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Research Article

Estimating the soil water retention curve from soil particle size distribution using the Arya and Paris model for Iranian soils

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Abstract: The direct measurement of the soil water retention curve (SWRC) in a laboratory is time-consuming, difficult, and costly. Thus, many attempts have been made to predict the water retention curve indirectly from the physical and chemical properties of soil. The particle size distribution curve is one of the indirect methods used to predict the water retention curve. The Arya and Paris (AP) model predicts the soil water retention curve from soil particle size distribution (PSD) data. The AP model estimates pore radius from the radius of spherical particles by using a scaling parameter (α). The objective of this study was to investigate the effect of predicting the scaling parameter with different methods to improve estimation of the SWRC. The evaluation of methods was done on 35 soil samples with different textures from the eastern region of Guilan Province in Iran. The results showed that estimated curves with different α values gave different results that depended highly on the scaling parameter. Therefore, α determination has a key role in estimating the soil water retention curve. The results also showed that a linear α and a constant α with a maximum coefficient of determination and minimum error are the best scaling parameters to estimate the SWRC.

Key words: Arya and Paris model, bulk density, particle size distribution, scaling parameter, soil water retention curve

Introduction

The soil water retention characteristic is an important property of soil and it is needed for the study of plant available water, infiltration, and solute movement. However, the high variability and the complexity of soil make direct determination of the soil water retention characteristic costly and time-consuming. Therefore, an alternative to measurement is to estimate this property indirectly using more easily available information, such as particle size distribution, bulk density, and organic matter content (*Zhuang* et al. 2001).

The Arya and Paris (1981) model is an indirect method used to estimate the soil water retention curve (SWRC) from soil particle size distribution (PSD) data. Arya and Paris presented a physicoempirical approach combining physical hypotheses with empirical representation. This approach is based mainly on the similarity between the shapes of the cumulative particle size distribution and the water retention curve. The Arya and Paris (AP) model treats the water flow paths in a soil as a bundle of capillary tubes and assumes that the size of the soil particles is related to the corresponding pore diameters of the capillary tubes. The capillary volume is taken to be a function of particle size, mass fraction of the particle size, and scaling parameter α. The AP model (1981) assumed that the scaling parameter (α)

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was a constant, whereas other researchers proposed alternative formulations for the calibration of this parameter. A value of α = 0.938 was proposed by Arya and Dierolf (1992). Similar models were later proposed by Haverkamp and Parlange (1986). Later investigations by Arya et al. (1982) showed that the average α varied among textural classes and ranged in value from 1.1 for finer textures to 2.5 for coarse-textured materials. A similar range of values was reported by Tyler and Wheatcraft (1989) for the fractal dimension. Tyler and Wheatcraft (1989) showed that the parameter α is equivalent to the fractal dimension of a tortuous fractal pore. The fractal dimension of the particle size distribution can be easily measured and related to the α parameter of the AP model. Several researchers (Schuh et al. 1988; Basil and D'Urso 1997; Nimmo 1997) have suggested that predictions of water retention curves would improve if α were formulated such that it varied over the range of particle sizes.

Arya et al. (1999) proposed alternative formulations for the AP scaling parameter (logistic method, linear method). Rezaei et al. (2004) presented a formulation for the estimation of α that estimated *a* and *b* indexes of the linear equation of the Arya et al. (1999) method. Vaz et al. (2005) evaluated the performance of the AP model using 3 constant α values (1.38, 0.938, and 0.977) and an expression for α as a function of the soil water content, (θ). Xu (2004) calculated SWRC and a hydraulic conductivity function from the fractal model for pore size distribution. Comparisons between the experimental results and the predictions of both the fractal model and the van Genuchten-Mualem model were performed and it was found that the predictions of the fractal model were better than those of the van Genuchten-Mualem model. Millan and Gonzalez (2005) used a fractal model to estimate SWRC. Almost all of the soil water retention data showed 2 fractal scaling regimes. The fit of a classical surface fractal model rendered poor results in terms of goodness of fit parameters with a large dispersion of predicted water content values at low tensions. Nimmo et al. (2007) developed alternative versions of the AP model that eliminate its interval size dependence and other problems. Nasta et al. (2009) explored the prediction of soil water retention and its variability from to soil texture and bulk density measurement, using a physically based scaling technique. They showed that the use of soil texture and scaling of the PSD curves using the AP model provided for adequate characterization of the mean and variance of SWRC, thus characterizing the soil's spatial variability. They also found that prior separation of soil textural classes provided for better scaling results with the different soil groups.

This study focuses on soil water retention and evaluates the AP prediction model to derive the scaling behavior of the soil water retention curve.

Materials and methods

Bulk density (Klute 1986), particle size distribution (by hydrometer method) (Klute 1986), saturated water content (θ_s) (Page et al. 1982), and organic carbon content (OC) (Page et al. 1982) of 35 soil samples were measured using standard techniques. The SWRC was obtained at 30, 100, 300, 600, 1000, and 1500 kPa. Basic statistical properties are presented in Table 1.

The AP model predicts the SWRC from particle size distribution and bulk density. The particle size distribution curve was divided into 20 fractions according to the method used by Arya and Paris (1981), with fraction boundaries at particle diameters of 1, 2, 3, 5, 10, 20, 30, 40, 50, 70, 100, 150, 200, 300, 400, 600, 800, 1000, 1500, and 2000 µm. In each fraction, the solid mass was assembled to form a hypothetical, cubic close-packed structure consisting of uniform-sized spherical particles. The pore volume in each assemblage was calculated from the bulk density and particle density measured on the natural structure soil. The water content was obtained from successive summations of water-filled pore volumes. The pore radius was related to the particle radius. Calculated pore radii were converted to equivalent pressure heads.

Basic relationships in the AP model are described by the following equations.

$$V_{vi} = \left[\frac{w_i}{r_s}\right] e = pr_i^2 l_i \quad [1]$$

$$e = (\rho_s - \rho_b)/\rho_b \quad [2]$$

$$\theta_i = \rho_b \sum_{j=1}^{i} V_{vj} \quad [3]$$

Table 1. Mean (M), maximum (max), and minimum (min) values of bulk density (ρ_b), saturated water content (θ_s), organic carbon content (OC), and soil textural classes.

Total no. of samples	Statistical indices	ρ_b (g cm^{-3})	θ_s(cm^3 cm^{-3})	OC (%)	Sand (%)	Silt (%)	Clay (%)
35	M	1.13	0.63	2.33	28.94	44.81	26.25
	max	1.55	0.76	5.36	99.00	70.55	49.30
	min	0.72	0.46	0.92	9.00	0.60	0.40

$$n_i = \frac{3w_i}{4\pi \rho_s R_i^3} \quad [4]$$

$$l_i = 2n_i^{\alpha} R_i \quad [5]$$

$$r_i = 0.816\, R_i \sqrt{e n_i^{(1-\alpha_i)}} \quad [6]$$

$$h_i = \frac{2\gamma \cos\beta}{\rho_w g r_i} \quad [7]$$

Here, V_{vi} is the pore volume (cm^3 g^{-1}), w_i is the fraction solid mass (g g^{-1}), ρ_s is the particle density (g cm^{-3}), e is the void ratio, r_i is the pore radius (cm), l_i is the pore length (cm g^{-1}), ρ_b is the bulk density (g cm^{-3}), θ_i is the water content (cm^3 cm^{-3}), n_i is the number of spherical particles, R_i is the mean particle radius (cm), α is the scaling parameter, h_i is the pressure head, ρ_w is the density of water (g cm^{-3}), g is the acceleration due to gravity (cm s^{-2}) γ is the surface tension (g s^{-2}), and β is the contact angle.

Estimating scaling parameter

The relationship between the number of spherical particles (n_i) and the number of spherical particles in the natural structure soil (N_i) is given by the following equation:

$$n_i^a = N_i \text{ or } \alpha_i = \frac{\log N_i}{\log n_i} \quad [8]$$

Arya et al. (1999) evaluated N_i with the following expression:

$$N_i = 7.37\, lw_i e \frac{h_{mi}^2}{\rho_s R_i} \quad [9]$$

where h_{mi} is the measured pressure head.

Constant α *(Arya and Paris 1981; Arya and Dierolf 1992; Vaz et al. 2005)*

Arya and Paris (1981) obtained a value of α = 1.38. Later, a value of α = 0.938 was proposed by Arya and Dierolf (1992), and α = 0.977 was proposed by Vaz et al. (2005) for Brazilian soil.

Estimating the scaling parameter with a logistic equation

The relationship between log N_i and log n_i is described by the following logistic equation (Arya et al. 1999):

$$(Y + \triangle Y) = \frac{Y_f\, Y_i}{Y_i + (Y_f - Y_i) \exp[\mu(X + \triangle X)]} \quad [10]$$

where Y is the dependent variable of log N_i, Y_f is the final value of log N_i, Y_i is the initial value of log N_i, μ is the rate coefficient, X is the independent variable log n_i, ΔY = Δlog N_i, and ΔX = Δlog n_i. These values are represented in Table 2.

Estimating the scaling parameter with a linear equation

Arya et al. (1999) evaluated a linear fit between log N_i and log (w_i/R_i^3):

$$\log N_i = a + b \log \frac{w_i}{R_i^3} \quad [11]$$

Combining Eq. [11] with Eqs. [4] and [8], α was calculated as:

$$\alpha_i = \left[\frac{a + b \log\left(\frac{w_i}{R_i^3}\right)}{\log n_i} \right] \quad [12]$$

Parameters for Eq. [12] for 5 different soil textures are represented in Table 3.

Table 2. Fitted values for $(\log N_i)_i$, $(\log N_i)_f$, μ, $\Delta\log N_i$ and $\Delta\log n_i$ for 5 textural classes (Arya et al. 1999).

Textural class	$(\log N_i)_i$	$(\log N_i)_f$	μ	$\Delta\log N_i$	$\Delta\log n_i$
Sand	0.996	16.602	0.609	1.734	0.00032
Sandy loam	0.559	16.983	0.553	2.492	1.849
Loam	0.628	16.614	0.510	2.242	1.977
Silty loam	0.719	19.686	0.457	1.902	0.684
Clay	1.993	21.685	0.289	4.766	2.648

Table 3. Parameters for Eq. [12] for 5 soil textures (Arya et al. 1999).

Textural class	*a*	*b*
Sand	-2.478	1.490
Sandy loam	-3.398	1.773
Loam	-1.681	1.395
Silt loam	-2.480	1.353
Clay	-2.600	1.305

Arya et al. (1999), with their comparison of 3 methods, showed that using the logistic α improved water retention data more than using the linear α and the constant α.

α *(Tyler and Wheatcraft 1989)*

Turcotte (1986) showed that the relationship between the number of particles and the mean particle radius is given by the equation:

$$n_i R_i^D = \text{Constant} \qquad [13]$$

The fractal dimension of the particle size distribution can be easily measured and related to the α parameter of the AP model. Instead of the straight capillary tube approach of Arya and Paris (1981), fractal measures can be used to evaluate the pore length as a function of measuring scale. The fractal dimensions (D) of the particle size distributions were calculated from the slope of the log particle size versus the log number of particles.

Tyler and Wheatcraft (1989) obtained the fractal dimension (D) using Eq. [13] based on the correlation between the mean particles radius (R_i) and the number of spherical particles (n_i):

$$n_i = aR_i^{-D} \qquad [14]$$

Tyler and Wheatcraft (1989) related the scaling parameter α in the AP soil water retention model to physical properties of the soil. Their results showed that by suggesting a physical significance of the coefficient (D), the universality of the model was greatly improved and the estimated water retention data closely matched the observed data.

α *(Vaz et al. 2005)*

Vaz et al. (2005) obtained an expression for α as a function of the soil water content,

$$\alpha = f(\theta)$$

$$\alpha_i = 0.947 + 0.427 \exp(-\theta_i/0.129) \qquad [15]$$

where θ is the water content ($cm^3\ cm^{-3}$).

Vaz et al. (2005) evaluated the performance of the AP model using 3 constant α values, 1.38, 0.938, and 0.977, and an α variable approach, eq. [15]. The exponential dependence of α on θ has improved the estimation of the retention curves of the AP model.

α *(Rezaei et al. 2004)*

Rezaei et al. (2004) obtained parameters of Eq. [11] as $a = \log(3/4\pi\rho_s)$ and $b = 1.0156e^{-0.953}$. They achieved better results for *a* and *b* than those reported by Arya et al. (1999).

Evaluation criteria

To determine the accuracy of the represented methods and the correlation between the measured and predicted water retention curve, statistical comparison of the results was carried out in terms of the coefficient of determination (R^2), mean error (ME), and normalized root mean square error (NRMSE). The formulae for calculating the R^2, NRMSE, and ME values are:

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (y_i - \widehat{y}_i)^2}{\sum_{i=1}^{N} (y_i - \overline{y}_i)^2} \qquad [16]$$

$$ME = \frac{\sum_{i=1}^{N} (\widehat{y}_i - y_i)}{N} \qquad [17]$$

$$NRMSE = \sqrt{\frac{\sum_{i=1}^{N} (y_i - \widehat{y}_i)^2}{N}} \Big/ \overline{y} \qquad [18]$$

where y_i is the measured water content, $\hat{y}_i$ is the estimated water content by different methods, $\bar{y}_i$ is the mean measured water content, and N is the number of experimental pairs of water content.

Results

The scaling parameter α allows the AP model to estimate the h(θ) relationship for structured soil. The best fits between the measured and estimated SWRC were obtained with the linear α method and AP constant α, respectively, with a maximum coefficient of determination (R^2), a minimum mean error (ME), and a normalized root mean square error (NRMSE), as shown in Table 4. The estimated soil water retention curve using the linear α and constant α methods agree well with the measured soil water retention curve for the finer and medium textures. All methods were evaluated by comparing measured and calculated water content at different pressure heads on a 1:1 plot (Figure 1).

The linear method showed good agreement between the measured and estimated water content. The linear regression had R^2 = 0.68 and the regression line differed only a little from the 1:1 line. Estimation with the AP constant α showed results similar to those for linear α. The use of linear α and constant α usually led to underestimation in the dry range and overestimation in the wet range. The AP model assumes complete desorption of all pores of a given class size at the critical pressure. At low tensions, this assumption appears reasonable; however, at high tensions, a significant percentage of water may be held as film and in poorly connected pores. As a result, the model will tend to underestimate the water content in the high-tension regions. The linear method of Arya et al. (1999) assumes a linear relationship between log N_i and log (w_i/R_i^3), so it may not be applicable in the dry range. The behavior of the linear α can create some errors in the wet and dry ranges. In the logistic method, large positive and large negative values of α occurred in the diameter range of 500-2000 μm (Figure 2) when logn_i was less than 0 (or negative). Arya et al. (1999) attributed this behavior to errors in the estimation of small numbers of particles.

Table 4. Values of R^2, ME, and NRMSE for the methods of estimating water retention of the soils.

Scaling parameter (α)	R^2	ME	NRMSE
α (linear, Arya et al. 1999)	0.681	-0.024	0.24
Constant α (Arya and Paris 1981)	0.616	-0.036	0.28
α (logistic, Arya et al. 1999)	0.591	0.042	0.30
α = f(θ) (Vaz et al. 2005)	0.683	-0.132	0.45
Fractal α (Tyler and Wheatcraft 1989)	0.303	0.103	0.47
α (Rezaei et al. 2004)	0.128	0.043	0.46

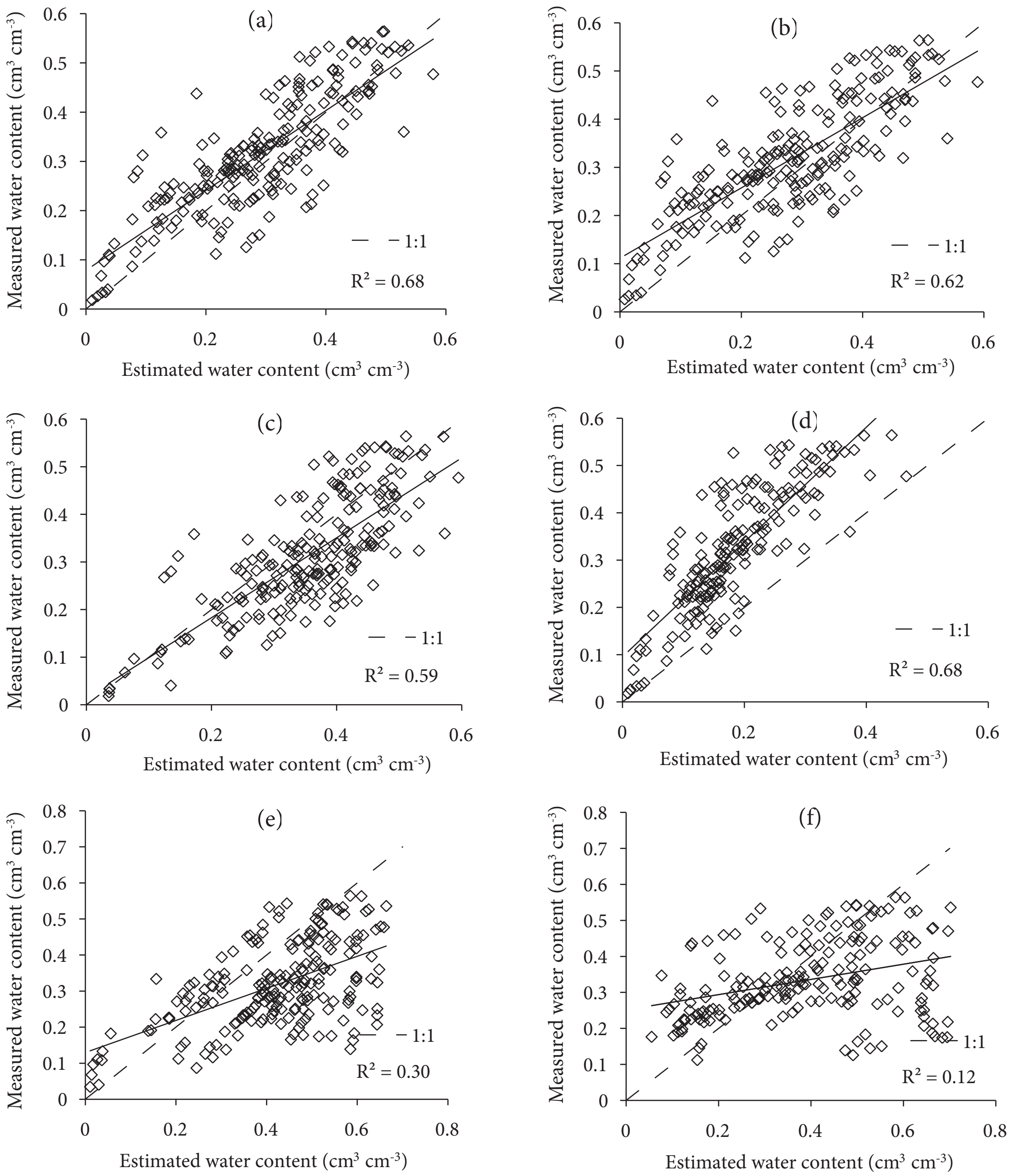

Figure 1. Comparison of estimated and measured water content by 6 methods for 35 soils: a) α (linear, Arya et al. 1999), b) constant α (Arya and Paris 1981), c) α (logistic, Arya et al. 1999), d) $\alpha = f(\theta)$(Vaz et al. 2005), e) fractal α (Tyler and Wheatcraft 1989), f) α (Rezaei et al. 2004).

The logistic method showed poor agreement between the measured and estimated water retention curves. Although the linear regression had $R^2 = 0.59$, it deviated from the 1:1 line in the wet range (Figure 1). For soils with finer textures, the α introduced by Vaz et al. (2005) could not estimate high tensions of 1000 and 1500 kPa (results not shown). In this method, the α value increased with sand content and decreased with clay content (Figure 3). When the α value decreased, this method could not predict

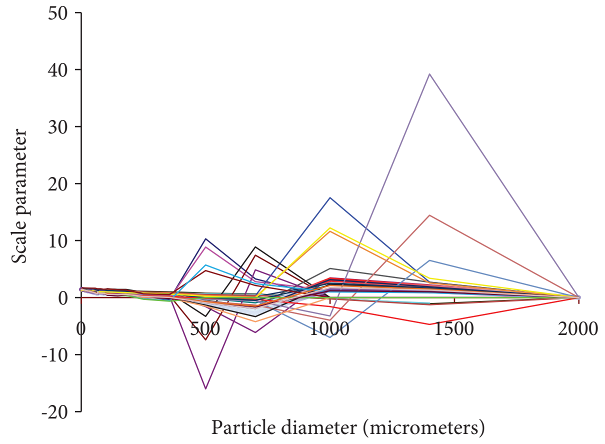

Figure 2. Alterations of scaling parameter with particle diameter in the logistic method.

high tension. For very low water content measured in very sandy soils in the water retention curve, the α value increased to about 1.37, that is very close to the α value introduced by Arya and Paris (1981). Although the regression appears to be good, with R^2 = 0.68, the method underestimated water content in the dry and wet ranges. The regression line deviated from the 1:1 line in the wet and dry ranges. In the present work, the α value proposed by Arya and Dierolf (1992), α = 0.938, and the constant α value proposed by Vaz et al. (2005), α = 0.977, were able to estimate water content in low pressure heads (30 and 100 kPa) in all soils, so these methods were not compared with the other introduced methods (results not shown).

In the Tyler and Wheatcraft method, the finer-textured soils that showed a wider distribution in particle sizes showed higher fractal dimensions, while the coarse-textured soils showed smaller fractal dimensions. Thus, α changed with alterations of fractal dimension. The results are consistent with those reported by Tyler and Wheatcraft (1989). The Tyler and Wheatcraft method (1989) caused a great overestimation of water content for the finer-textured soils (results not shown). The regression line deviated from the 1:1 line in the wet range. The value of R^2 in this case was 0.30. The fractal dimension increased with clay content and decreased with sand content (Figure 4). The α introduced by Rezaei et al. (2004) showed poor agreement with the measured retention data. In this method, the water content at high tensions was overestimated, especially for the finer textured soils (results not shown). The deviation of the regression line from the 1:1 line was much greater than in other methods and the linear regression had R^2 = 0.12 (Figure 1).

Discussion

Determination of the soil water retention curve is required for many applications. However, the necessary measurements are especially time-consuming and tedious. A better estimation of the water retention curve is presented using the AP model based on particle size distribution. We showed that PSD data can be used to characterize the spatial

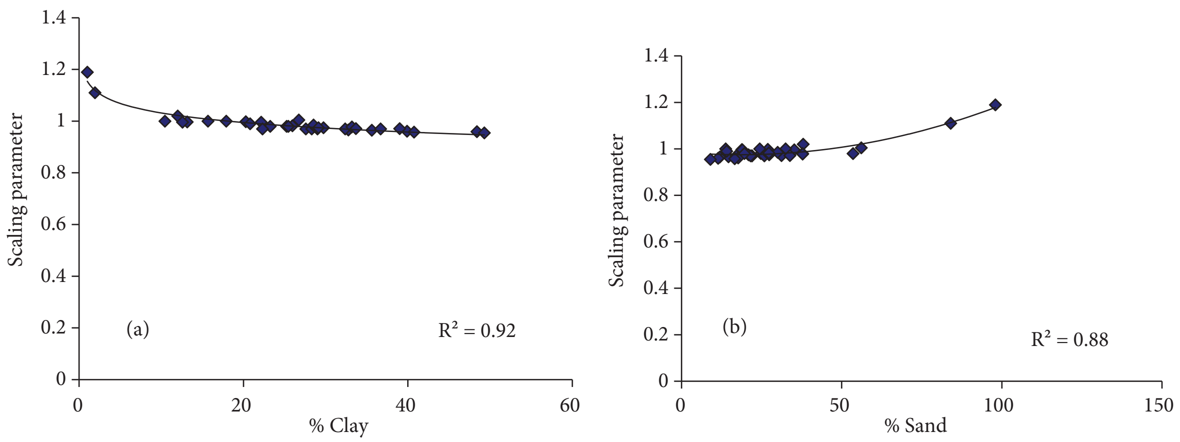

Figure 3. Dependence of average α value with a) clay and b) sand contents for soil samples according to the method of Vaz et al. (2005).

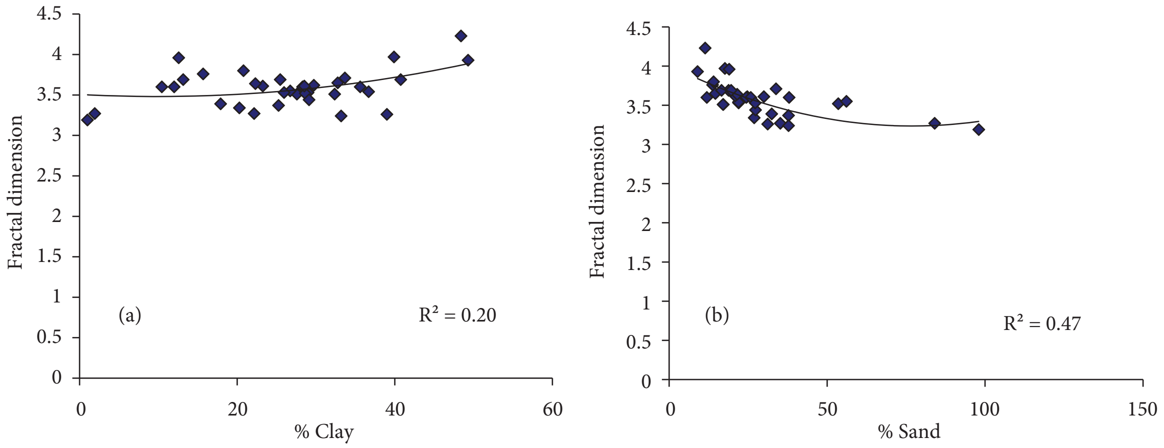

Figure 4. Dependence of average fractal dimension value (D) with a) clay and b) sand contents for 35 soil samples.

variability of SWRC. This would be advantageous since calculating PSD measurements is much simpler and less time-consuming than performing hydraulic measurements. This study compared 8 methods of estimating a scaling parameter (α) in the AP (1981) model for prediction of the SWRC, which translates a particle size distribution curve into a corresponding SWRC. These methods were evaluated on 35 soil samples. From the study in this paper, the main conclusions obtained are that the linear α as reported by Arya et al. (1999) and the constant α as reported by Arya and Paris (1981) exhibited good agreement between the measured and estimated SWRCs. The other methods showed poor agreement with the measured SWRC. Between the linear α and constant α, using the constant α is better than the linear α, because this method was shown to be easy for efficient estimation of the water retention curve, especially when laboratory measurements are not available. However, this method must be tested on numerous soils until reliable results are obtained.

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