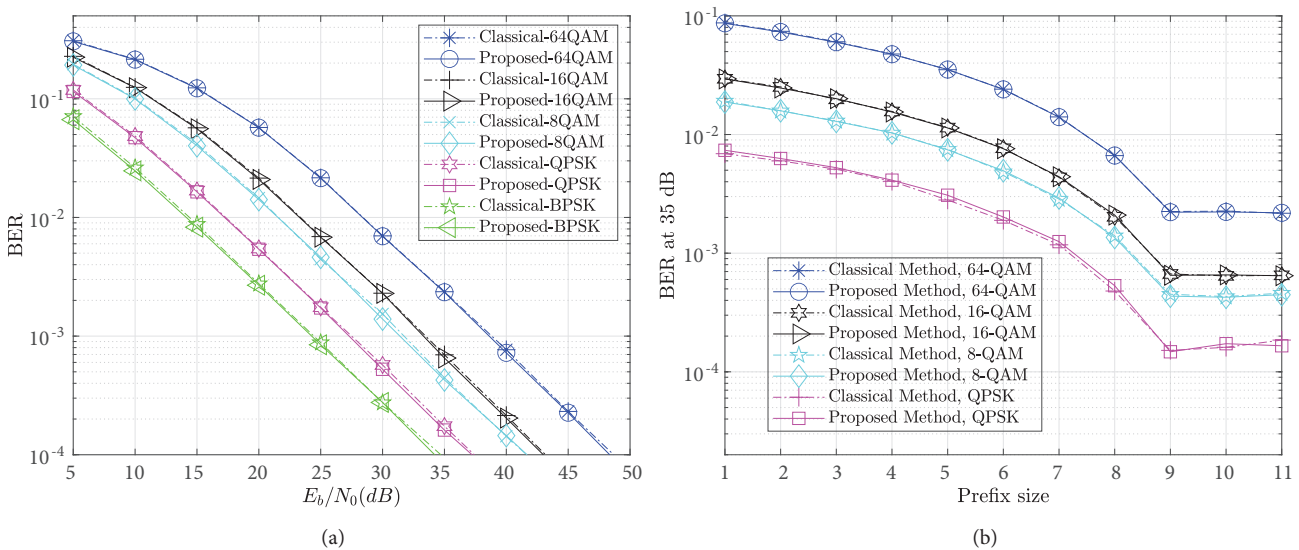
Let us evaluate the complexity of the proposed simple algorithm (Algorithm-2). The computational complexity of *Max-Min* operation in steps 1 and 2 is  $\mathcal{O}(N_\alpha)$ . Moreover, it is clear that steps 3 and 4 require

only one addition and one subtraction operation. Hence, the computational complexity is given as  $\mathcal{O}(1)$  for steps 3 and 4. Decimal to binary conversion in step 5 has a complexity of  $\mathcal{O}(\log_2(N_\alpha)) \approx \mathcal{O}(1)$ . It is obvious that the overall additional computational complexity order is  $\sim \mathcal{O}(N_\alpha)$ . As a result, the computational complexity of the proposed system is slightly higher than the CP-based OFDM system.

### 8. Simulation examples

The BER performance of the proposed method is evaluated for different modulation schemes by assuming frequency-selective Rayleigh channels with length  $L = 10$  and CP size  $N_{CP} = 16$ . SNR is described as  $E_b/N_0$ , where  $N_0$  is the noise power and  $E_b$  is energy per bit. CP is used for the first OFDM symbol to determine the reference channel, then GP is applied to the remaining OFDM symbols.

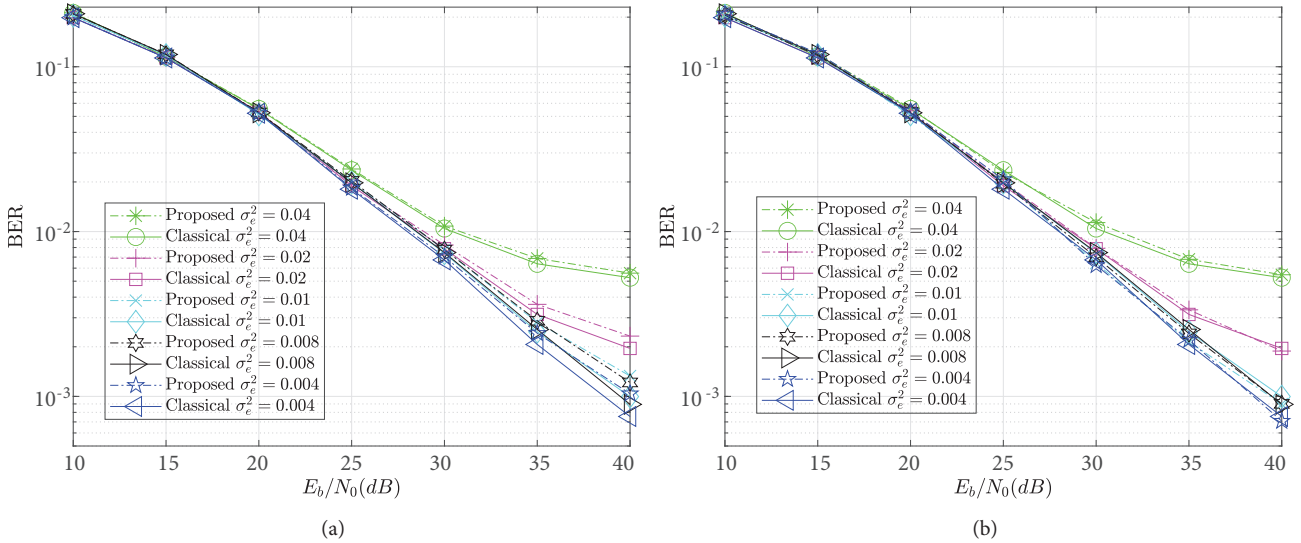
The BER curves of CP-based conventional OFDM and the proposed scheme are given in Figure 3a. As illustrated in Figure 3a, the proposed method achieves the same performance as the currently well-known OFDM with different modulation schemes such as BPSK, QPSK, 8QAM, 16QAM, and 64QAM with higher spectral efficiency for Algorithm-2. Furthermore, the same results are obtained for Algorithm-1. While having the same BER performance for the BPSK modulation scheme, the proposed method and the classical OFDM scheme have spectral efficiencies of 0.9375 bpcu and 0.8888 bpcu, respectively.



**Figure 3.** BER performances of the proposed scheme (with Algorithm-2) and conventional OFDM (a) for  $N = N_\alpha = 128$  (b) for different prefix sizes  $N_{CP}$  at SNR= 35 dB.

Figure 3b illustrates the BER curves of different prefix sizes  $N_{CP}$  for the CP-based OFDM and the proposed method with Algorithm-2. This figure compares the BER performances for  $N = N_\alpha = 128$ ,  $L = 10$ , SNR=35dB with different modulation schemes. It can be seen that as the prefix size decreases, the BER performance gets worse for both methods. It is clear that a prefix size higher than 9 is enough for a given channel profile. Moreover, the same results are obtained for Algorithm-1. Consequently, it is shown that the performance of the proposed prefix structure also has the same performance as the classical OFDM technique with any prefix size.

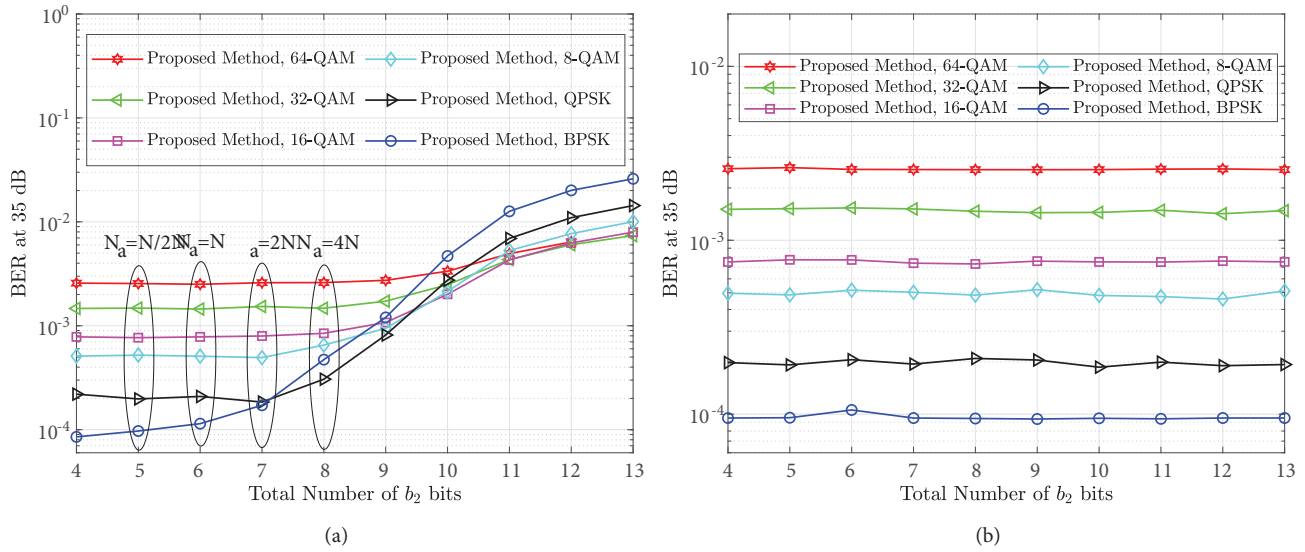
With channel estimation, impairments in channel coefficients will occur and their effect on the proposed scheme will be significant. Hence, in Figure 4, for imperfect CSI, the BER performances with various power values of the estimation error for Algorithm-1 and Algorithm-2 are presented. It is assumed that the estimated channel of  $\bar{\mathbf{H}}$  and  $\bar{\mathbf{H}}_\beta$  are  $\hat{\mathbf{H}} = \bar{\mathbf{H}} + \mathbf{e}$  and  $\hat{\mathbf{H}}_\beta = \bar{\mathbf{H}}_\beta + \mathbf{e}$ , respectively. The channel vector  $\bar{\mathbf{H}}$  is independent of estimation errors  $\mathbf{e}$  and  $E\{\mathbf{e}\mathbf{e}^H\} = \sigma_e^2 \mathbf{I}_n$  [37]. It is also assumed that the power of the estimation error is constant. Figure 4a shows that when  $\sigma_e^2$  decreases from 0.004 to 0.04, an error floor occurs. In terms of estimation error, the BER performance of the classical OFDM is slightly higher than the proposed scheme for Algorithm-2. In Figure 4b, unlike Algorithm-2, CCM-based adaptive algorithm (Algorithm-1) is used at the receiver with threshold value  $10^{-7}$ . It is shown that the proposed method with CCM-based adaptive algorithm has almost the same performance as the classical OFDM. Thus, the performance of the CCM-based adaptive algorithm is slightly better than the proposed algorithm under channel estimation error with more complexity.



**Figure 4.** BER performances of the proposed scheme and conventional OFDM with 64-QAM and  $N = N_\alpha = 1024$  under imperfect channel estimation error (a) for Algorithm-2 (b) for Algorithm-1.

Figure 5a, provides the BER curves of different  $N_\alpha$  values for the proposed technique with Algorithm-2 at SNR= 35dB,  $\sigma_e = 0.01$ , and  $N = 64$ . It is clear that increasing  $N_\alpha$  increases the spectral efficiency of the proposed method. By increasing the length of  $N_\alpha$ , increased spectral efficiency is achieved at the expense of worse BER performance. As shown in this figure, the BER performance of the proposed method with the proposed algorithm (Algorithm-2) is satisfactory for higher-order modulation with maximum  $N_\alpha = 4N$  (i.e.  $\Delta = 1/4$  and number of bits is  $\log_2(4N) = 8$ ) and lower order modulation with maximum  $N_\alpha = N$  (i.e.  $\Delta = 1$  and number of bits is  $\log_2(N) = 6$ ).

In Figure 5b, CCM-based adaptive algorithm (Algorithm-1) is used at the receiver with the same parameters given in Figure 5a. It is illustrated that the proposed method with the CCM-based adaptive algorithm has the same performance as the classical OFDM with more computational complexity compared to Algorithm-2. Higher length of  $N_\alpha$  yields more spectral efficiency, but this comes at the expense of increased computational load. Therefore, the proposed algorithm is more sensitive in case of channel estimation error and



**Figure 5.** BER performances of the proposed scheme and conventional OFDM for different  $N_\alpha$  sizes at SNR= 35 dB (a) with Algorithm-2 (b) with Algorithm-1.

small  $\Delta$  values while having low computational complexity compared to CCM-based adaptive algorithm. As a result, it is demonstrated that the proposed method with Algorithm-2 has high spectral efficiency and almost the same BER performance with tolerable complexity compared to the currently best known CP-based OFDM scheme.

### 9. Conclusions

In practice, in order to eliminate ISI, OFDM systems widely use CP. However, CP reduces the number of symbols that can be transmitted in the OFDM symbol. In this work, a new prefix design approach is proposed to enhance the spectral efficiency of conventional OFDM systems. This prefix fulfills the functionality of the classical CP method. To obtain high spectral efficiency, the proposed design uses GP with additional bits. In addition, a simple algorithm is proposed to detect the additional information bits conveyed by the proposed GP technique. It is shown by simulation results that the proposed method has high spectral efficiency and almost the same BER performance with tolerable complexity compared to the currently best known CP-based OFDM scheme. To further increase the spectral efficiency of the proposed method, various approaches, such as subblock structure, will be applied for future consideration.

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