

Estimating wetting front coordinates under surface trickle irrigation

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Received: 28.02.2012 • Accepted: 30.04.2012

Abstract: In this study, wetting front or wetted bulb coordinates in soil under surface trickle irrigation were measured for 1 loam soil and 2 sandy loam soils with 2 different emitter discharges of 2 and 4 L h⁻¹ by using the trenching method. A model is presented for estimating wetted bulb coordinates with a function of emitter discharge, water application time, average variation in volumetric water content, and saturated hydraulic conductivity of soil. For calculating the distance of the maximum wet surface, relationships are presented based on saturated hydraulic conductivity and water application time. By comparison of measured values of wetting front coordinates, the presented model shows good reliability. The goodness of fit ratio and root mean square error of the model were 0.82 and 17.85 mm, respectively. The model for predicting surface trickle irrigation wetting front coordinates can be applicable for the emitter with 2 and 4 L h⁻¹ discharges.

Key words: Incomplete sphere, saturated hydraulic conductivity, wetted bulb, wetting front coordinates

Introduction

Trickle irrigation is considered to be an appropriate method for areas of limited water resources due to high efficiency of water use under good management. For design and management of a trickle irrigation system, the shape of the wetted bulb should be known, and it can be predicted by solving numerical equations governing flow (Bristow et al. 2000). One of the basic factors in the design of trickle irrigation systems is the availability of information about soil texture (Philip 1984; Cote et al. 2003). To increase water use and nutrition efficiencies in trickle irrigation, there should be uniformity between emitter distances, emitter discharges,

and soil moisture profile, as well as the duration of water application (Thorburn et al. 2003b). Sezen et al. (2006) found that having information about the wetted bulb of soil is necessary for trickle irrigation system design. One effective method to optimize trickle irrigation system design is using numerical simulation, resulting in moisture distribution in the soil (Schmitz et al. 2002). Thorburn et al. (2003a) used the equation of Philip (1984) to obtain wetted bulb dimensions in surface and subsurface trickle irrigation. Sepaskhah and Chitsaz (2004) studied the analysis of Green and Ampt (1911) to determine the wet radius and depth of surface trickle irrigation. Lazarovitch et al. (2007) studied the characteristics

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of wetted soil volume under surface and subsurface trickle irrigation. Researchers of the estimation of the dimensions of the wetted bulb in a number of methods have used Richards' equation from 1931. This equation requires many inputs (Chu 1994).

The purpose of this study was to provide a new and simple model considering the wetted bulb under conditions of an incomplete sphere. The estimation of wetting front coordinates will be possible with minimum soil hydraulic parameters in the presented model.

Materials and methods

Field experiments were performed in 3 different locations of the Tabriz suburbs, namely Khalatpooshan, Arpadarasi, and Karkaj. Physical properties of the soils are presented in Table 1.

Experiments

Several emitters with spacing of 1 m were installed on lateral pipes of 16 mm in diameter on the experimental soils. These laterals were connected with a 200-L water reservoir through a main line. To decrease the turbulence in the reservoir, the water was first conveyed into a preliminary reservoir before entering the main reservoir. The main reservoir was equipped with a spillway to supply a constant head in emitters during the tests (Figure 1). The emitter discharges were set at 2 rates of 2 and 4 L h⁻¹. The volumes of applied water during each test by an emitter were, in total, 4 or 8 L. Coordinates of the wetting front for different times were measured from emission points by trenching.

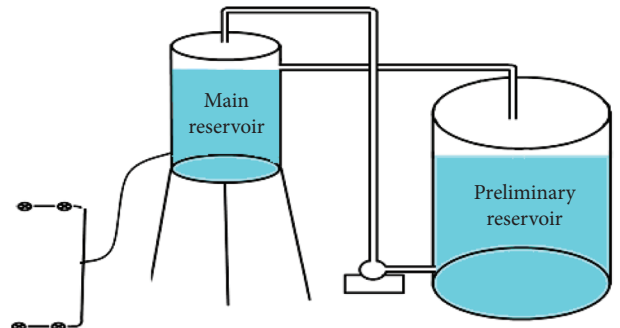


Figure 1. The layout of the experimental setup.

Theory

If the wetted bulb shape resulting from an emitter according to Figure 2 is considered in the form of an incomplete sphere, noting triangles ABO and ABC, we can express the following relationships.

$$\text{Cos}(\varphi) = \frac{R \cdot \text{Sin}(\beta) - d}{R_{\text{max}}^h} \tag{1}$$

$$R^2 = H^2 + Rh_{\text{max}}^2 \text{Sin}^2(\varphi) \tag{2}$$

$$R^2 = d^2 + Rh_{\text{max}}^2 \text{Cos}^2(\varphi) + 2d \cdot Rh_{\text{max}}^2 \text{Cos}(\varphi) + Rh_{\text{max}}^2 \text{Sin}^2(\varphi) \tag{3}$$

$$R^2 = d^2 + Rh_{\text{max}}^2 + 2d \cdot Rh_{\text{max}}^2 \text{Cos}(\varphi) \tag{4}$$

By combining Eqs. (1) and (4), the direct relationships between β and R will be as follows:

$$R^2 = Rh_{\text{max}}^2 + 2d \cdot R \text{Sin}(\beta) - d^2, \tag{5}$$

Table 1. Physical properties of experimental soils.

Experiment sites	Sand (%)	Clay (%)	Silt (%)	Soil texture	θ _s [*] (%)	θ ₀ [*] (%)	ρ _b [*] (g cm ⁻³)	Ks (m day ⁻¹)
Khalatpooshan	70	12	18	Sandy loam	38	10	1.62	0.3878
Arpadarasi	44	24	32	Loam	43.5	14.2	1.53	0.1874
Karkaj	71	8	21	Sandy loam	37	7	1.58	0.5106

*ρ_b = bulk density (g cm⁻³); θ₀ and θ_s = initial and saturated soil water content [L³ L⁻³], respectively.

or:

$$R = (Rh_{\max}^2 - d^2 \cos^2(\beta))^{1/2} + \sin(\beta), \quad (6)$$

where R is the radial distance of the wetting front (m), d is the distance of the maximum wetted width to ground surface (m), Rh_{\max} is maximum wetted width (m), and β is the angle between the soil surface and any radial distance R .

Eq. (6) presents the relation of R with the angle between the radial distance and the soil surface (β), the maximum wetted width, and its position toward the ground (d). To use Eq. (6) for estimation of the wetting front coordinates of the wetted bulb, Rh_{\max} and d should be known. If the wetted bulb resulting from the surface emitter is considered to form an incomplete sphere (Figure 2), the wetted bulb volume can be calculated by:

$$V = \pi(4 Rh_{\max}^3 - (Rh_{\max} - d)^2(2Rh_{\max} + d))/3, \quad (7)$$

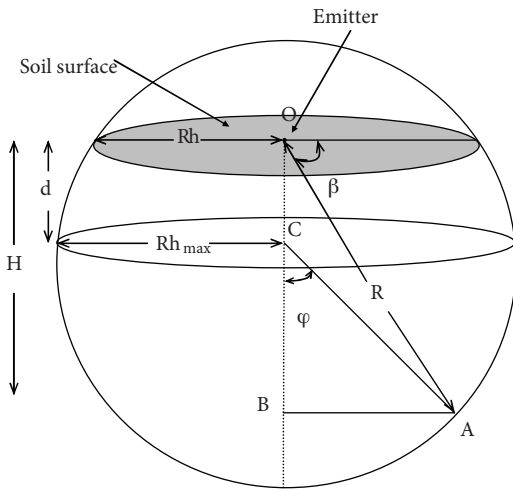


Figure 2. General hypothetical wetted bulb for incomplete sphere for constant surface emitter discharge

where V is the volume of the wetted bulb. Assuming that the average soil moisture content after irrigation of the θ_0 in θ_v is increased, we have:

$$\pi(4Rh_{\max}^3 - (Rh_{\max} - d)^2(2Rh_{\max} + d))/3 = \frac{Qt}{\Delta\theta}, \quad (8)$$

where $\Delta\theta$ is the average change in volumetric water content in the soil ($L^3 L^{-3}$).

From Eq. (8), Rh_{\max} is obtained as follows:

$$Rh_{\max} = -d/2 = \frac{d^2}{\lambda^{1/3}} + 1/2\lambda^{1/3}. \quad (9)$$

In this equation:

$$\lambda = \frac{1.91Qt}{\Delta\theta} + d^3 + \quad (10)$$

$$2\left(0.9128\left(\frac{Qt}{\Delta\theta}\right)^2 + \frac{0.955Qd^3t}{\Delta\theta}\right)^{1/2}.$$

As is evident from Eqs. (9) and (10), Rh_{\max} is the function of flow rate (Q), $\Delta\theta$, d , and the time of water application (t):

$$Rh_{\max} = f(\Delta\theta, d, t, Q). \quad (11)$$

Parameter d can be estimated for each discharge based on the following relationship:

$$d = \alpha k_s^\sigma t^\gamma \quad (12)$$

where d , k_s , and t have already been defined, and α , γ , and σ are coefficients of the equation. By combining Eqs. (6), (9), and (10) and with consideration of Eq. (12), the following model can be used to calculate the radial distance of the wetting front from the center emitter at any time t of the start of irrigation:

$$R = d_{(t,k_s)} \sin(\beta) + \left(\frac{-d_{(t,k_s)}/2 + \frac{d_{(t,k_s)}^2}{\left(\frac{1.91Qt}{\Delta\theta} + d_{(t,k_s)}^3 + 2\left(0.9128\left(\frac{Qt}{\Delta\theta}\right)^2 + \frac{0.955tQd_{(t,k_s)}^3}{\Delta\theta}\right)^{1/2}\right)^{1/3}}}{1/2\left(\frac{1.91Qt}{\Delta\theta} + d_{(t,k_s)}^3 + 2\left(0.9128\left(\frac{Qt}{\Delta\theta}\right)^2 + \frac{0.955tQd_{(t,k_s)}^3}{\Delta\theta}\right)^{1/2}\right)^{1/3}} \right)^2 - d_{(t,k_s)}^2 \cos^2(\beta) \right)^{1/2}, \quad (13)$$

where $\Delta\theta$ is obtained as (Ben-Asher et al. 1986):

$$\Delta\theta = \frac{\theta_s}{2}, \tag{14}$$

where θ_s is the saturated volumetric water content of the soil. Other parameters have been described previously. Using the above equation at any angle β from the ground, the radial distance of the wetting front and the center emitter can be estimated, and using , wetting front coordinates can be estimated. It is clear that with regard to $\beta = 0^\circ$ and $\beta = 90^\circ$, Eq. (13) will yield the vertical and horizontal advances, respectively.

To evaluate the model, the root mean square error (RMSE) and goodness of fit ratio (R^2) were used as follows:

$$RMSE = \left(\sum_{i=1}^n (E_i - M_i)^2 / n \right)^{1/2}, \tag{15}$$

$$R^2 = \left(\sum_{i=1}^n (M_i - \bar{M})^2 - \sum_{i=1}^n (E_i - \bar{M})^2 \right) / \left(\sum_{i=1}^n (M_i - \bar{M})^2 \right)^{-1}, \tag{16}$$

where \bar{M} is the average of the measured values of the wetting front radial distance, and M and E are the measured and calculated radial distances of the wetting front, respectively.

Model sensitivity analysis

We assume that each variable has different effects on the model results; therefore, it is necessary to assess the effects of parameters t , Q , d , and $\Delta\theta$ using sensitivity analysis before using the model. To do this, a base case was considered such that the radial distances at different values of angle β were estimated using the presented model and were then compared with those resulting from changes in the quantity of the parameters in 16 modes. To analyze the results of changes in input parameters from those of the base case, the RMSE was used as in Eq. (15), where M and E are estimated values of the radial distance from the model in the base case and the mode to increase or decrease the amount of input parameters, respectively.

Results

As was mentioned in theory, based on saturated hydraulic conductivity and water application time for flow rates of 2 and 4 L h⁻¹, for calculation of the position of the maximum wet width of the ground surface, the empirical relations are as follows.

$$d = 4.5327k_s^{0.03765} t^{0.5187}, \quad Q = 2L \text{ h}^{-1} \quad R^2 = 0.93 \tag{17}$$

$$d = 1.9038k_s^{-0.1282} t^{0.58248}, \quad Q = 4L \text{ h}^{-1} \quad R^2 = 0.933 \tag{18}$$

Here, d is the position of the maximum wet width of the ground (mm), k_s is the saturated hydraulic conductivity (mm s⁻¹), and t is the water application time (min). The presented model sensitivity analysis was performed for the change in parameters, and also in a base case with values $t = 120$ min, $Q = 2$ L h⁻¹, $d = 44$ mm, and $\Delta\theta = 0.2$. Results are available in Figure 3 and Table 2.

Table 2 and Figure 3 show that the presented model has less sensitivity to parameter d compared to other parameters. High model sensitivity in the mode for reducing quantities is related to parameter $\Delta\theta$, and the mode for increasing quantities is equally more effective in 2 parameters, Q and t .

Measured values of wetting front coordinates for 1 loamy (Arpadarasi region) and 2 sandy loam (Khalatpooshan and Karkaj regions) soils with 2 flow rates of 2 and 4 L h⁻¹ were compared with those of the presented model from Eq. (13). The results are presented in Tables 3 and 4 and Figures 4–6.

The graphs in Figure 4 show, for the sandy loam soil of Karkaj region, that the measured values of radial distance and those of the presented model are in good agreement with all applied water volumes in mode $Q = 2$ and 4 L h⁻¹.

In mode $Q = 2$ L h⁻¹ and $t = 240$ min, agreement between the measured values of the wetting front and the estimated values is high; the RMSE and R^2 values were estimated to be 11.506 mm and 0.976, respectively.

Considering the graphs in Figure 5 (loamy soil), it is evident that for $Q = 2$ L h⁻¹ with applications of both 4 and 8 L, the values of the presented model are

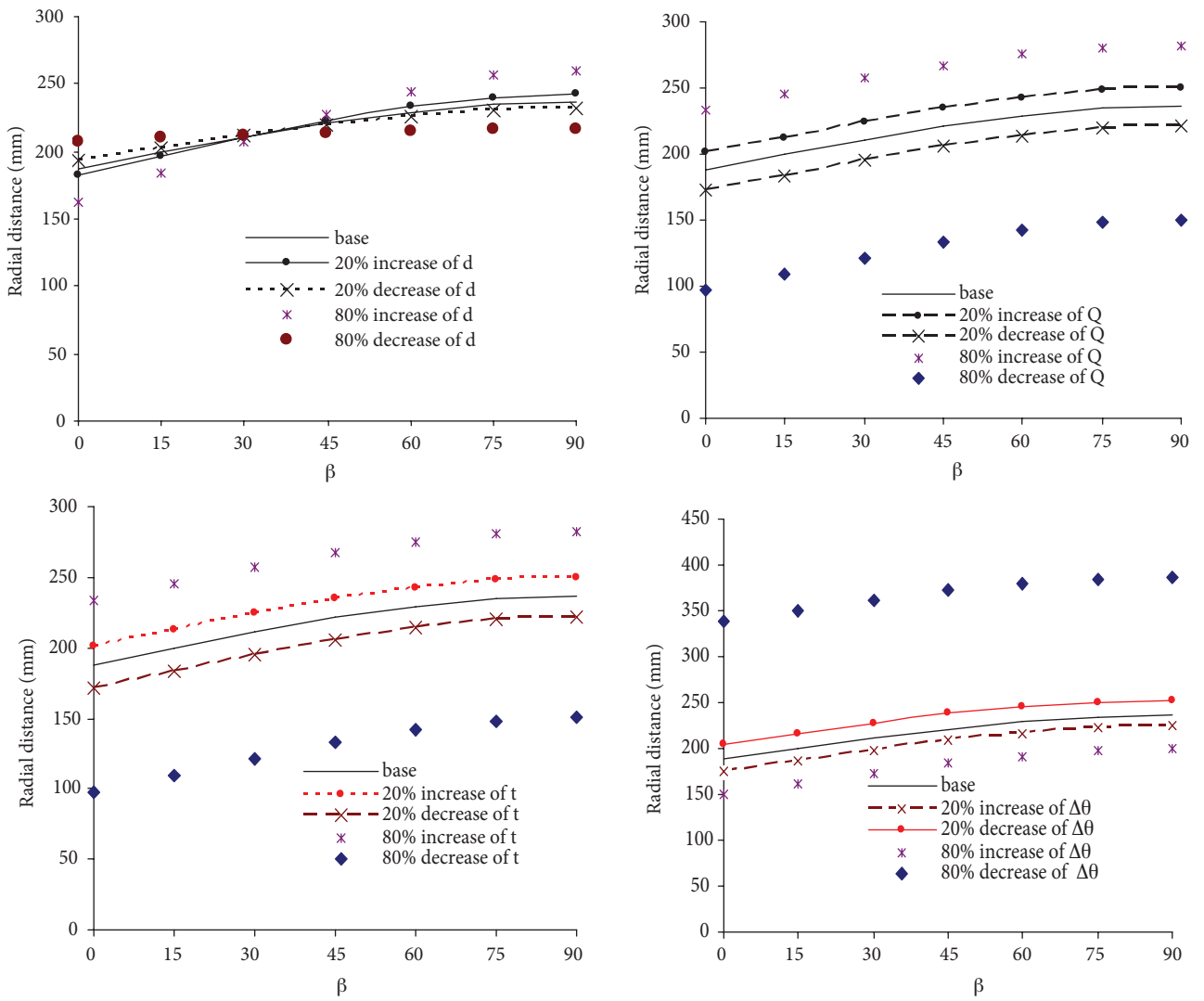


Figure 3. Radial distances resulting from base mode of presented model and different modes of the parameters.

Table 2. RMSE of model for different modes of parameters.

Parameters	20% increase	20% decrease	80% increase	80% decrease	In all 4 modes
d	4.08	3.91	17.54	3.91	17.54
Q	13.31	15.23	45.97	88.38	50.83
$\Delta\theta$	12.53	16.4	37.82	150.73	78.38
t	13.31	15.23	45.97	88.38	50.83

Table 3. R² and RMSE for Q = 4 L h⁻¹ of model for sandy loam soil (Khalatpooshan).

T (min)	Point	1	2	3	4	5	6	7	RMSE (mm)	R ²
120	Measured	240	232.43	250.59	240.83	266.27	291.10	300	22.75	0.71
	Presented model	235.9	258.93	268.38	281.11	293.66	304.17	307		
60	Measured	200	213.77	206.15	209.34	208.08	199.24	230	18.17	0.13
	Presented model	192.4	198.34	207.23	220.73	228.43	236.54	239		
60, 120	Presented model								20.59	0.818

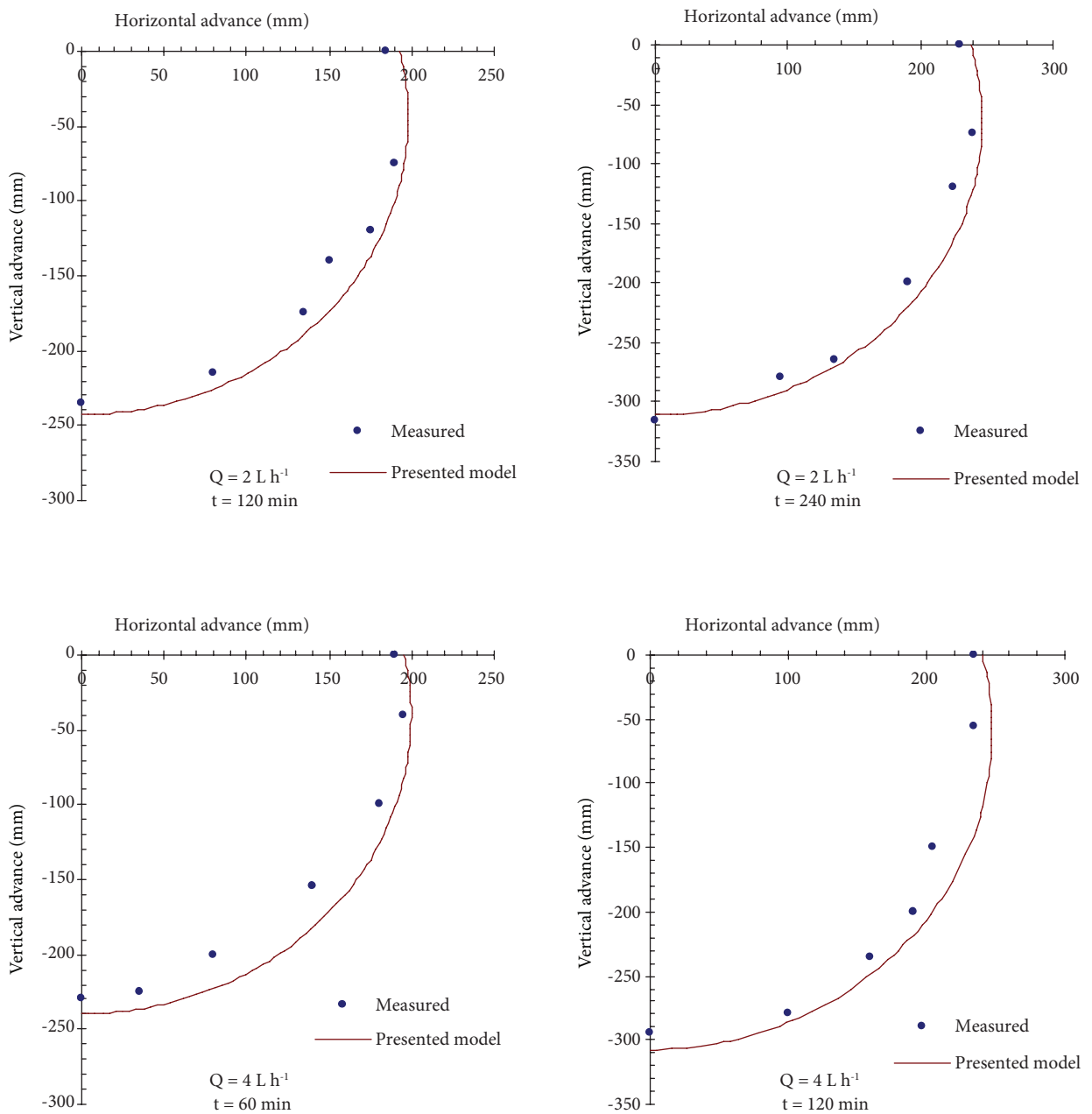


Figure 4. Measured and calculated wetting fronts in sandy loam soil (Karkaj region).

Table 4. R² and RMSE values of presented model.

Emitter discharge (L h ⁻¹)	Sandy loam (Karkaj)		Sandy loam (Khalatpooshan)		Loam		In all 3 soils	
	R ²	RMSE (mm)	R ²	RMSE (mm)	R ²	RMSE (mm)	R ²	RMSE (mm)
2	0.982	9.66	0.931	10.43	0.879	13.25	0.921	11.22
4	0.976	12.07	0.818	20.59	0.475	30.52	0.787	22.39
2,4	0.976	11.506	0.836	16.89	0.65	23.53	0.82	17.85

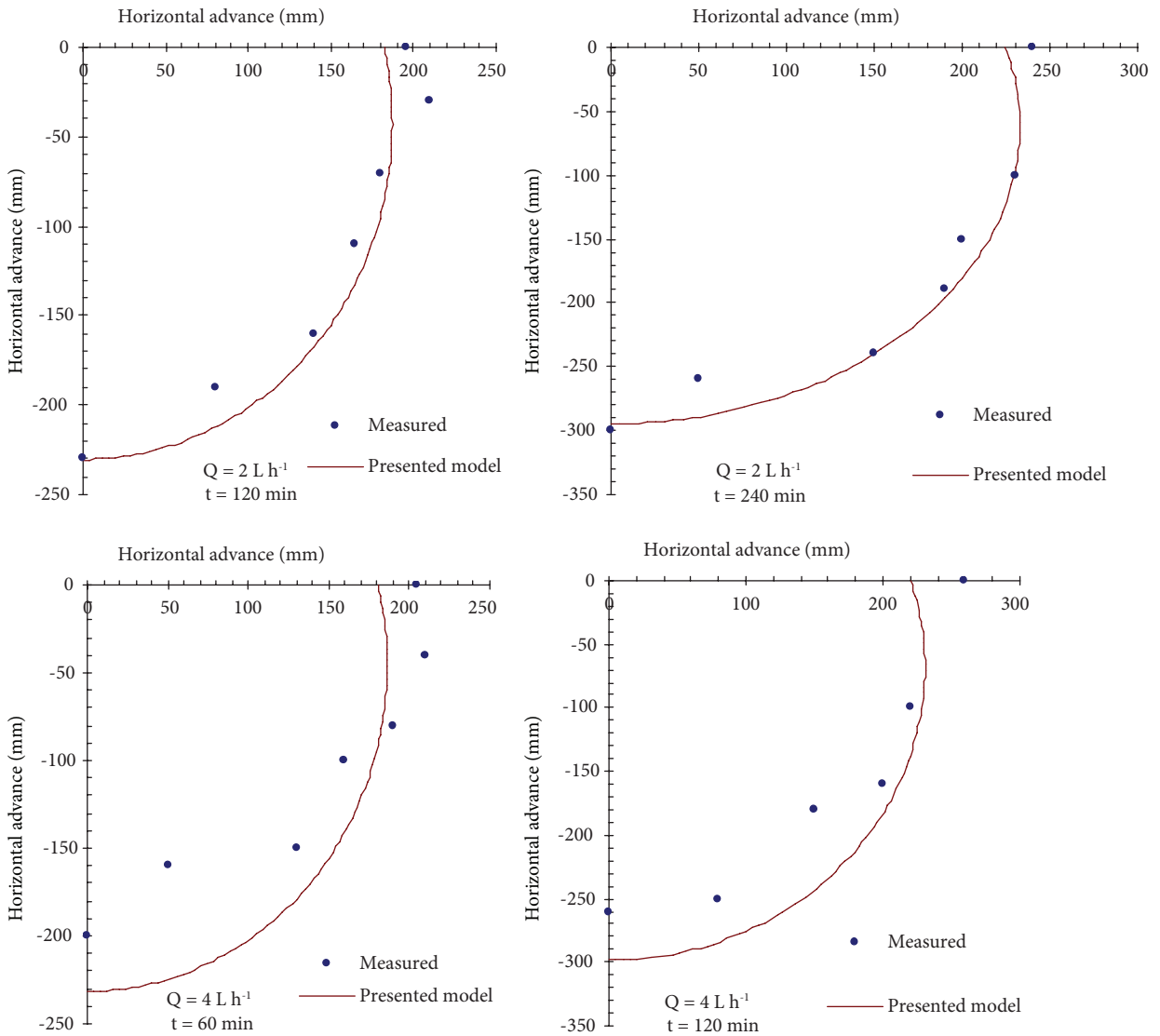


Figure 5. Measured and calculated wetting fronts in loam soil.

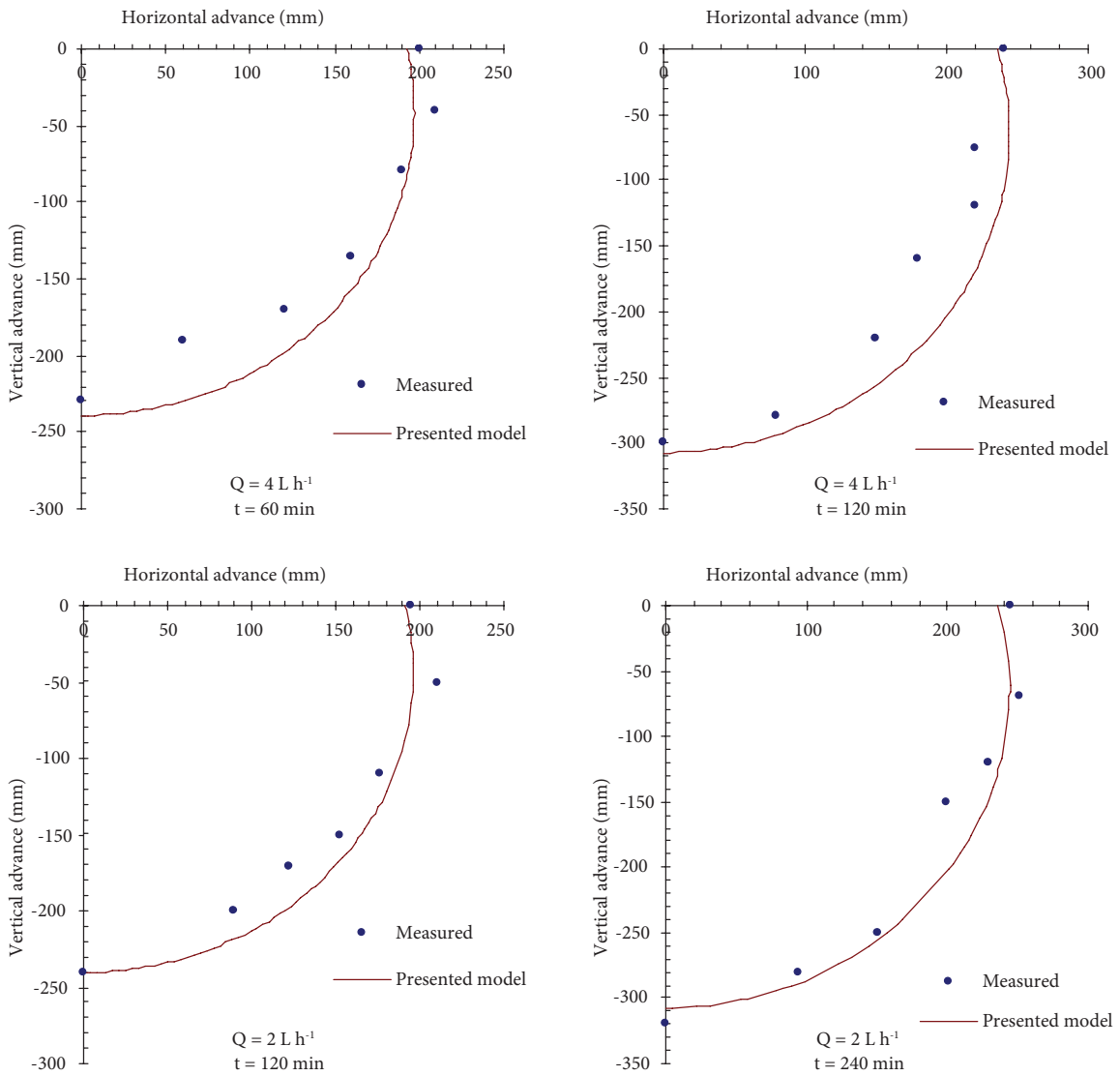


Figure 6. Measured and calculated wetting fronts in sandy loam soil (Khalatpooshan region).

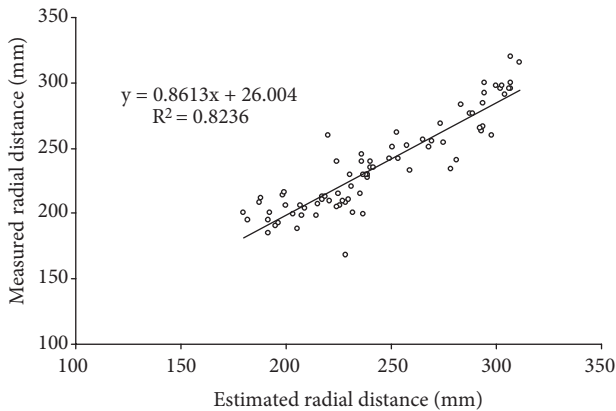


Figure 7. Comparison of measured and estimated radial distances of wetting fronts in all 3 soils.

in high accordance with the measured data of the wetting front coordinates. However, for $Q = 4 \text{ L h}^{-1}$, the accordance is not as good as in the application mode of $Q = 2 \text{ L h}^{-1}$. In application mode $Q = 4 \text{ L h}^{-1}$ with 4 L of volume, less matching is seen, and the R^2 and RMSE of the model in this soil are 0.65 and 23.53 mm, respectively.

Results in the sandy loam soil (Khalatpooshan region) show good agreement between the measured data of the wetting front coordinates and those of the presented model for $Q = 2$ and 4 L h^{-1} for both application volumes of 4 and 8 L. In the application mode of $Q = 2 \text{ L h}^{-1}$ with a volume of 8 L, there is

particularly high accordance and R^2 and RMSE are estimated to be 0.836 and 16.89 mm, respectively. The advantage of using the presented model as compared with other methods such as those of Schwartzman and Zur (1986), Li et al. (2004), Thabet and Zayani (2008), and Amin and Ekhmaj (2006) is that the presented model is a geophysical model that anticipates the profile of the wetting front, while the other mentioned methods are often experimental and can only estimate the width and depth of the wetting front.

Discussion

Results of these 3 soils showed that the measured values of wetting front coordinates for both $Q = 2$ and 4 L h^{-1} and for both 4 and 8 L of water volume

had very good accordance with those of the presented model.

The R^2 and RMSE values of the model for these soil types were estimated to be 0.82 and 17.85 mm, respectively (Figure 7). The results in Table 2 show that the presented model in the sandy loam soils (Karkaj and Khalatpooshan) has higher R^2 and lower RMSE values than in the loamy soil (Arpadarasi).

Model inputs are the average change in volumetric water content, soil saturated hydraulic conductivity, flow rate of emitter, and water application time.

The present high accordance and ease of the presented model are recommended for estimating the coordinates of the wetting front in surface trickle irrigation for sandy loam and loamy soils for an emitter with 2 and 4 L h^{-1} discharges.

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