

1-1-2019

## The ground state of pentaquarks with cluster structure in the few-body system approach

ZARI BINESH

MOHAMMAD REZA SHOJAEI

BEHNAM AZADEGAN

Follow this and additional works at: <https://journals.tubitak.gov.tr/physics>



Part of the [Physics Commons](#)

---

### Recommended Citation

BINESH, ZARI; SHOJAEI, MOHAMMAD REZA; and AZADEGAN, BEHNAM (2019) "The ground state of pentaquarks with cluster structure in the few-body system approach," *Turkish Journal of Physics*: Vol. 43: No. 4, Article 2. <https://doi.org/10.3906/fiz-1807-33>

Available at: <https://journals.tubitak.gov.tr/physics/vol43/iss4/2>

This Article is brought to you for free and open access by TÜBİTAK Academic Journals. It has been accepted for inclusion in Turkish Journal of Physics by an authorized editor of TÜBİTAK Academic Journals. For more information, please contact [academic.publications@tubitak.gov.tr](mailto:academic.publications@tubitak.gov.tr).

## The ground state of pentaquarks with cluster structure in the few-body system approach

Zari BINESH<sup>1</sup> , Mohammad Reza SHOJAEI<sup>1,\*</sup> , Behnam AZADEGAN<sup>2</sup> 

<sup>1</sup>Department of Physics, Shahrood University of Technology, Shahrood, Iran

<sup>2</sup>Hakim Sabzevari University, Sabzevar, Iran

Received: 14.07.2018

Accepted/Published Online: 03.05.2019

Final Version: 02.08.2019

**Abstract:** In this study, an analytical model is presented based on the Nikiforov–Uvarov method for the  $\Theta^+$  exotic KN resonance as a kind of pentaquark. The radial Schrödinger equation for the central Yukawa and modified Yukawa potentials was solved for  $\Theta^+$  by use of the Nikiforov–Uvarov method. The energy eigenvalues and corresponding wave functions of  $\Theta^+$  were obtained in terms of potential coefficients and  $n, l$  quantum numbers. Finally, the potential coefficients were determined according to the binding energy of  $\Theta^+$  exotic KN resonance. The potentials and wave functions of  $\Theta^+$  exotic KN resonance for different sets of potential coefficients are also shown schematically.

**Key words:** Pentaquark, diquark-triquark model, modified Yukawa potential, Nikiforov–Uvarov method, radial Schrödinger equation

### 1. Introduction

Several experimental and theoretical reports have claimed the observation of  $\Theta^+$  pentaquarks with mass of 1540 MeV and small width of 20 MeV [1–5]. Pentaquarks are exotic hadrons consisting of four quarks and one antiquark and have been described by several models such as the quark model and bag models [1,2]. The exotic pentaquarks have an antiquark whose flavor is different from any of its four flavors. The  $uudd\bar{s}$  pentaquark with strangeness quantum number of  $S = +1$  was predicted by the chiral quark soliton model [4] and many other theoretical studies [4–10] before its experimental observation [11]. From the first day of observation, many experimental activities have been performed to prove and characterize these exotic states [12–17]. Various groups around the world have published reports on the existence of the pentaquark state. Five quarks bag [5], diamond [6], triquark-diquark [7], diquark-diquark-antiquark [8,9], and hadronic molecule [10] are the most famous structures proposed for the pentaquark. The chiral soliton model has predicted spin value of 1/2 and positive parity for  $\Theta^+$  pentaquarks, with a light mass of about 1540 MeV/ $c^2$  and width of  $\sim 20$  MeV [1]. The hadronic molecule model is a kind of diquark-triquark model, which includes a baryon such as  $uud$  or  $udd$  and a meson like  $d\bar{s}$  or  $u\bar{s}$  [18]. The mass of  $\Theta^+$  as presented in experiments ranged from 1520 to 1560 MeV/ $c^2$  [19].

In this work, we present an analytical method to obtain the binding energy of a diquark-triquark model of the  $\Theta^+$  pentaquark. In this regard, the radial Schrödinger equation was solved by Nikiforov–Uvarov (NU) method applying Yukawa and modified Yukawa potentials [20] to describe diquark–triquark clusters interaction.

\*Correspondence: binesh\_z@yahoo.com

## 2. Cluster–cluster interaction with the Yukawa potential and modified Yukawa potentials

In nonrelativistic quantum physics, the solution of the radial Schrödinger equation with a physical potential is of crucial importance for the definition of a quantum system [21]. Various analytical methods such as super symmetry [22] and the NU method [23] have been developed in this context. For a two-cluster system, the Schrödinger equation for the central potential  $V(r)$  has the following form [24]:

$$\frac{-\hbar^2}{2\mu} \left( \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} \right) \psi(r) + \left( V(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2} \right) \psi(r) = E\psi(r), \quad (1)$$

where  $\psi(r)$  is the wave function,  $V(r)$  shows the potential energy between two clusters,  $E$  denotes the energy eigenvalues,  $\hbar$  represents the Planck constant, and  $\mu$  is the reduced mass. By introducing the reduced radial wave function  $u(r) = r\psi(r)$ , Eq. (1) is reduced to an equivalent one-dimensional problem, namely:

$$\frac{-\hbar^2}{2\mu} \frac{d^2 u(r)}{dr^2} + \left( V(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2} \right) u(r) = Eu(r), \quad (2)$$

with the boundary condition of  $u(r)|_{r=0} = 0$ . The core–cluster interaction may be constructed from a nucleon–nucleon interaction [25]. The Yukawa potential [26] as a short-range nucleon–nucleon interaction may be used as the interacting potential between two clusters, which has the following form:

$$V(r) = -V_0 \frac{e^{-\alpha r}}{r}, \quad \alpha, V_0 > 0, \quad (3)$$

where  $V_0$  is the depth of the potential well and  $\sigma$  is related to the range of the potential. Due to nuclear force saturation at lower distances, one may add a repulsive term to the Yukawa potential [27]:

$$V(r) = -V_0 \frac{e^{-\alpha r}}{r} + V_1 \frac{e^{-\alpha r}}{r^2}, \quad \alpha, V_0, V_1 > 0, \quad (4)$$

where  $V_1$  is a positive constant for the repulsive part of the interaction potential. In the next section, we transform the equivalent radial Schrödinger equation of Eq. (2) for the potentials in Eqs. (3) and (4) into the form of the NU differential equation, which will be solved analytically.

## 3. Application of the NU method

The equivalent radial Schrödinger equation of Eq. (2) can be converted into a hypergeometric type second-order differential equation by suitable variable transformation. The NU method can be used to solve the hypergeometric differential equation with an appropriate coordinate transformation  $s = s(r)$  [23]:

$$\psi''(s) + \frac{\bar{\tau}(s)}{\sigma(s)} \psi'(s) + \frac{\bar{\sigma}(s)}{\sigma(s)^2} \psi(s) = 0, \quad (5)$$

where  $\sigma(s)$  and  $\bar{\sigma}(s)$  are polynomials of at most second order, and  $\bar{\tau}(s)$  is a first-order polynomial. Eq. (5) can be solved by separation of variables as:

$$\psi(s) = \phi(s) y(s). \quad (6)$$

By inserting Eq. (6) into Eq. (5), the hypergeometric differential equation will be:

$$\sigma(s)y''(s) + \tau(s)y'(s) + \lambda y(s) = 0, \tag{7}$$

$$\frac{\phi'(s)}{\phi(s)} = \frac{\pi(s)}{\sigma(s)}, \tag{8}$$

in which the function of  $\pi(s)$ , the parameter of  $\lambda$ , and the function of  $\tau(s)$  have the following form:

$$\pi(s) = \frac{\sigma'(s) - \bar{\tau}(s)}{2} \pm \sqrt{\left(\frac{\sigma'(s) - \bar{\tau}(s)}{2}\right)^2 - \bar{\sigma}(s) + k\sigma(s)} \tag{9}$$

$$\lambda = k + \pi'(s) \tag{10}$$

$$\lambda = \lambda_n = -n\tau'(s) - \frac{n(n+1)}{2}, \quad n = 0, 1, 2, \dots, \tag{11}$$

$$\tau(s) = 2\pi(s) + \bar{\tau}(s), \tag{12}$$

where  $\tau'(s)$  has negative value and  $k$  can be found such that the square root in Eq. (9) must be the square of a polynomial that is at most a first-degree polynomial.  $y(s)$  can be obtained using the Rodrigues relation as:

$$y_n(s) = \frac{B_n}{\rho[s]} \frac{d^n}{ds^n} [\sigma(s)^n \rho(s)], \tag{13}$$

where  $B_n$  is a normalization constant and the function  $\rho(s)$  is obtained by:

$$(\sigma(s)\rho(s))' = \tau(s)\rho(s). \tag{14}$$

#### 4. Solutions of the Schrödinger equation for the Yukawa and modified Yukawa potential

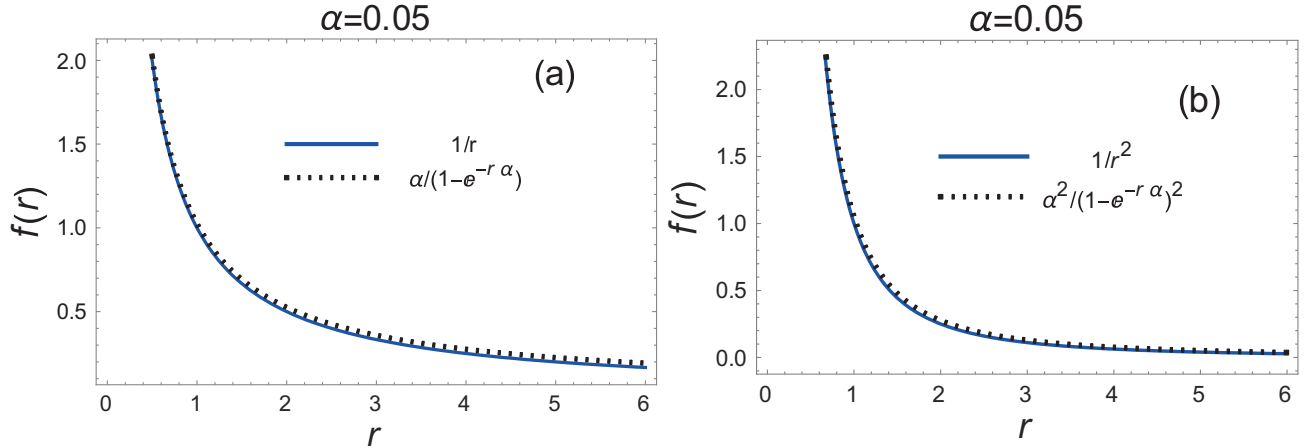
In this section, we employ the NU method for the equivalent radial Schrödinger equation with Yukawa and modified Yukawa potentials to obtain the eigenenergies and eigenfunctions of the pentaquark in the diquark-triquark cluster scheme. The exact solution of Eq. (2) with Yukawa and modified Yukawa potentials in Eqs. (3) and (4) is difficult; therefore, it is necessary to introduce a new variable and make some approximations. The approximate expansions of  $1/r$  and  $1/r^2$  using a Pekeris approximation scheme [28] are given as:

$$\frac{1}{r} = \frac{\alpha}{(1 - e^{-\alpha r})}, \quad \frac{1}{r^2} = \frac{\alpha^2}{(1 - e^{-\alpha r})^2}. \tag{15}$$

The behavior of  $1/r$  and  $1/r^2$  and their approximations are plotted in Figure 1 for  $\sigma = 0.05$ . It can be seen that for small  $\sigma$ , the Pekeris approximation is valid.

By introducing a new variable,  $s = e^{-\alpha r}$ , and using the Pekeris approximation, the equivalent Schrödinger in Eq. (2) can be transformed to the hypergeometric differential equation in Eq. (5) with the following expressions:

$$\bar{\tau}(s) = 1 - s, \quad \sigma(s) = s(1 - s), \quad \bar{\sigma}(s) = As^2 + Bs + C. \tag{16}$$



**Figure 1.** (a)  $1/r$  and its approximation; (b)  $1/r^2$  and its approximation.

These expressions can be used to determine two functions of  $\pi(s)$ ,  $\tau(s)$ , and constant  $k$ . Applying the expressions  $\pi'(s)$ ,  $\tau'(s)$ ,  $k$ , and  $\sigma''(s)$  in Eqs. (10) and (11), the following eigenvalue relation is obtained as a function of quantum number  $n$ :

$$2(B + 2C) = \left(1 + \sqrt{1 - 4A - 4B - 4C}\right) \left(1 + 2\sqrt{-C}\right) + 2n(3 + \sqrt{1 - 4A - 4B - 4C} + 2\sqrt{-C} + n). \quad (17)$$

In order to determine the wave functions in the NU method, we must first obtain the two functions of  $\phi(s)$  and  $\rho(s)$  from Eqs. (8) and (14):

$$\phi(s) = (1 - s)^{\frac{1}{2}(1 + \sqrt{1 - 4A - 4B - 4C})} s^{\sqrt{-C}}, \quad (18)$$

$$\rho(s) = (1 - s)^{\sqrt{1 - 4A - 4B - 4C}} s^{2\sqrt{-C}}. \quad (19)$$

Substituting Eq. (19) into the Rodrigues relation given in Eq. (13), we obtain  $y_n(s)$ . By multiplying  $\phi(s)$  in  $y_n(s)$ , the wave function can be written as:

$$\psi(s) = B_n (1 - s)^{\frac{1}{2}(1 + \sqrt{1 - 4A - 4B - 4C}) - \sqrt{1 - 4A - 4B - 4C}} s^{-\sqrt{-C}} \frac{d^n}{ds^n} \left( (1 - s)^{\sqrt{1 - 4A - 4B - 4C}} s^{2\sqrt{-C}} ((1 - s)s)^n \right). \quad (20)$$

## 5. Diquark-triquark cluster scheme of the $\Theta^+$ pentaquark

The  $\Theta^+$  pentaquark structure is a complicated five-body system of  $uudd\bar{s}$ . In the chiral soliton model and the hadronic molecule model,  $\Theta^+$  is considered as a system consisting of two clusters, one baryon with three quarks and one meson with a quark and one antiquark [29–32]. In a chiral soliton model, the two clusters, a  $ud$  diquark and a  $ud\bar{s}$  triquark, are in a relative P-wave with  $J^P = 1/2^+$  state [8]. In [8] the authors roughly estimated  $m_{diq} = 720 \text{ MeV}$ ,  $m_{triq} = 1260 \text{ MeV}$ , and  $\mu_{di-tri} = 458 \text{ MeV}$  where  $\mu_{di-tri}$  denotes the reduced mass for the

relative motion of the diquark-triquark system. In this model, the binding energy of the two clusters is roughly  $(m_{triq} + m_{diq} - m_{pentaq}) = 440 \text{ MeV}$ .

One may assume that  $\Theta^+$  consists of five quarks in an S-wave and therefore has negative parity,  $J^P = 1/2^-$  (a baryon ( $uud$ ) and a meson ( $d\bar{s}$ )) [33]. The mass of the  $d\bar{s}$  meson with spin of zero is  $m_d = 497.72 \text{ MeV}$ , and the mass of the proton is  $m_{uud} = 938.28 \text{ MeV}$ . This configuration is not bound and the repulsive terms of the interaction force may then overcome the attractive terms and the S-wave pentaquark rearranged into the usual  $KN$  system. Other configuration of S-wave pentaquark may be the  $d\bar{s}$  meson with spin of one and mass of  $892 \text{ MeV}$ . Hence, the reduced mass of  $\Theta^+$  is  $\mu_{di-tri} = 457.277 \text{ MeV}$  and the binding energy will be  $290.28 \text{ MeV}$ .

In what follows, we assume that any cluster behaved like a particle and ignored the internal structure of each cluster. Moreover, the interactions operated only between the two clusters. The two clusters consisted of  $d\bar{s}$  diquark with spin one and mass of  $892 \text{ MeV}$  and  $uud$  triquark with spin of  $1/2$  and mass of  $938.28 \text{ MeV}$  in a relative S-wave.

## 6. Results

By introducing the exponential variable of  $s = e^{-\alpha r}$  and using Pekris approximation, the equivalent Schrödinger equation of Eq. (2) was transformed to the hypergeometric differential equation of Eq. (5) with the three constants of A, B, and C and Yukawa potential:

$$A = \frac{2(E_n - V_0\alpha)\mu}{\hbar^2\alpha^2} B = \frac{2(-2E_n + V_0\alpha)\mu}{\hbar^2\alpha^2} C = -l(1+l) + \frac{2E_n\mu}{\hbar^2\alpha^2}. \quad (21)$$

By inserting the expressions of Eq. (21) into Eq. (17) and solving it for the eigenenergy of  $E_n$  we obtain:

$$\begin{aligned} E_n = & \frac{1}{16\hbar^2(l-n)^2(1+l+n)^2\mu} (-\hbar^4(4l^3(1+4n(3+n)) + l^4(2+8n(3+n)) - 2ln(-1+3\sqrt{(1+2l)^2} \\ & + n(4+7\sqrt{(1+2l)^2} + n(10+2\sqrt{(1+2l)^2}+3n))) + n^2(5-3\sqrt{(1+2l)^2} + n(20-4\sqrt{(1+2l)^2} \\ & + n(31+5\sqrt{(1+2l)^2} + 2n(7+\sqrt{(1+2l)^2}+n)))) - 2l^2(-1+n(-13+3\sqrt{(1+2l)^2} + n(7\sqrt{(1+2l)^2} \\ & + n(10+2\sqrt{(1+2l)^2}+3n))))\alpha^2 + 4\hbar^2(8l^3+4l^4-l^2(-5+\sqrt{(1+2l)^2} + 2n(-1+\sqrt{(1+2l)^2}+n)) \\ & - l(-1+\sqrt{(1+2l)^2} + 2n(-1+\sqrt{(1+2l)^2}+n)) + 2n(1-\sqrt{(1+2l)^2} + n(3-2\sqrt{(1+2l)^2} \\ & + n(4+n))))V_0\alpha\mu + 4(-1-2l(1+l) + \sqrt{(1+2l)^2} - 2n(1-\sqrt{(1+2l)^2}+n))V_0^2\mu^2). \end{aligned} \quad (22)$$

Eq. (22) for  $l = 0$  will take a simple form:

$$E_n = -\frac{(\hbar^2(1+4n+n^2)\alpha - 2V_0\mu)^2}{8\hbar^2(1+n)^2\mu}, \quad (23)$$

and the radial wave function for  $n = 0$  and  $l = 0$  is:

$$\psi(r) = \frac{B_n(e^{-r\alpha})^{1+\sqrt{2}\sqrt{-\frac{E_0\mu}{\alpha^2\hbar^2}}(-1+e^{r\alpha})}{r}. \quad (24)$$

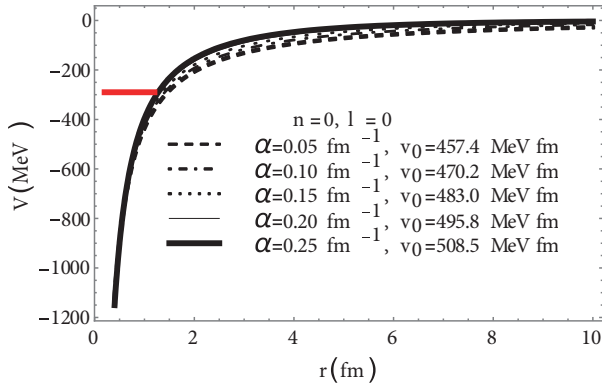
The normalization factor  $B_n$  in Eq. (24) can be obtained by the normalization condition of the radial wave function:

$$\int_0^\infty \psi^*(r)\psi(r)r^2dr = 1. \tag{25}$$

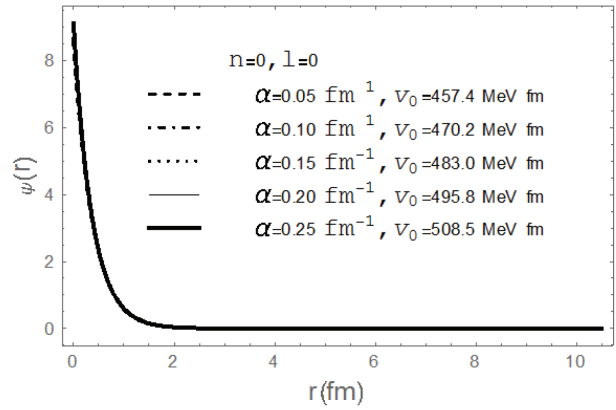
The calculated binding energy of the  $\Theta^+$  pentaquark and coefficients of Yukawa potential in the diquark-triquark model are listed in Table 1. The graph of potentials for different values of  $\alpha$  and  $V_0$  are also shown in Figure 2. The horizontal red line shows the binding energy of the  $\Theta^+$  pentaquark. The radial wave functions of the  $\Theta^+$  pentaquark for different values of  $\alpha$  and  $V_0$  for  $n = 0$  and  $l = 0$  are illustrated in Figure 3. As shown in Figure 3, different sets of  $\alpha$  and  $V_0$  give the same radial wave function for the ground state of the  $\Theta^+$  pentaquark.

**Table 1.** The calculated binding energy and coefficients of Yukawa potential in the diquark-triquark model of the  $\Theta^+$  pentaquark.

$\alpha (fm^{-1})$	$V_0(MeV fm)$	$E_0(MeV)$	$E_0 = -B (MeV)$
0.05	457.4	-290.22	-290.28 MeV
0.10	470.2	-290.25	
0.15	483.0	-290.29	
0.20	495.8	-290.33	
0.25	508.5	-290.23	



**Figure 2.** The Yukawa potential of the  $\Theta^+$  pentaquark for different values of  $\alpha$  and  $V_0$ . The horizontal red line shows the binding energy of the  $\Theta^+$  pentaquark.



**Figure 3.** The radial wave function of the  $\Theta^+$  pentaquark for different values of  $\alpha$  and  $V_0$  for  $n = 0$  and  $l = 0$  for Yukawa potential in the diquark-triquark cluster model.

For the case of modified Yukawa potential, which involves the repulsive term for short distances, the three constants of A, B, and C are defined as:

$$A = \frac{2(E_n - V_0\alpha)\mu}{\hbar^2\alpha^2} \quad B = -\frac{2(2E_n + \alpha(-V_0 + V_1\alpha))\mu}{\hbar^2\alpha^2} \quad C = -l(1+l) + \frac{2E_n\mu}{\hbar^2\alpha^2}. \tag{26}$$

By inserting the expressions of Eq. (26) into Eq. (17) and solving it for the eigenenergy of  $E_n$  for  $l = 0$ , we

obtain:

$$\begin{aligned}
 E_n = & -\frac{1}{16\mu(\hbar^2 n(1+n) - 2V_1\mu)^2} (16V_1(V_0 - V_1\alpha)^2\mu^3 + 4\hbar^2\mu^2(8n^2V_0V_1\alpha + V_0^2(1 - \sqrt{1 + \frac{8V_1\mu}{\hbar^2}} \\
 & + 2n(1+n - \sqrt{1 + \frac{8V_1\mu}{\hbar^2}})) + V_1^2\alpha^2(1 + \sqrt{1 + \frac{8V_1\mu}{\hbar^2}} + 2n(3-n + \sqrt{1 + \frac{8V_1\mu}{\hbar^2}}))) \\
 & - 4\hbar^4 n\alpha\mu(2V_0(1 - \sqrt{1 + \frac{8V_1\mu}{\hbar^2}} + n(3+n(4+n) - 2\sqrt{1 + \frac{8V_1\mu}{\hbar^2}})) + V_1\alpha(1 + \sqrt{1 + \frac{8V_1\mu}{\hbar^2}} \\
 & + n(6+n^2 + 3\sqrt{1 + \frac{8V_1\mu}{\hbar^2}} + 2n(5 + \sqrt{1 + \frac{8V_1\mu}{\hbar^2}}))) + \hbar^6 n^2\alpha^2(5 - 3\sqrt{1 + \frac{8V_1\mu}{\hbar^2}} \\
 & + n(-4(-5 + \sqrt{1 + \frac{8V_1\mu}{\hbar^2}}) + n(31 + 5\sqrt{1 + \frac{8V_1\mu}{\hbar^2}} + 2n(7+n + \sqrt{1 + \frac{8V_1\mu}{\hbar^2}}))))), \tag{27}
 \end{aligned}$$

and the radial wave function for  $n = 0$  and  $l = 0$  is:

$$\psi(r) = \frac{B_n(e^{-r\alpha})^{\sqrt{2}\sqrt{-\frac{E_0\mu}{\alpha^2\hbar^2}}}(1 - e^{-r\alpha})^{\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{8V_1\mu}{\hbar^2}}}}{r}. \tag{28}$$

The calculated binding energy of the  $\Theta^+$  pentaquark as well as coefficients of modified Yukawa potential in the diquark-triquark model are presented in Table 2. The graph of potentials for different values of  $\alpha$ ,  $V_0$ , and  $V_1$  are shown in Figure 4. The horizontal red line shows the binding energy of the  $\Theta^+$  pentaquark. The radial wave functions of the  $\Theta^+$  pentaquark for different values of  $\alpha$ ,  $V_0$ , and  $V_1$  for  $n = 0$  and  $l = 0$  are provided in Figure 5.

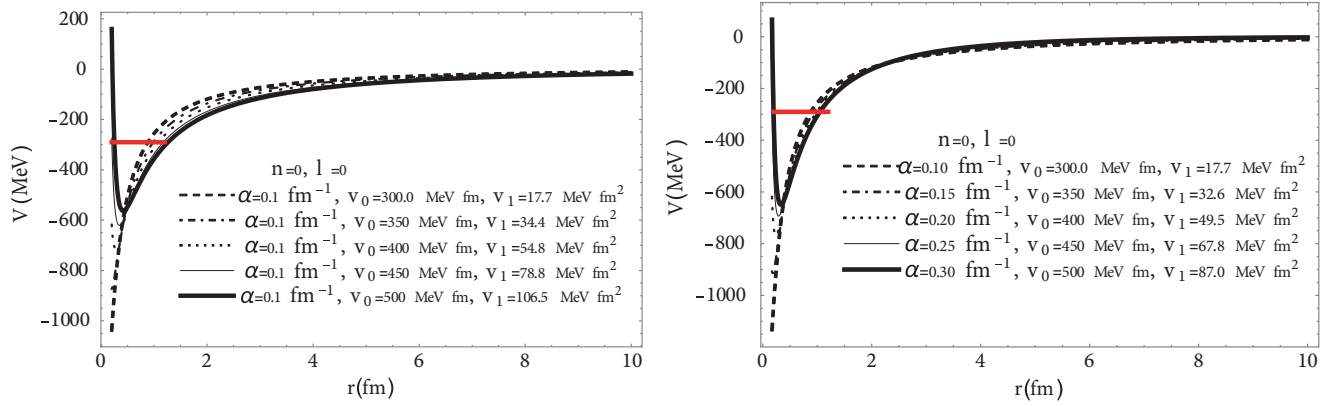
**Table 2.** The calculated binding energy and coefficients of modified Yukawa potential in diquark-triquark cluster model of the  $\Theta^+$  pentaquark.

$\alpha (fm^{-1})$	$V_0 (MeV fm)$	$V_1 (MeV fm^2)$	$E_0 (MeV)$	$E_0 = -B (MeV)$
0.10	300	17.7	-290.37	-290.28 MeV
0.10	350	34.4	-290.60	
0.10	400	54.8	-290.70	
0.10	450	78.8	-290.53	
0.10	500	106.5	-290.50	
0.15	350	32.6	-290.68	
0.20	400	49.5	-290.73	
0.25	450	67.8	-290.72	
0.30	500	87.0	-290.65	

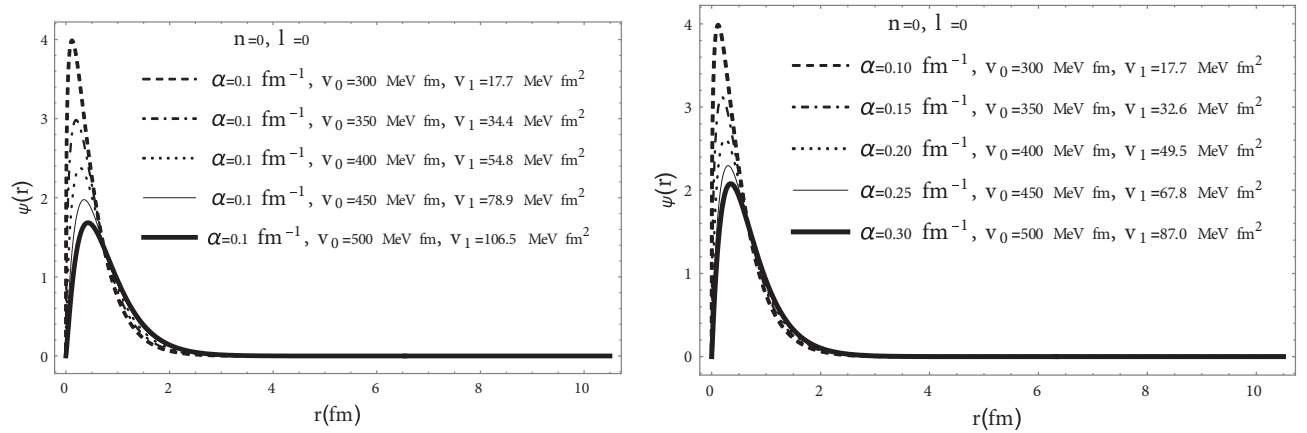
## 7. Conclusion

In this study, a cluster model was considered for the  $\Theta^+$  pentaquark by dividing the system into two clusters: a  $uud$  triquark and a  $d\bar{s}$  diquark, which are in a relative S-wave. Considering the Yukawa potential and modified Yukawa potential as the interaction potential between clusters and solving the radial Schrödinger equation by the NU method, the values of potential coefficients were determined, which predicted the binding energy of





**Figure 4.** The modified Yukawa potential of the  $\Theta^+$  pentaquark for different values of  $\alpha$ ,  $V_0$ , and  $V_1$ . The horizontal red line shows the binding energy of the  $\Theta^+$  pentaquark.



**Figure 5.** The radial wave function of the  $\Theta^+$  pentaquark for different values of  $\alpha$ ,  $V_0$ , and  $V_1$  for  $n = 0$  and  $l = 0$  for modified Yukawa potential in the diquark-triquark cluster model.

the  $\Theta^+$  pentaquark. As shown in Figures 3 and 5, the radial wave functions of the  $\Theta^+$  pentaquark vanished at about  $2 \text{ fm}$  for both cases of Yukawa and modified Yukawa potentials, but in the case of modified Yukawa potential, it vanished near the origin while for the Yukawa potential it increased at the origin.

## References

- [1] Gell-Mann M. A schematic model of baryons and mesons. *Physics Letters* 1964; 8: 214-215. doi: 10.1016/S00321-9163(64)92001-3
- [2] Hicks KH. On the conundrum of the pentaquark. *European Physical Journal H* 2012; 37: 1-31. doi: 10.1140/epjh/e2012-20032-0
- [3] Schulte-Wissermann M, Czodrowski P, Lehnert B, Morgenstern M, Freiesleben H. On determining the cross section of the supposed pentaquark  $\Theta^+$  in the reaction  $pp \rightarrow \Theta^+\Sigma^+ \rightarrow pK^0\Sigma^+$ . *European Physical Journal A* 2014; 50: 109. doi: 10.1140/epja/i2014-14109-7
- [4] Diakonov D, Petrov V, Polyakov M. Exotic anti-decuplet of baryons: prediction from chiral solitons. *Zeitschrift für Physik A Hadrons and Nuclei* 1997; 359: 305-314.
- [5] Strottman, D. Multiquark baryons and the MIT bag model. *Physical Review D* 1979; 20: 748-767.

- [6] Song XC, Zhu SL. The narrow  $\Theta_5$  pentaquark as the first non-planar hadron with the diamond structure and negative parity. *Modern Physics Letters A* 2004; 19: 2791-2797. doi: 10.1142/S0217732304015154
- [7] Karliner M, Lipkin HJ. A diquark-triquark model for the KN pentaquark. *Physics Letters B* 2003; 575: 249-255. doi: 10.1016/j.phsletb
- [8] Jaffe RL, Wilczek F. Diquarks and exotic spectroscopy. *Physical Review Letters* 2003; 91: 232003. doi: 10.1103/PhysRevLett.91.232003
- [9] Chandra A, Bhattacharya A, Chakrabarti B. Heavy pentaquarks and doubly heavy baryons in quasiparticle approach. *Modern Physics Letters A* 2012; 27: 1250006
- [10] Mironov A, Morozov A. Is the pentaquark doublet a hadronic molecule? *Journal of Experimental and Theoretical Physics* 2015; 102: 271-273. doi: 10.1134/S0021364015170099
- [11] Inoue T, Lyubovitskij VE, Gutsche T, Faessler A. Mass spectrum of the  $J^P=1/2^-$  and  $3/2^-$  pentaquark antidecuplets in the perturbative chiral quark model. *International Journal Modern Physics E* 2005; 14: 995-1016. doi: 10.1142/S0218301305003752
- [12] Nakano T, Ahn DS, Ahn JK, Akimune H, Asano Y et al. LEPS Collaboration. Evidence for a narrow  $S=+1$  baryon resonance in photoproduction from the neutron. *Physical Review Letters* 2003; 91: 012002. doi: 10.1103/PhysRevLett.91.012002
- [13] Barmin VV, Borisov VS, Davidenko GV, Dolgolenko AG, Guaraldo C et al. DIANA Collaboration. Observation of a baryon resonance with positive strangeness in  $K^+$  collisions with Xe nuclei. *Physics of Atomic Nuclei* 2003; 66 (9): 1715-1718. doi: 10.1134/1.1611587
- [14] Stepanyan S, Hicks K, Carman DS, Pasyuk E, Schumacher RA et al. CLAS Collaboration. Observation of an exotic  $S=+1$  baryon in exclusive photoproduction from the deuteron. *Physical Review Letters* 2003; 91: 252001
- [15] Barth J, Brauna W, Ernst J, Glander KH, Hannappel J et al. SAPHIR Collaboration. Evidence for the positive-strangeness pentaquark  $\Theta^+$  in photoproduction with the SAPHIR detector at ELSA. *Physics Letters B* 2003; 572: 127-132. doi: 10.1016/j.phsletb.2003.08.019
- [16] Akopov N, Akopov Z, Augustyniak W, Avakian R, Avetissian A et al. HERMES Collaboration. Pentaquark  $\Theta^+$  search at HERMES. *Physical Review D* 2015; 91: 057101. doi: 10.1103/PhysRevD.91.057101
- [17] Tomonori NT, Satoshi A, Michelangelo A, Shuhei A, Kanae A et al. Pentaquark  $\Theta^+$  search in hadronic reaction at J-PARC (J-PARC E19 Collaboration) 022011. *JPS Conference Proceedings* 2015; 8: 022011. doi: 10.7566/JPS-SCP.8.022011
- [18] Zhu R, Qiao CF. Pentaquark states in a diquark-triquark model. *Physics Letters B* 2016; 756: 259. doi: 10.1016/j.phsletb.2016.03.022
- [19] Hicks K. Experimental search for pentaquarks. *Progress in Particle and Nuclear Physics* 2005; 55: 647-676. doi: 10.1016/j.pnpnp.2005.05.002
- [20] Yukawa H. On the interaction of elementary particles I. *Proceedings of the Physico-Mathematical Society of Japan* 1935; 17: 48-57. doi: 10.1143/PTPS.1.1
- [21] Mousavi M, Shojaei MR. Mirror nuclei of  $^{17}\text{O}$  and  $^{17}\text{F}$  in relativistic and nonrelativistic shell model. *Advances in High Energy Physics* 2017; 2017: 5841701. doi: 10.1155/2017/5841701
- [22] Rezvani F, Shojaei MR. Bound states of the Klein-Gordon equation for spin symmetry with exponential-type potentials by the supersymmetry method. *Journal of Theoretical Physics and Cryptography* 2014; 6: 42-45.
- [23] Shojaei MR, Mousavi M. Solutions of the Klein-Gordon equation for  $l \neq 0$  with position-dependent mass for modified Eckart potential plus Hulthen potential. *International Journal of Physical Sciences* 2015; 10 (9): 324-328. doi: 10.5897/IJPS2015.4303
- [24] Zoghi-Foumani N, Shojaei MR, Rajabi AA. A new approach for cluster structures in  $^{16}\text{O}$  and  $^{20}\text{Ne}$ . *Pramana Journal of Physics* 2017; 88: 68. doi: 10.1007/s12043-016-1324-5

- [25] Shojaei MR, Roshanbakht N. Calculation of energy spectrum of  $^{12}\text{C}$  isotope with modified Yukawa potential by cluster models. *Pramana Journal of Physics* 2016; 87: 54. doi: 10.1007/s12043-016-1261-3
- [26] Shojaei MR, Rajabi AA, Farrokh M, Zoghi-Foumani N. Energy levels of spin-1/2 particles with Yukawa interaction. *Journal of Modern Physics* 2014; 5 (9): 773-780. doi: 10.4236/jmp.2014.59087
- [27] Rezvani F, Shojaei MR. Energy levels of weakly bound nuclei with relativistic effects. *Scientific Bulletin-University Politehnica of Bucharest Series A* 2017; 79 (3): 193-202.
- [28] Onate CA, Ojonubah JO. Eigensolutions of the Schrödinger equation with a class of Yukawa potentials via supersymmetric approach. *Journal of Theoretical and Applied Physics* 2016; 10: 21-26. doi: 10.1007/s40094-015-0196-2
- [29] Stancu F, Riska DO. Stable uudds-pentaquarks in the constituent quark model. *Physics Letters B* 2003; 575: 242-246. doi:10.1016/j.physletb.2003.09.061
- [30] Ramachandran R. Chiral soliton model vs. pentaquark structure for  $\Theta(1540)$ . *Pramana Journal of Physics* 2005; 65 (3): 381-391
- [31] Kopeliovich VB. Pentaquarks in chiral soliton models. *Physics of Particles and Nuclei* 2006; 37 (4): 623-645. doi: 10.1134/S106377960604006X
- [32] Dudek J. Triquark-diquark models of a  $\Theta(1540)$ . *Journal of Physics Conference Series* 2005; 9: 209-212. doi: 10.1088/1742-6596/9/1/038
- [33] Zhu SL. Understanding pentaquark States in QCD. *Physical Review Letters* 2003; 91 (23): 232002. doi: 10.1103/PhysRevLett.91.232002