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THIRUVOTHUR VENKATESAN RAJA

ACHINTYA KUMAR DAS

SATISH KUMAR RATHEE

UMESH SINGH

SUSHIL KUMAR

See next page for additional authors

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Comparative efficiency of five mathematical functions in modelling the first lactation milk yield of Kankrej cattle

Thiruvothur Venkatesan RAJA[1,*], **Achintya Kumar DAS**[1], **Satish Kumar RATHEE**[2], **Umesh SINGH**[1], **Sushil KUMAR**[1], **Bharat Singh RATHOD**[3], **Harshad PANCHASARA**[3]

[1]Cattle Genetics and Breeding Division, ICAR-Central Institute for Research on Cattle, Meerut, Uttar Pradesh State, India
[2]Scientist-Division of Animal Genetics and Breeding, ICAR-National Dairy Research Institute, Karnal, Haryana State, India
[3]Livestock Research Station, Sardarkrushinagar Dantiwada Agricultural University, Gujarat State, India

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Abstract: Modelling of first lactation milk yield of Kankrej cattle was done using five different non-linear mathematical functions viz., exponential decline function (EDF), gamma function (GF), inverse polynomial function (IPF), mixed log function (MLF) and parabolic exponential function (PEF). We compared their efficiency in describing variations in first lactation milk yield of animals maintained under field conditions. A total of 3994 fortnightly daily milk records of 203 Kankrej cattle calved during the years between 2011 and 2017 at farmers' herds in the Banaskantha region of Gujarat State, India were utilized for the study. The PROC NLIN procedure of Statistical Analysis Software (SAS) using Newton method of iteration was applied to generate the model parameters and corresponding standard errors. Different parameters like adjusted R^2-value, root mean square error (RMSE), Durbin–Watson (DW) statistic, Akaike information criterion (AIC) and Bayesian information criterion (BIC) were used to compare the efficiency of different models. Results inferred that mixed log function was the best non-linear mathematical function for explaining variations in the first lactation milk yield of Kankrej cattle followed by gamma function and parabolic exponential function. The exponential decline function was the least efficient in fitting the lactation curve in Kankrej cattle, followed by inverse polynomial function.

Key words: Kankrej Cattle, Lactation curve, Non-linear mathematical models, Test day yields

1. Introduction

Kankrej is the important dual purpose cattle breed of India known for higher milk production and draft capabilities. This breed originates from Banaskantha District in Gujarat State of India [1] and have abilities to withstand the harsh tropical climatic conditions of the region and survive in poor plane of nutrition. Even though Kankrej cattle is classified as a dual purpose breed, it has high genetic potential for milk production as the elite Kankrej cows produce more than 4200 kg of milk in a standard mature lactation of 300 days. In order to improve milk production potential of this breed, many genetic improvement programmes are implemented in the home tract of the breed. ICAR-Central Institute for Research on Cattle, Meerut, a nodal institution for cattle research in India is also implementing the Indigenous Breeds Project (IBP) under All India Co-ordinated Research Project on Cattle (AICRP on Cattle) in collaboration with Sardarkrushinagar Dantiwada Agricultural University, Dantiwada, Gujarat through which the genetic improvement of this breed is targeted. The Livestock Research Station (LRS), Dantiwada is identified as the germplasm (GP) unit, which maintains elite females as bull mothers of young bulls for their testing in the data recording (DR) units. Kankrej animals in the farmer herds, NGOs and Gaushalas are registered under the project as DR units. Genetic evaluation of young bulls is done through Associated Herd Progeny Testing Programme based on their expected breeding values (EBVs) estimated on the basis of the first lactation milk yield of their daughters.

Advancement in the field of animal breeding has resulted in the use of different test day regimes for genetic evaluation of animals. Recent studies suggest that selection of animals on the basis of daily milk yield is more accurate than the selection on the basis of standard lactation milk yield. With this view, many developed countries have shifted their bull selection criteria to test day yields, as it accounts for the variations even in daily milk yield of individual cows. Daily variations in the lactation milk yield can also be analysed graphically in the form of lactation curve using mathematical functions, which may help to understand the biological efficiency of the dairy animals and the pattern of

* Correspondence: venkatesanraja09@gmail.com

milk production. It was also suggested [2] that lactation curves are useful in designing the management strategies for better exploitation of the genetic potential of dairy animals, prediction of lactation milk yield, developing a suitable breeding programme for increasing the milk production and accurate genetic evaluation. Different mathematical models have been developed by many researchers to describe the lactation milk yield of dairy cows. Many studies have been conducted to fit different lactation curve of different breeds of cattle maintained under organized farm and field conditions. However, no detailed study has been reported so far on the modelling of first lactation milk yield of Kankrej cattle reared under field conditions. Hence, the study was conducted on modelling of first lactation milk yield in Kankrej cattle using five different non-linear mathematical functions viz., exponential decline function (EDF), gamma function (GF), inverse polynomial function (IPF), mixed log function (MLF) and parabolic exponential function (PEF) and to compare their efficiency in describing variations in first lactation milk yield of animals maintained under field conditions.

2. Materials and methods

2.1. Study materials

Kankrej is an important dual purpose cattle breed having the home tract Kankrej Tehsil in Banaskantha region of Gujarat State, India. The present study was conducted on 203 Kankrej cattle maintained by the farmers in field conditions and calved during the years from 2011 to 2017. The average age of the cows included in the study was 41.79 months with the range of 20.28–63.12 months. All the animals were maintained under similar management conditions. Grazing is mainly practiced and animals are fed occasionally with concentrates. The climatic condition of the region is tropical semi-arid with fairly cold winter season (November–February), hot dry summer (March–June) and extremely humid hot monsoon season (July–October) [3]. Records of cows with abnormal calving, disease, mastitis, etc. were not included in the study. Finally, 3994 fortnightly daily milk records starting from 15th day of post calving were utilized.

2.2. Mathematical models

The standard lactation curve is generally characterized by the ascending and descending phases. The duration of ascending phase starts from the date of calving to the peak daily milk yield, and the descending phase is from the peak production to the date of drying. The slope of descending phase describes the persistency of lactation milk yield.

Five different mathematical functions were used to develop the lactation curves of first lactation milk yield as listed below:

1. Exponential decline function [4]: $Y_t = ae^{-ct}$

Where,

Y_t = Average daily yield in the t^{th} fortnight of lactation
a = initial milk yield after calving
c = descending slope parameter
t = length of time since calving
e = residual error

This is the first mathematical model proposed to describe the lactation curve of dairy cattle. Even though this model describes the descending phase of the lactation curve effectively, it could not explain the increase in milk production from the date of calving to the peak milk production, as it does not estimate the ascending slope parameter "b". As it describes the descending phase, this model could explain the persistency of cows in milk production.

2. Parabolic exponential model [5]: $Y_t = a \exp(bt - ct^2)$

Where,

Y t = Average daily yield in the t^{th} fortnight of lactation
a = initial milk yield after calving
b = ascending slope parameter up to the peak yield
c = descending slope parameter
t = test day or length of time since calving
e = residual error

This model produces a truncated bell shaped curve for milk yield and provides good fit for the lactation milk yield of primiparous animals but less effective in multiparous animals.

3. Inverse polynomial model [6]: $Y_t = t(a + bt + ct^2)^{-1}$

Where,

Y_t = Average milk yield in t^{th} fortnight of lactation
a = Initial milk yield after calving
b = Ascending slope parameter up to peak yield
c = Descending slope parameter
t = test day or length of time since calving
e = residual error

This model was found to be more efficient in fitting the lactations starting with low yield and reaching earlier to the peak yield.

4. The gamma function [7]: $Y_t = at^b e^{-ct}$

Where,

Y_t = Average daily yield in the t^{th} fortnight of lactation
a = initial milk yield after calving
b = ascending slope parameter up to the peak yield
c = descending slope parameter
t = length of time since calving
e = residual error

The constants were derived by solving the equation after log scale transformation $\mathrm{In}(Y_t) = \mathrm{In}(a) + b\mathrm{In}(t) - ct$

The milk yields up to fortnight t was given by

$$Y_t = a \int_{0 to 1} t^b \exp(-ct)dt$$

A total of 305-days milk yield (20 fortnights) was obtained as the integral of the average fortnightly milk yields.

Even though the gamma function provides acceptable fit of the milk yield data, the basic problem experienced by researchers was that it underpredicts the mid lactation yields and overpredicts the early and late lactation yields.

5. Mixed log function [8]: $Y_t = a + bt^{1/2} + c\log_t + e_t$
Where,

Y_t = Average milk yield in t^{th} fortnight of lactation
a = Initial milk yield after calving
b = Ascending slope parameter up to peak yield
c = Descending slope parameter
t = Test day or length of time since calving
e_t = Residual error

This model can be considered as the extension of the linear cum log model proposed by Singh and Gopal wherein the term "t" has been substituted by the square root of "t". The basic drawback of this model is that it underestimates the peak yield and overestimate the post-peak yield.

The PROC NLIN procedure of SAS [9] was used to obtain the curve functions (a, b and c), and the Newton method of iteration was applied to generate the model parameters and the corresponding standard errors. According to the convergence criterion of model, number of iterations varied. The model parameters viz., adjusted R^2-value, root mean square error (RMSE), Durbin Watson (DW) statistic, Akaike information criterion (AIC) and Bayesian information criterion (BIC) were used to compare the efficiency of different models in explaining the variations of first lactation fortnightly test day milk yields in Kankrej cattle. The formulae used for estimation of different parameter functions are as follows:

$$R^2_{adj} = 1 - \lfloor (n-1)/(n-p) \rfloor * (1-R^2) \text{ or } 1 - \lfloor MS_E/MS_T \rfloor$$

$$R^2 \text{ value} = 1-(RSS/TSS)$$

$$RMSE = \sqrt{RSS/(n-p-1)}$$

$$AIC = n * \log(RSS/n) + 2p$$

$$BIC = n * \log(RSS/n) + p * (\log(n))$$

$$\text{DW statistic} = \frac{\sum_{t}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

Where RSS = residual sum of squares, n = number of test days, p = number of parameters estimated in the model, MS_E = error mean square, MS_T = total mean square, e_t is residual at time t, and e_{t-1} is residual at time t – 1.

Models with highest R^2 value, lowest RMSE, AIC and BIC values indicate the best fit of the model. The Durbin–Watson statistic explains the extent of autocorrelation among the residual values obtained in the regression analysis. DW statistic ranges from 0 to 4 wherein 0 indicates positive autocorrelation and 4 indicates negative autocorrelation, while 2 indicates the absence of autocorrelation. R-software was used to estimate the degree of correlation between the actual and predicted test day milk yields.

3. Results and discussion

Twenty average first lactation fortnightly test day milk yields recorded in Kankrej cattle were used to fit five non-linear lactation models viz., exponential decline function (EDF), gamma function (GF), inverse polynomial function (IPF), mixed log function (MLF) and parabolic exponential function (PEF), and the curve parameters were obtained. The estimated lactation curve functions or curve parameters (a, b and c) and the corresponding standard errors and different measures of goodness of fit of the models are given in Table 1. The parameters of all the five functions except "b" parameter of MLF were found to be positive. The negative parameter obtained in the present study is in accordance with the negative "b" parameter of –9.69 for MLF reported [10] in Gir crossbred cattle. Similarly, the positive parameters for other functions such as EDF, PEF, IPF and GF obtained in the study are supported by the results reported [11] in Iranian Holstein cows.

Perusal of the goodness of fit estimates obtained for different lactation curve functions revealed the range of adjusted R^2 percentage from 77.01% for EDF to 98.53% for MLF. The GF had the estimate of 98.34%, while PEF and IPF had 98.12 and 90.24%, respectively. Contrary to the wide discrimination in adjusted R^2 estimates among the different models obtained in the present study, [11] reported similar accuracy estimates of 93 and 94% for all the four models viz., EDF, GF, IPF and PEF developed in single and twin calved Holstein cows. Comparatively, lower adjusted R^2 estimates ranging from 67.9 to 89.3% were reported in primiparous Gir crossbred cows [10], which was also supported in [12]. The higher adjusted R^2 values obtained in the present study indicate higher accuracy of the models fitted in Kankrej cattle. RMSE estimates of the functions revealed the highest estimate of 0.8788 for EDF and the lowest value of 0.2218 for MLF. As expected, the models with the lowest adjusted R^2 value had the highest RMSE value. The AIC and BIC estimates of all the models also showed similar trend to the RMSE estimates. The BIC ranged from –55.7145 for MLF to –0.6437 for EDF, and AIC ranged from 5.7606 for MLF and 8.5142 for EDF signifying the superiority of MLF in fitting the first lactation test day yields in Kankrej cattle. The Durbin–

Table 1. Lactation curve parameters and goodness of fit estimated by different functions for the prediction of first lactation fortnightly test day yields in Kankrej cattle.

No.	Functions	Parameters of functions (SE)			Goodness of fit				
		a	b	c	Adjusted R^2	RMSE	AIC	BIC	DW
1.	EDF	10.6134 (0.4475)	-	0.0341 (0.0042)	77.01	0.8788	8.5142	−0.6437	0.2297
2.	PEF	8.5871 (0.1825)	0.0333 (0.0052)	0.0036 (0.0003)	98.12	0.2510	6.0081	−50.7645	0.6686
3.	IPF	0.0688 (0.0143)	0.0568 (0.0072)	0.0066 (0.0006)	90.24	0.5726	7.6574	−17.7787	0.4712
4.	GF	8.6014 (0.1821)	0.3216 (0.0233)	0.0798 (0.0035)	98.34	0.2362	5.8863	−53.2020	0.6667
5	MLF	13.6475 (0.1771)	−5.7312 (0.2346)	5.4682 (0.2952)	98.53	0.2218	5.7606	−55.7145	0.9332

EDF: Exponential decline function; PEF: Parabolic exponential function; IPF: Inverse polynomial function; GF: Gamma function; MLF: Mixed log function; MSE: Mean square error; RMSE: Root mean square error; AIC: Akaike's information criteria; BIC: Bayesian Information Criteria; DW: Durbin–Watson statistic.

Watson statistic estimates for all the five models fitted were positive ranging from 0.2297 for EDF to 0.0.9332 for MLF. The positive DW estimates signify the chances of positive autocorrelation among the residual estimates, which may result in positive error in the predictions using the models developed in the study. The highest R^2 value (98.53%) and lowest RMSE (0.2218), AIC (5.7606) and BIC (-55.7145) values indicated that the mixed log function (MLF) was the best non-linear mathematical function for explaining the variations in the first lactation milk yield of Kankrej cattle.

The actual average first lactation fortnightly milk yield and the yields predicted by different lactation curve functions are presented in Table 2 and depicted in Figure 1. Perusal of the results revealed the actual average peak yield of 9.560 kg observed in 5th fortnight. Similar to present findings, a peak yield of 9.72 kg of milk attained in 60 days of lactation was reported [13] in Dhofari cattle. However, [14] reported higher milk yield of 10.53 kg attained in 55.75 days in the herd of elite Kankrej cows maintained under organized farm conditions. [10] also reported slightly higher average peak yield of 10.08 kg recorded during the second month of lactation in Gir crossbred cattle. Perusal of the actual lactation curve reveals gradual increase in the test day yields during the initial phase of lactation up to the peak period, which later gradually declined up to drying indicating the persistency of first lactation milk yield in Kankrej cattle. Perusal of the lactation curve obtained in the present study showed a gradual increase in the milk yield from 8.27 kg at the first fortnight to 9.56 in the 5th fortnight followed by a fairly low rate of decrease of around 0.40 kg per fortnight up to 14th fortnight. This finding is similar to the result reported in Iranian Holstein heifers [15]. On the other hand, inconsistent lactation curve with intermittent increase and decrease in the test day yields (peak yield of 9.72 kg in 60 days, 5.11 kg in 120 days, 2.69 kg in 210 days, 3.91 kg in 270 days and 0.50 kg in 305 days of lactation) was reported in Dhofari cattle [13]. Many earlier studies [15–18] attributed the variations in shape of lactation curves to various factors such as breed, herd, parity, management conditions, health status of animal, calving season etc.

Different lactation curve models developed by various mathematical functions along with the predicted fortnightly test day yields are depicted Figure 1. Unlike the other functions, the EDF developed typical straight line showing a gradual decline in the predicted milk yield from first (10.258 kg) to last fortnight (5.367 kg). This finding is in accordance with the report of [19] who also found the poor efficiency of EDF in explaining the variations in test day yields in Rathi cattle. This poor efficiency of EDF might be attributed to the exclusion of inclining function (b) in the model. However, in the present study, MLF model explained the variations in first lactation fortnightly milk yields effectively in Kankrej cattle. Similar to the present findings, [20] also adjudged MLF as an excellent model for prediction of weekly test day milk yields in Holdeo crossbred cattle.

In the present study, five non-linear mathematical functions were used to model the first lactation milk yield

Table 2. Estimated daily milk yield and prediction errors for fortnightly test day milk yields using different functions.

Test day	Actual	EDF		PEF		IPF		GF		MLF	
		Predicted	Error	Predicted	Error	Predicted	Error	Predicted	Error	Predicted	Error
1	8.270	10.258	1.988	8.846	0.576	7.562	–0.708	7.942	–0.328	7.916	–0.354
2	9.050	9.914	0.864	9.048	–0.002	9.581	0.531	9.163	0.113	9.333	0.283
3	9.430	9.582	0.152	9.190	–0.240	10.054	0.624	9.638	0.208	9.728	0.298
4	9.500	9.261	–0.239	9.268	–0.232	9.972	0.472	9.761	0.261	9.766	0.266
5	9.560	8.950	–0.610	9.280	–0.280	9.671	0.111	9.683	0.123	9.633	0.073
6	9.410	8.650	–0.760	9.227	–0.183	9.288	–0.122	9.480	0.070	9.407	–0.003
7	9.250	8.360	–0.890	9.109	–0.141	8.882	–0.368	9.197	–0.053	9.125	–0.125
8	9.030	8.080	–0.950	8.929	–0.101	8.480	–0.550	8.864	–0.166	8.808	–0.222
9	8.610	7.809	–0.801	8.691	0.081	8.095	–0.515	8.500	–0.110	8.469	–-0.141
10	8.220	7.548	–0.672	8.399	0.179	7.733	–0.487	8.118	–0.102	8.115	–0.105
11	7.830	7.295	–0.535	8.060	0.230	7.393	–0.437	7.729	–0.101	7.751	–0.079
12	7.440	7.050	–0.390	7.680	0.240	7.077	–0.363	7.338	–0.102	7.382	–0.058
13	7.010	6.814	–0.196	7.266	0.256	6.783	–0.227	6.952	–0.058	7.009	–0.001
14	6.610	6.586	–0.024	6.825	0.215	6.510	–0.100	6.573	–0.037	6.634	0.024
15	6.420	6.365	–0.055	6.367	–0.053	6.256	–0.164	6.205	–0.215	6.259	–0.161
16	5.980	6.151	0.171	5.896	–0.084	6.020	0.040	5.849	–0.131	5.884	–0.096
17	5.720	5.945	0.225	5.422	–0.298	5.800	0.080	5.507	–0.213	5.510	–0.210
18	5.100	5.746	0.646	4.951	–0.149	5.594	0.494	5.179	0.079	5.137	0.037
19	4.660	5.553	0.893	4.489	–0.171	5.402	0.742	4.865	0.205	4.767	0.107
20	3.930	5.367	1.437	4.041	0.111	5.222	1.292	4.567	0.637	4.398	0.468

Actual: Actual observed test day yields; EDF: Exponential decline function; PEF: Parabolic exponential function; IPF: Inverse polynomial function; GF: Gamma function; MLF: Mixed log function.

of Kankrej cows using 20 fortnightly test day yields. The actual and predicted yields along with their deviations calculated as prediction errors are given in Table 2. Among the different models, the predicted test day yields were overestimated during the first (ascending) and third (declining) stages of lactation while underestimated in the second phase of lactation by the EDF. This poor efficiency of prediction by EDF might be due its inherent problem of describing the second phase, i.e. the stagnation phase of lactation. This model also produced a consistently declining lactation curve in the form of a straight line. On the other hand, MLF had the lowest prediction errors among the functions modelled indicating its superiority in explaining the variations present in the fortnightly milk yields of Kankrej cows. Similar to the present findings, [21] also reported that the MLF was the best fitted lactation curve model in Sahiwal cattle. The result that exponential decline function was the least efficient in fitting the lactation curve for first lactation milk yield is also supported by the results of [22] in crossbred dairy cattle. Contrary to the present finding, [19] reported that the PEF proposed by [5] provided a good fit for the curves of the first lactation cows. In the present study, three functions viz., MLF, GF, and PEF, respectively, had almost similar efficiency of fitting. The residual error (kg) estimates of different fortnightly milk yields were the highest for the EDF followed by IPF, while GF, MLF and PEF had comparatively lower and more or less similar prediction errors. The peak yield of 9.761 kg predicted by GF was closest to the actual peak of 9.560 kg observed in the present study. MLF had the lowest prediction errors compared to other models as the data points plotted were close to the X-axis, while GF and PEF had more or less similar data points indicating the similarity in their prediction efficiency (Figure 2).

Figure 3 depicts the Pearson's product moment correlation coefficients among actual and predicted daily milk yields. All the correlation estimates were positive and high ranging from 0.89 to 0.99. In general, all the actual and predicted test day yields had highly significant correlations ($p < 0.001$); however, EDF had the lowest correlation with

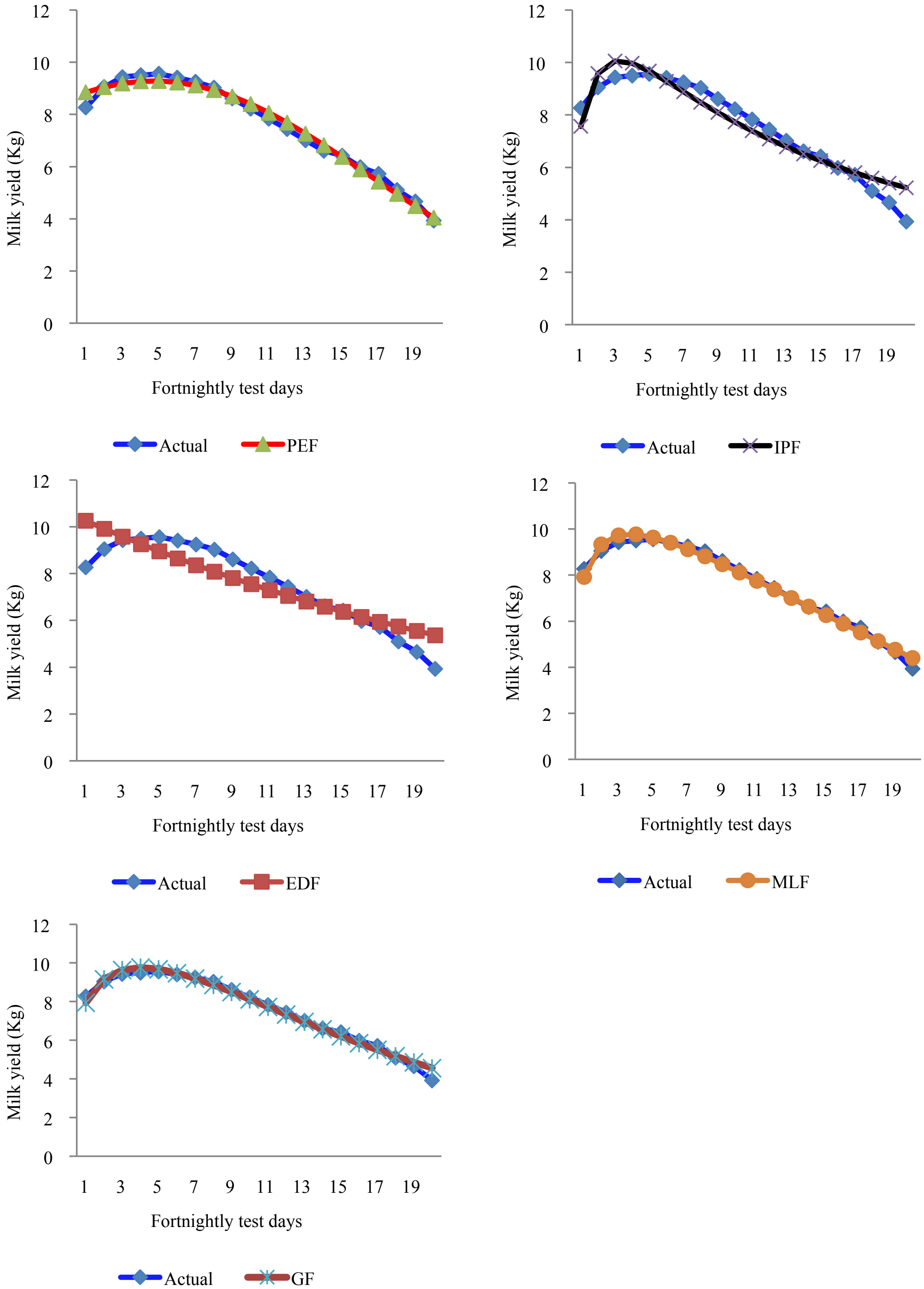

Figure 1. Actual and predicted fortnightly test day milk yields for different lactation curve functions in Kankrej cattle. EDF: Exponential decline function; PEF: Parabolic exponential function; GF: Gamma function; MLF: Mixed log function; IPF: Inverse polynomial function.

the actual yield and yields predicted by other methods, reflecting its comparatively lower efficiency in explaining the variations in the first lactation milk yield of Kankrej cattle.

4. Conclusion

The highest R^2 value (98.53%) and lowest RMSE (0.2218), AIC (5.7606) and BIC (–55.7145) values indicated that the mixed log function (MLF) was the best non-linear

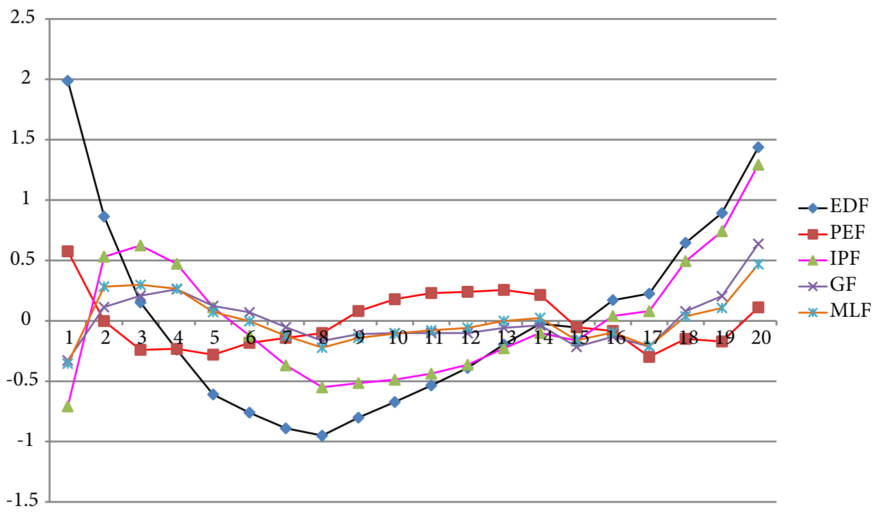

Figure 2. Residuals (kg) of predicted WTDMY for different functions. EDF: Exponential decline function; PEF: Parabolic exponential function; GF: Gamma function; MLF: Mixed log function; IPF: Inverse polynomial function.

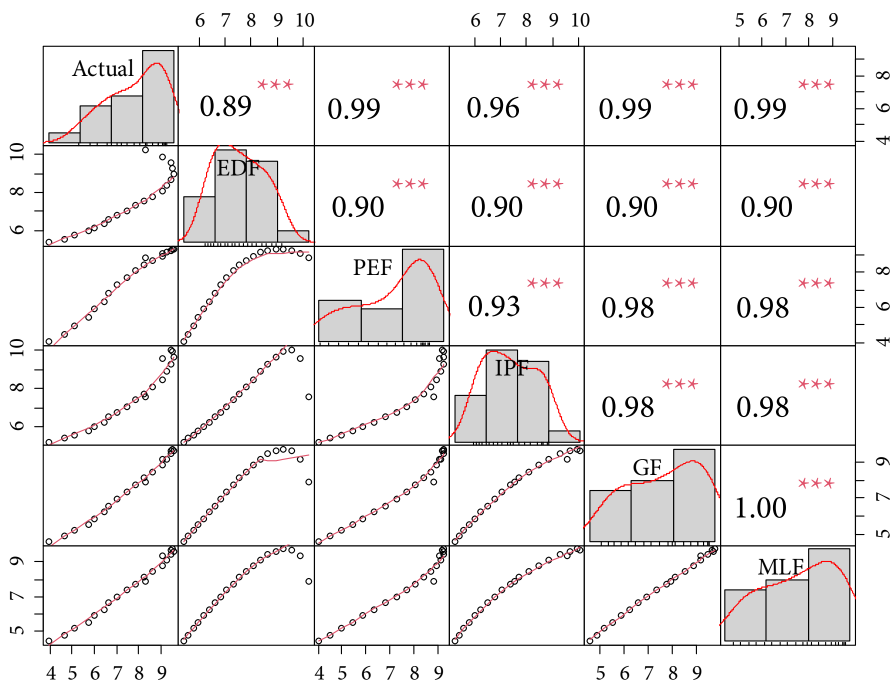

Figure 3. Correlation coefficients between actual observed daily milk yields and milk yields predicted by different functions in Kankrej cattle.

mathematical function for explaining the variations in the first lactation milk yield of Kankrej cattle followed by gamma function (GF) and parabolic exponential function (PEF). The exponential decline function (EDF) was the least efficient in fitting the lactation curve in Kankrej cattle followed by inverse polynomial function (IPF).

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