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Tunneling radiation and quantum entropy of a massive gravity black hole

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Abstract: By using the Parikh-Wilczek (PW) quantum tunneling method, the Hawking radiation of black holes in massive gravity is investigated, the emission rate of particles and the black hole entropy are calculated. It is shown that the emission spectrum is not purely thermal, depends on the increment of the black hole entropy, consists with an accurate unitary theory and supports the standpoint of information conservation. Unlike other modified gravities, the entropy of the massive gravity black hole unexpectedly conforms to the area law just as that of Einstein gravity black hole.

Key words: Tunneling radiation, massive gravity, quantum entropy

1. Introduction

Black hole is a mysterious object. Because of its big density and strong gravity, any objects (including light) close to it will be swallowed up. As a result, it cannot be directly observed by the naked eye or optical equipment. Taking black hole as a thermodynamic system, the study of the thermodynamic properties of black holes has always been one of the frontiers of physics. As early as the 1970s, the Hawking radiation was discovered [1] as an accurate thermal spectrum. Since then, a series of studies had been done to prove that the radiation spectra are purely thermal [2–6]. However, the purely thermal property of the black hole radiation is inconsistent with both the property of time inversion and the underlying unitary theory in quantum mechanics. This result reveals the incompatibility between general relativity and quantum mechanics and leads to the ”information paradox”. The information paradox of black hole has been listed as one of the top 10 physics problems of the century. But the physicists always insist that information is never lost. More than two decades later, Parikh and Wilczek suggested that Hawking radiation should be treated as a quantum tunnelling effect and thought that the barrier is determined by the energy of the emitting particle itself so that the energy conservation is satisfied when a particle radiates from a black hole. They used the method to calculate the modified radiation spectra of particles emitted from the Schwarzschild black hole and the Reissner–Nordstrom one [7–9], and found that the spectra are not purely thermal and satisfy an accurate unitary theory which supports the conclusion of information conservation. Not long after then, in 2004, Hawking changed his view at the 17th conference of general relativity and gravity and accepted the idea that information of black holes cannot be lost. Subsequently, the PW method was used to calculate the corrected radiation spectra of various Einstein gravity black holes [11–23]. Later, the method was also used to calculate the radiation spectra of black holes in various modified gravities including $f(\mathrm{R})$, Gauss–Bonnet, Lovelock–Born–Infeld, Hořava–Lifshitz and conformal anomaly [24–28]. Not only were

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the impurely thermal spectra were given, but also the various forms of the Bekenstein–Hawking (BH) entropy were displayed. It was also shown that the area law of the entropy was broken as a result of the existence logarithmic correction terms and other terms or factors [24–28]. Of course, a few especial cases were proven to exist, for which the PW method cannot convert isothermal Hawking radiation resulting from the emission of the uncharged particles of the linear dilaton black hole to a nonthermal one unless the quantum gravity corrections are considered [29–32].

A graviton is defined as a zero mass particle in the theory of Einstein gravity known as general relativity. One can ask whether a self-consistent gravity theory may be established if the mass of the graviton is not zero. It turns out that it is not easy. But, researchers have been trying hard so that a series of research results were made and some strange properties of this modified gravity theory were discovered [33–39]. Among them, a class of nonlinear massive gravity is the most representative one proposed by Rham, Gabadadze and Tolley [33–34]. Within this framework, a black hole solution with negative cosmological constant was proven to exist in the massive gravity theory [35–36]. The study of the thermodynamic properties of massive gravity black holes has attracted the attention of many researchers [36–41]. In this paper, we extend the PW method to a four-dimensional massive gravity black hole to investigate the emission rate of a particle from the event horizon. Not only will the impurely thermal property of Hawking radiation be verified but also the effect of the graviton mass on the black hole radiation and on its thermodynamic quantities such as quantum entropy will be discussed.

2. Radial emission equation of particles

From Ref. [36], the line element of a four-dimensional spherically symmetric and static space-time in massive gravity can be written as follows:

$$\begin{aligned} &\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2), \\ &f(r) = 1 - \tfrac{\Lambda r^2}{3} - \tfrac{2M}{r} + \tfrac{4Q^2}{r^2} + \tfrac{c_1 m^2 r}{2} + c_2 m^2, \end{aligned} \tag{1}$$

where Λ is the cosmological constant, m is the graviton mass, c_1, c_2 are constants corresponding to the first and second massive potentials, and M, Q are the mass and charge of the black hole.

By solving the equation $f(r) = 0$, the event horizon radius of the black hole r_+ can be obtained. Then, the mass and Hawking temperature of the black hole can be expressed as follows:

$$M = \frac{1}{2}\left(r_+ - \frac{\Lambda r_+^3}{3} + \frac{4Q^2}{r_+} + \left(\frac{c_1 r_+^2}{2} + c_2 r_+\right) m^2\right) \tag{2}$$

and

$$T_H = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi}\left(\frac{1}{r_+} - \Lambda r_+ - \frac{4Q^2}{r_+^3} + \left(c_1 + \frac{c_2}{r_+}\right) m^2\right). \tag{3}$$

There are two key points to calculate the black hole emissivity by using the PW method. One is that the particle-black hole system satisfy the energy conservation during the process of tunneling radiation. The other is that a good coordinate system needs to be selected to make the singularity of the metric at the event horizon not to exist. We make a coordinate transformation as follows:

$$\mathrm{d}t = \mathrm{d}T - f^{-1}(r)\sqrt{\frac{r_+}{r}}\mathrm{d}r. \tag{4}$$

Then, the line element (1) becomes

$$ds^2 = -f(r)dT^2 + 2\sqrt{\frac{r_+}{r}}dTdr + \left(1 - \frac{r_+}{r}\right) f^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{5}$$

Formula (5) is called Painleve–Gullstrand line element, which is well-behaved at the event horizon and has at least the following properties: (a) space-time is steady; (b) all constant-time slices are just flat Euclidean space; (c) the metric (5) satisfies Landau's condition of coordinate clock synchronization, which is given by:

$$\frac{\partial}{\partial x^j}(-\frac{g_{0i}}{g_{00}}) = \frac{\partial}{\partial x^i}(-\frac{g_{0j}}{g_{00}}), \quad (i, j = 1, 2, 3). \tag{6}$$

Since the tunnelling of particles is an instantaneous process in quantum mechanics, the condition (6) is very necessary [12].

From (5), it is easy to get the radial light-like geodesic equation at the event horizon, that is, the motion equation of the outgoing particles

$$\dot{r} = \frac{dr}{dT} = f(r)\left(1 + \sqrt{\frac{r_+}{r}}\right)^{-1}. \tag{7}$$

3. Eemission rate and BH entropy

The total energy of a stationary space-time should be conserved in the course of the radiation of a particle outward from a black hole. When a particle of energy ω is emitted, the black hole mass will be reduced to $M - \omega$, and the horizon radius, Hawking temperature and so on all will be changed. Therefore, when the equations related with M are used, M should be replaced by $M - \omega$. Since the metric is spherically symmetric, the outgoing particle can be regarded as a spherical energy layer, that is, as a de Broglie spherical wave. According to the WKB approximation approach, the relationship between the tunneling probability and the the action is [42]

$$\Gamma \sim \exp\left(-2ImZ\right). \tag{8}$$

During the process of a particle passing through the barrier, the horizon radius of the black hole changes from the initial value r_i (corresponding to the mass M) to the final value r_f (corresponding to the mass $M - \omega$), and the imaginary part of the action reads

$$ImZ = Im\int_{r_i}^{r_f} p_r dr = Im\int_{r_i}^{r_f}\int_0^{p_r} dp_r dr, \tag{9}$$

where p_r is the canonical momentum conjugate to r. Utilizing the Hamiltonian equation $\dot{r} = \frac{dH}{dp_r}\Big|_r = \frac{dM}{dp_r}$ and substituting Eq. (7) into Eq. (9), we have

$$ImZ = Im\int_{M_i}^{M_f}\int_{r_i}^{r_f}\frac{dr}{\dot{r}}dM = Im\int_{M_i}^{M_f}\int_{r_i}^{r_f}\left(1 + \sqrt{\frac{r_+}{r}}\right) f^{-1}(r)drdM \ , \tag{10}$$

where $M_i = M; M_f = M - \omega$. It is obvious that the integrand diverges at $r = r_+$. Therefore, we must make use of the integral method of complex variable function to calculate the r integral in Eq. (10). By deforming the contour around the single pole, we evaluate the r integral and obtain

$$ImZ = -2\pi \int_{M_{\rm i}}^{M_{\rm f}} \frac{1}{f'(r_+)} {\rm d}M. \tag{11}$$

From Eq. (2), we have

$$\frac{dM}{{\rm d}r_+} = \frac{1}{2}\left(1 - \Lambda r_+^2 - \frac{4Q^2}{r_+^2} + (c_1 r_+ + c_2)\, m^2\right). \tag{12}$$

Substituting Eqs. (3) and (12) into Eq. (11), we can easily finish the integral and get

$$ImZ = \frac{\pi}{2}\left(r_{\rm i}^2 - r_{\rm f}^2\right). \tag{13}$$

According to Eq. (8), the emission probability of the particle and the emission spectrum can be obtained

$$\Gamma \sim \exp\left[\pi\left(r_{\rm f}^2 - r_{\rm i}^2\right)\right] = \exp\left(\Delta S_{\rm BH}\right), \tag{14}$$

where $\Delta S_{\rm BH} = \pi\left(r_{\rm f}^2 - r_{\rm i}^2\right)$ is the increment of the black hole entropy before and after the particle radiation. Obviously, the emission spectrum is no longer a pure thermal one and satisfies an accurate unitary theory, it has the same functional form as that of black holes in Einstein gravity and other modified gravities. Since the horizon area of the black hole is $A = 4\pi r_+^2$, the BH entropy given by this quantum tunneling method can be expressed as follows:

$$S_{\rm BH} = \pi r_+^2 = \frac{A}{4}. \tag{15}$$

Compared with the results of other modified gravity black holes [28–32], the entropy of the massive gravity black hole satisfies unexpectedly the area formula.

Further, we expand $\Delta S = {\rm S}(M) - S(M-\omega)$ in terms of ω, namely

$$\Delta S = a_1\omega + O(\omega), \tag{16}$$

where

$$a_1 = \left.\frac{d(\Delta S)}{d\omega}\right|_{\omega=0}. \tag{17}$$

It is not difficult to demonstrate that $-a_1 = \beta = \frac{1}{T_{\rm H}}$ is the inverse of the Hawking temperature. Therefore, the spectrum (14) can be written as follows:

$$\Gamma \sim \exp(\Delta S) = \exp(-\beta\omega + O(\omega)). \tag{18}$$

In (18), the leading-order term is the thermal Boltzmann factor $e^{-\beta\omega}$ for the emitting radiation, the others are the corrections due to the response of the background geometry to the particle radiation, which describes the reaction of the quantum radiation and the deviation of the emission spectrum from the purely thermal one.

4. Conclusion

We took energy conservation into account and considered the radiation particle as a de Broglie spherical wave, made use of the PW quantum tunnelling method to investigate the tunnelling Hawking radiation of black holes in massive gravity. The corrected emission rate and quantum entropy were obtained and the universality was proved again that the radiation spectrum is not purely thermal. It was shown that the modified spectrum keeps the completely same form as that of existing results [7–28] and is further illustrated that other information than temperature can be carried in the process of black hole radiation. Surprisingly, the BH entropy of the massive gravity black hole is equal to a quarter of the horizon area and satisfies the area law. The result differs from that of other modified gravities [28–32].

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