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Electroweak decays of unflavored mesons in Poincaré covariant quark model

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Abstract: This work presents the procedure for obtaining integral representation of the radiative decay constant in the Poincaré-covariant quark model, based on Poincaré-invariant quantum mechanics. In the course of the work the authors give numerical estimates of magnetic moments of the light quarks (u, d, s) from the $V \to P\gamma$ decay, based on the calculation method of the observer that was previously developed. This technique is generalized in the case of the decay $P \to V\gamma$ with further numerical calculation of mixing angles $\eta - \eta'$ of pseudoscalar mesons. Within the framework of the offered approach, we obtain a self-consistent model that describes radiative and leptonic decays of pseudoscalar and vector mesons.

Key words: Poincaré-invariant, point-form, meson, quark model, mixing schemes, gluonium content

1. Introduction

The interest in studying the decays of pseudoscalar and vector mesons with γ-quantum emission has recently highly increased. This is caused by getting experimental data on such decays with pinpoint accuracy: the collaborations KLOE [1–3] and MAMI [4] obtained numerical values of phenomenological electromagnetic form-factors ϕ, ω, and η-mesons for different transmitted momentums. These and other preexisting experimental works [5, 6] were used as a basis for working out a number of studies on similar processes within the bounds of various models and approaches. Nevertheless, no approach requiring further development for the mechanism of quark interaction in low-energy areas has been completely studied.

Among the variety of processes involving hadrons, real and virtual γ-quantum decay processes $V(P) \to P(V)\gamma^* \to P(V)\ell^-\ell^+$ are of particular interest. Such electromagnetic transitions are the simplest ones and allow us to find various phenomenological characteristics of hadrons such as magnetic moments, different form-factors, mixing angle combinations of quarks and gluons in their wave functions, etc. As a result, such processes are a favorable testing ground for any theory that describes the structure of strongly interacting particles, and they provide a fuller picture of quarks' interaction than purely hadronic interactions.

Also, such decays give the opportunity not only to obtain the behavior of form-factors of meson transitions but also to estimate the mixing angles for the mesons of pseudoscalar and vector sectors. This problem has become particularly topical due to the latest experimental data on the decays of pseudoscalar η–η' mesons [7, 8], where some cases of data analysis with additional mixing angles for gluonium content are known. This led to the appearance of a series of papers devoted to mixing angles' evaluation in different approaches (see below). However, the results in these models have significant differences, which makes the further study of such processes a topical task of bound-state physics.

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In addition to the characteristics mentioned above, the processes with γ-quantum emission make it possible not only to investigate the behavior of hadron form-factors, but also the electromagnetic structure of quarks as well. For light mesons this problem is especially important as such systems are purely relativistic, which makes it possible to obtain more accurate results. Despite a number of works based on light-front dynamics [9–12], where quarks were assumed as structureless particles, in the instant form of dynamics [13, 14], the anomalous quark magnetic moment was introduced for theoretical calculations and experimental data agreement. This assumption is consistent with [15–17], where model calculations for baryons led to the fact that the anomalous quark magnetic moment is nonzero.

Despite the fact that a number of papers are devoted to the calculations of relativistic two-particle bound systems in light-front and instant-form dynamics (see above), the point-form approach of Poincaré-invariant quantum mechanics (PiQM) is rarely used for such kinds of calculations. In spite of the developed theoretical basis of the electromagnetic forms-factors description [18–20] and its successful use for the calculation of the nucleon form-factors [21], there are some significant differences between theoretical calculations and experimental data that led to the appearance of various modifications of this form of dynamics such as the Dirac point-form of dynamics [22] and spectator-model for operators in the point-form of PiQM [23]. Thus, the further development of this form of dynamics is a topical problem of hadron physics.

This article is devoted to the calculation of the observed $V(P) \to P(V)\gamma$ decay processes within the point-form of PiQM with further calculation of the anomalous magnetic moments for light quarks sector. Besides that, the authors using experimental data on these decays and estimate the values of the mixing angles for the pseudoscalar $\eta - \eta'$–meson sector taking into account gluonium content (keeping in mind the requirement of agreement between the theoretical calculations and experimental data).

The article is organized as follows. Section 2 is devoted to a brief description of the procedure for obtaining the integral representation of the radiative decay constant, taking into account the anomalous magnetic moments of the quarks. Section 3 presents our model parameters that were obtained earlier using current quark masses and a pseudoscalar density constant [24]. It should be noted that these results were compared with other models based on front-form and instant-form dynamics of PiQM. We therefore do not analyze these values. In Section 4 we discuss possible mixing schemes for unflavored pseudoscalar and vector mesons with the following analysis of the mixing scheme with gluonium content for the pseudoscalar $\eta - \eta'$-sector. Section 5 is devoted to the numerical calculation with the following brief analysis of the obtained model parameters: we compare our results with various approaches and discuss numerical differences. Conclusions and outlooks are presented in Section 6.

2. Radiative delay calculations scheme in point-form of PiQM

Parameterization of the matrix element for the vector (pseudoscalar) meson $V(P)$ transition into a pseudoscalar (vector) meson $P(V)$ with 4-momentums $Q = \{Q_0, \mathbf{Q}\}$ $\left(Q^2 = M^2\right)$ and $Q' = \{Q'_0, \mathbf{Q}'\}$ $\left(Q'^2 = M'^2\right)$ by emitting a virtual γ^* is given by [10, 11]:

$$\langle \mathbf{Q}', M' | \hat{J}^{\mu}_{h}(0) | \mathbf{Q}, M \rangle = i\,\sqrt{4\pi\alpha}\; g_{VP\gamma^*}(q^2) \frac{\epsilon^{\mu\nu\rho\sigma}\varepsilon_{\nu}(\lambda_V) Q_{\rho} Q_{\sigma}}{(2\pi)^3\sqrt{4Q_0 Q'_0}}, \tag{2.1}$$

where $\varepsilon_{\nu}(\lambda_V)$ is the polarization vector of meson V, $\alpha = e^2/4\pi$ is a fine structure constant, and $q = Q - Q'$.

The experimental value of decay constant $g_{VP\gamma} = g_{VP\gamma^*}(q^2 = 0)$ is defined by

$$\Gamma(V \to P\gamma) = \alpha\, \frac{g^2_{VP\gamma}}{3} \left(\frac{M_V^2 - M_P^2}{2\; M_V} \right)^3 \tag{2.2}$$

for $V \to P\gamma$ decay and

$$\Gamma(P \to V\gamma) = \alpha\; g^2_{VP\gamma} \left(\frac{M_P^2 - M_V^2}{2\; M_P} \right)^3 \tag{2.3}$$

for $P \to V\gamma$ decay.

The scheme for obtaining the decay constants in PiQM is as follows: in the case of a two-particle system with the masses m_q, $m_{\bar{Q}}$ and respectively, 4-momentums $p_1 = \left(\omega_{m_q}(\mathrm{p}_1), \mathbf{p}_1\right)$, $p_2 = \left(\omega_{m_{\bar{Q}}}(\mathrm{p}_2), \mathbf{p}_2\right)$ basis of direct product of two noninteracting particles,

$$|\mathbf{p_1}, \lambda_1\,\rangle \otimes |\mathbf{p_2}, \lambda_2\,\rangle \equiv |\mathbf{p_1}, \lambda_1, \mathbf{p_2}, \lambda_2\rangle \tag{2.4}$$

defines a reducible representation of the Poincaré group. For irreducible representation that characterizes the entire system, we introduce a full momentum

$$\mathbf{P}_{12} = \mathbf{p_1} + \mathbf{p_2} \tag{2.5}$$

and the relative momentum $\mathbf{k}$ of particles [25].

The requirement that the operators for a free and bound $q\bar{Q}$ system satisfy the algebra of the Poincaré group leads to the fact that the state vector of the meson with mass M, total angular momentum J, and projection μ is defined as the direct product of the state vectors of free particles (quarks) (2.4) with the wave function (WF) $\Phi^J_{\ell S}\left(\mathrm{k}, \beta_{q\bar{Q}}\right)$ [25]:

$$\begin{aligned} |\mathbf{Q}, J\mu, M\rangle = \sum_{\lambda_1,\lambda_2} \sum_{\nu_1,\nu_2} \int d\mathbf{k} \sqrt{\frac{\omega_{m_q}(\mathrm{p}_1)\, \omega_{m_{\bar{Q}}}(\mathrm{p}_2)\, M_0(\mathrm{k})}{\omega_{m_q}(\mathrm{k})\, \omega_{m_{\bar{Q}}}(\mathrm{k})\, \omega_{M_0}(\mathrm{P}_{12})}}\; \Phi^J_{\ell S}\left(\mathrm{k}, \beta_{q\bar{Q}}\right) \mathbf{C}\left\{ {}^{s_1\;\; s_2\;\;\;\; S}_{\nu_1, \nu_2, \nu_1+\nu_2} \right\} \times \\ \times \mathbf{C}\left\{ {}^{\ell \qquad\quad S \quad J}_{\mu-(\nu_1+\nu_2), \nu_1+\nu_2, \mu} \right\} Y_{\ell, \mu-(\nu_1+\nu_2)}(\theta_k, \phi_k)\; D^{1/2}_{\lambda_1,\nu_1}(\mathbf{n}_{W_1})\; D^{1/2}_{\lambda_2,\nu_2}(\mathbf{n}_{W_2})\, |\mathbf{p_1}, \lambda_1, \mathbf{p_2}, \lambda_2\rangle\;. \end{aligned} \tag{2.6}$$

Note that in Eq. (2.6) the functions $\mathbf{C}\left\{ {}^{s_1\;\, s_2\;\, S}_{\nu_1, \nu_2, \lambda} \right\}$, $\mathbf{C}\left\{ {}^{\ell\;\; S\;\, J}_{m, \lambda, \mu} \right\}$ are Clebsh–Gordan coefficients of the $SU(2)$ group, $Y_{\ell,m}(\theta_k, \phi_k)$ is the spherical harmonic, and $D_{\lambda,\nu}(\mathbf{n}_W)$ is the Wigner function [25]. For brevity, we use the following notations:

$$M_0(\mathrm{k}) = \omega_{m_q}(\mathrm{k}) + \omega_{m_{\bar{Q}}}(\mathrm{k}),\;\; \omega_m(\mathrm{k}) = \sqrt{\mathrm{k}^2 + m^2},\;\; \mathrm{k} = |\mathbf{k}|, \tag{2.7}$$

and WF $\Phi^J_{\ell S}\left(\mathrm{k}, \beta_{q\bar{Q}}\right)$ in (2.6) is subject to the normalization condition

$$\sum_{\ell,S} \int d\mathrm{k}\; \mathrm{k}^2\; \left|\Phi^J_{\ell S}\left(\mathrm{k}, \beta_{q\bar{Q}}\right)\right|^2 = 1. \tag{2.8}$$

Using the equality of 4-velocities of the bound system V_Q ($V_{Q'}$) and the system of two particles $V_{P_{12}}$ in the point-form of PiQM [25],

$$V_Q = \frac{Q}{M} \equiv V_{P_{12}} = \frac{P_{12}}{M_0(\mathrm{k})}, \tag{2.9}$$

and in our approach the decay constant of $V(P) \to P(V)\gamma^*$ from Eq. (2.1) is defined by

$$g_{VP\gamma^*}(q^2) = (2\pi)^3 \langle \mathbf{Q}', M'| \frac{\sqrt{4V_0'V_0}}{\sqrt{M_0(\mathrm{k}')\ M_0(\mathrm{k})}} \frac{\left(\hat{J}_h(0)\cdot K^*(\lambda_V)\right)}{(K^*(\lambda_V)\cdot K(\lambda_V))} |\mathbf{Q}, 1\lambda_V, M\rangle, \tag{2.10}$$

where $K^\mu(\lambda_V) = i\epsilon^{\mu\nu\rho\sigma}\varepsilon_\nu(\lambda_V)V_\rho V_\sigma'$.

We assume that the considered decay process is caused by interaction of constituent quarks, which are included in the meson $V(P)$, and virtual γ-quantum: the transition current between initial and final conditions, taking into account the relativistic impulse approximation [26],

$$\hat{J}_h^\mu(0) \approx \hat{J}_{quark}^\mu(0) = \sum_{q,q'=u,d,s} e_q \bar{\psi}_{q'} \Gamma^\mu \psi_q, \tag{2.11}$$

in the quark basis for this type of decay leads to

$$\left\langle \mathbf{p}_1', \lambda_1', \mathbf{p}_2', \lambda_2' \left| \hat{J}_{quark}^\mu(0) \right| \mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2 \right\rangle =$$

$$\frac{1}{(2\pi)^3}\left(\frac{e_q \bar{u}_{\lambda_1'}(\mathbf{p}_1', m_q')\Gamma_q^\mu u_{\lambda_1}(\mathbf{p}_1, m_q)}{\sqrt{2\omega_{m_q'}(\mathrm{p}_1')}\sqrt{2\omega_{m_q}(\mathrm{p}_1)}} + \frac{e_{\bar{Q}} \bar{\upsilon}_{\lambda_2}(\mathbf{p}_2, m_{\bar{Q}})\Gamma_{\bar{Q}}^\mu \upsilon_{\lambda_2'}(\mathbf{p}_2', m_{\bar{Q}}')}{\sqrt{2\omega_{m_{\bar{Q}}'}(\mathrm{p}_2')}\sqrt{2\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}} \right), \tag{2.12}$$

where

$$\Gamma_{q,\bar{Q}}^\mu = F_1(q^2)\gamma^\mu + \frac{1}{2m_{q,\bar{Q}}} F_2(q^2)\sigma^{\mu\nu}q_\nu. \tag{2.13}$$

We apply further calculation in the generalized Breit system, where $\mathbf{V}_Q + \mathbf{V}_{Q'} = 0$, assuming that the studied decay process is caused by electromagnetic interaction of constituent quarks and photon, and so

$$m_q = m_q', \quad m_{\bar{Q}} = m_{\bar{Q}}', \tag{2.14}$$

and from relations (2.6,2.10), and (2.12) one can obtain the integral representation of the $V \to P\gamma^*$ decay constant [27]:

$$\begin{aligned} g_{VP\gamma^*}(q^2) = & \frac{1}{4\pi} \sum_{\nu_1,\nu_1'} \int d\mathbf{k}\, \sqrt{\frac{3+4\nu_1(\lambda_V-\nu_1)}{4}} \frac{\nu_1'}{\sqrt{M_0(\mathrm{k})}} \Phi(\mathrm{k}, \beta_{q\bar{Q}}^V) \\ & \times \sqrt{\frac{1}{\omega_{m_q}(\mathrm{k})\omega_{m_{\bar{Q}}}(\mathrm{k})}} \left(e_q \sqrt{\frac{\omega_{m_{\bar{Q}}}(\mathrm{k}_2)}{\omega_{m_q}(\mathrm{k}_2)}} \bar{u}_{\nu_1'}(\mathbf{k}_2, m_q) B(\boldsymbol{\upsilon}_Q) \left(K^*(\lambda_V)\cdot\Gamma_q\right) u_{\nu_1}(\mathbf{k}, m_q) \right. \\ & \times \frac{1}{\sqrt{\varpi_{12}^2(\mathrm{k},t)-1}} \frac{\Phi^*(\mathrm{k}_2, \beta_{q\bar{Q}}^P)}{\sqrt{M_0(\mathrm{k}_2)}} D_{-\nu_1', \lambda_V-\nu_1}(\mathbf{n}_{W_2}(\mathbf{k}, \boldsymbol{\upsilon}_Q)) + e_{\bar{Q}} \sqrt{\frac{\omega_{m_q}(\mathrm{k}_1)}{\omega_{m_{\bar{Q}}}(\mathrm{k}_1)}} \\ & \times \bar{\upsilon}_{\lambda_V-\nu_1}(\mathbf{k}, m_{\bar{Q}}) B(-\boldsymbol{\upsilon}_Q) \left(K^*(\lambda_V)\cdot\Gamma_{\bar{Q}}\right)\big) \upsilon_{-\nu_1'}(\mathbf{k}_1, m_{\bar{Q}}) \frac{1}{\sqrt{\varpi_{12}^2(\mathrm{k},t)-1}} \\ & \left. \times \frac{\Phi^*(\mathrm{k}_1, \beta_{q\bar{Q}}^P)}{\sqrt{M_0(\mathrm{k}_1)}} D_{\nu_1',\nu_1}(\mathbf{n}_{W_1}(\mathbf{k}, \boldsymbol{\upsilon}_Q)) \right). \end{aligned} \tag{2.15}$$

In Eq. (2.15) the function $B(\boldsymbol{v}_Q)$ is a boost operator, which depends on

$$\boldsymbol{v}_Q = \frac{\mathbf{V}_Q}{V_0},\ \mathbf{n}_{W_{2,1}}(\mathbf{k},\boldsymbol{v}_Q) = -\frac{[\mathbf{k}\times\mathbf{V}_Q]}{\omega_{m_{q,\bar{Q}}}(k)+m_{q,\bar{Q}}-(\mathbf{k}\mathbf{V}_Q)},$$

$$\mathbf{k}_{1,2} = \mathbf{k} \pm \boldsymbol{v}_Q\left((\varpi_{12}(\mathrm{k},t)+1)\,\omega_{m_{q,\bar{Q}}}(\mathrm{k}) - \mathrm{k}\sqrt{\varpi_{12}^2(\mathrm{k},t)-1}\cos\theta_k\right) \tag{2.16}$$

and

$$K(\lambda_V) = \sqrt{\frac{\varpi_{12}^2(\mathrm{k},t)-1}{2}}\{0,\lambda_V,i,0\},\ \varpi_{12}(\mathrm{k},t) = \left(V_{P_{12}}\cdot V_{P'_{12}}\right). \tag{2.17}$$

After some calculation of the spinor part of (2.15) and limiting $q^2\to 0$, which for form-factors from (2.13) leads to

$$F_1(0)+F_2(0) = \mu_{q,\bar{Q}},\ \mu_{q,\bar{Q}} = \frac{e_{q,\bar{Q}}}{2m_{q,\bar{Q}}}(1+\kappa_{q,\bar{Q}}), \tag{2.18}$$

one can obtain the integral representation of the radiative decay constant for $V\to P\gamma$ decay:

$$g_{VP\gamma} = \int \mathrm{dk}\ \mathrm{k}^2\ \Phi(\mathrm{k},\beta^V_{q\bar{Q}})\Phi^*(\mathrm{k},\beta^P_{q\bar{Q}})\ \left(e_q f_1\left(\mathrm{k},m_q,m_{\bar{Q}}\right)+\right. \tag{2.19}$$

$$\left.+\frac{e_q\kappa_q}{2\ m_q}f_2\left(\mathrm{k},m_q,m_{\bar{Q}}\right) - e_{\bar{Q}}f_1\left(\mathrm{k},m_{\bar{Q}},m_q\right) - \frac{e_{\bar{Q}}\kappa_{\bar{Q}}}{2\ m_{\bar{Q}}}f_2\left(\mathrm{k},m_{\bar{Q}},m_q\right)\right),$$

where

$$f_1\left(\mathrm{k},m_q,m_{\bar{Q}}\right) = \frac{1}{3\ \omega_{m_q}(\mathrm{k})}\left(\frac{m_q+m_{\bar{Q}}}{M_0(\mathrm{k})}+\frac{m_q}{\omega_{m_q}(\mathrm{k})}+1\right) \tag{2.20}$$

and

$$f_2\left(\mathrm{k},m_q,m_{\bar{Q}}\right) = -\frac{2}{3}\left(\frac{m_q^2+\omega_{m_q}(\mathrm{k})\left(m_q+\omega_{m_q}(\mathrm{k})\right)}{\omega^2_{m_q}(\mathrm{k})}\right). \tag{2.21}$$

3. Parameters of the model based on the point-form of PiQM

The procedure for obtaining the parameters of the model is described in detail in [24]; therefore, here we give only the results for the oscillator wave function for pseudoscalar ($I=P,\ \ell=0$) and vector mesons ($I=V,\ \ell=0$):

$$\Phi(\mathrm{k},\beta^I_{q\bar{Q}}) = \frac{2}{\pi^{1/4}\left(\beta^I_{q\bar{Q}}\right)^{3/2}}\exp\left[-\frac{\mathrm{k}^2}{2\left(\beta^I_{q\bar{Q}}\right)^2}\right],\ I=V,P. \tag{3.1}$$

Using experimental values [28] for leptonic decays, one can obtain the following constituent quark masses and $\beta^I_{q\bar{Q}}$-parameters of wave function (3.1):

$$m_u = (219.48\pm 9.69)\ \mathrm{MeV},\ m_d = (221.97\pm 9.69)\ \mathrm{MeV},\ m_s = (416.95\pm 61.22)\ \mathrm{MeV}, \tag{3.2}$$

$$\beta^P_{u\bar{d}} = (367.93\pm 25.10)\ \mathrm{MeV},\ \beta^V_{u\bar{d}} = (311.95\pm 2.14)\ \mathrm{MeV},$$

$$\beta^P_{u\bar{s}} = (375.53 \pm 19.66)\ \text{MeV},\ \beta^V_{u\bar{s}} = (313.62 \pm 24.22)\ \text{MeV}.$$

For further calculations, using weak isotopic symmetry violation, we assume that

$$\beta^V_{u\bar{u}} = \beta^V_{u\bar{d}} - \triangle\beta_{u\bar{d}},\ \beta^V_{d\bar{d}} = \beta^V_{u\bar{d}} + \triangle\beta_{u\bar{d}},\ \beta^V_{d\bar{s}} = \beta^V_{u\bar{s}} + \triangle\beta_{u\bar{d}},\ \beta^P_{d\bar{s}} = \beta^P_{u\bar{s}} + \triangle\beta_{u\bar{d}}, \tag{3.3}$$

where $\triangle\beta_{u\bar{d}} \simeq m_d - m_u = (2.5 \pm 0.2)$ MeV.

The analysis of the data (Table 1) shows that the basic parameters of the proposed model (see (3.2)) correlate with the results of the works based on different forms of PiQM.

Table 1. Basic parameters of other models, based on different forms of PiQM.

	[9, 10]	[12]	[13, 14]
m_u, MeV	220	250±5	220
m_d, MeV	220	250±5	220
m_s, MeV	450	370±20	–

Fixing values of quarks (u, d, and s), magnetic moment will be carried out from the requirement for coincidence of theoretical calculations with experimental data [28]. Using ρ^+, K^{*+}, and K^{*0} decay values from (2.19), one can obtain the following values: $\kappa_u = (-0.123 \pm 0.084)$, $\kappa_d = (-0.088 \pm 0.015)$, and $\kappa_s = (-0.198 \pm 0.011)$ (we use natural quark units). The analysis shows that values of anomalous magnetic moments in our work are in good agreement with other models and assumptions: the work in [29] used values of $\tilde{\kappa}_u = -0.064$, $\tilde{\kappa}_d = 0.017$; comparing to our results, one can get $\tilde{\kappa}_u = \kappa_u/e_u = -0.082$, $\tilde{\kappa}_d = \kappa_d/e_d = 0.029$.

4. Mixing scheme of mesons

Due to various mixing effects mesons are usually a linear combination of state vectors (2.6), which differ from each other by their quark composition, so in this section we briefly discuss the mixing scheme scenario for the unflavored pseudoscalar and vector mesons.

In physical applications mixing schemes with basis are widely used [30, 31]:

$$\begin{cases} \psi_1 = (1/\sqrt{2})|u\bar{u} - d\bar{d}\rangle, \\ \psi_q = (1/\sqrt{2})|u\bar{u} + d\bar{d}\rangle, \\ \psi_s = |s\bar{s}\rangle, \end{cases} \quad \text{and} \quad \begin{cases} \psi_1 = (1/\sqrt{2})|u\bar{u} - d\bar{d}\rangle, \\ \psi_8 = (1/\sqrt{6})|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle, \\ \psi_0 = (1/\sqrt{3})|u\bar{u} + d\bar{d} + s\bar{s}\rangle. \end{cases} \tag{4.1}$$

By means of (4.1) physical states of the vector meson are defined by

$$\begin{pmatrix} |\phi\rangle \\ |\omega\rangle \\ |\rho^0\rangle \end{pmatrix} = U(\phi_V, \phi_{\rho\phi}, \phi_{\rho\omega}) \begin{pmatrix} \psi_q \\ \psi_s \\ \psi_1 \end{pmatrix} = U(\theta_V, \theta_{\rho\phi}, \theta_{\rho\omega}) \begin{pmatrix} \psi_8 \\ \psi_0 \\ \psi_1 \end{pmatrix}, \tag{4.2}$$

where $U(\alpha, \beta, \gamma)$ is the rotation matrix.

It is known that the $\phi_{\rho\omega}$-mixing angle is due to QCD processes, which is a complex number [32], the numerical value of which is obtained in [33, 34] with large error of $\sim 30\%$, so we assume that $\phi_{\rho\omega} = 0$; $\rho - \phi-$ mixing also can be neglected. In the final analysis we will use only one ϕ_V-angle for the vector meson sector, for which from (4.2) it follows that

$$\begin{cases} |\phi\rangle = \cos\phi_V\psi_q - \sin\phi_V\psi_s, \\ |\omega\rangle = \sin\phi_V\psi_q + \cos\phi_V\psi_s, \\ |\rho^0\rangle = \psi_1, \end{cases} \tag{4.3}$$

where ϕ_V-angle is related to the θ_V-angle [31] by

$$\theta_V = \phi_V - \arctan\sqrt{2}\,. \tag{4.4}$$

For the pseudoscalar sector, respectively,

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |\pi^0\rangle \end{pmatrix} = U(\phi_P, \phi_{\pi\eta}, \phi_{\pi\eta'}) \begin{pmatrix} \psi_q \\ \psi_s \\ \psi_1 \end{pmatrix}, \tag{4.5}$$

where mixing effects for $\pi^0 - \eta$ and $\pi^0 - \eta'$ are extremely small [35, 36], so a naive mixing scheme form (4.5) leads to

$$\begin{cases} |\eta\rangle = \cos\phi_P\psi_q - \sin\phi_P\psi_s, \\ |\eta'\rangle = \sin\phi_P\psi_q + \cos\phi_P\psi_s, \\ |\pi^0\rangle = \psi_1. \end{cases} \tag{4.6}$$

However, the decay $\eta - \eta'$ analysis shows that scheme (4.6) with one mixing angle is not enough for describing the experimental data. Therefore, the number of independent mixing parameters $\eta - \eta'$ was increased due to the gluonium content [7, 37, 38]. As a result, physical states for pseudoscalar mesons could be written as [30]

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \end{pmatrix} = U(\phi_P, \alpha_G, \phi_G) \begin{pmatrix} \psi_q \\ \psi_s \\ \psi_G \end{pmatrix}, \tag{4.7}$$

or, using the explicit form $U(\phi_P, \alpha_G, \phi_G)$-matrix,

$$\begin{cases} |\eta\rangle = X_\eta\psi_q + Y_\eta\psi_s + Z_\eta\psi_G, \\ |\eta'\rangle = X_{\eta'}\psi_q + Y_{\eta'}\psi_s + Z_{\eta'}\psi_G, \\ |G\rangle = X_G\psi_q + Y_G\psi_s + Z_G\psi_G, \end{cases} \tag{4.8}$$

where $|G\rangle$ is gluonium content and

$$X_\eta = \cos\phi_P\cos\alpha_G,\ Y_\eta = -\sin\phi_P\cos\alpha_G,\ Z_\eta = -\sin\alpha_G, \tag{4.9}$$

$$X_{\eta'} = \cos\phi_P\sin\alpha_G\sin\phi_G + \sin\phi_P\cos\phi_G,\ Y_{\eta'} = \cos\phi_P\cos\phi_G - \sin\phi_P\sin\alpha_G\sin\phi_G,$$

$$Z_{\eta'} = \cos\alpha_G\sin\phi_G\ .$$

Table 2. Decay analysis in PiQM.

Radiative decay	Decay constant representation in PiQM
$\rho^0 \to \pi^0\gamma$	$e_u I(u\bar{u}) + e_d I(d\bar{d})$
$\rho^0 \to \eta\gamma$	$X_\eta (e_u I(u\bar{u}) - e_d I(d\bar{d}))$
$\phi \to \pi^0\gamma$	$\cos\phi_V (e_u I(u\bar{u}) - e_d I(d\bar{d}))$
$\phi \to \eta\gamma$	$\cos\phi_V X_\eta (e_u I(u\bar{u}) + e_d I(d\bar{d})) - 2\ e_s I(s\bar{s}) \sin\phi_V Y_\eta$
$\phi \to \eta\prime\gamma$	$\cos\phi_V X_{\eta'} (e_u I(u\bar{u}) + e_d I(d\bar{d})) - 2\ e_s I(s\bar{s}) \sin\phi_V Y_{\eta'}$
$\omega \to \pi^0\gamma$	$\sin\phi_V (e_u I(u\bar{u}) - e_d I(d\bar{d}))$
$\omega \to \eta\gamma$	$\sin\phi_V X_\eta (e_u I(u\bar{u}) + e_d I(d\bar{d})) + 2\ e_s I(s\bar{s}) \cos\phi_V Y_\eta$
$\eta\prime \to \rho^0\gamma$	$X_{\eta'} (e_u I(u\bar{u}) - e_d I(d\bar{d}))$
$\eta\prime \to \omega\gamma$	$X_{\eta'} \sin\phi_V (e_u I(u\bar{u}) + e_d I(d\bar{d})) + 2\ e_s I(s\bar{s}) \cos\phi_V Y_{\eta'}$

5. Numerical results and discussion

Let us define the numerical values of the mixing angles' $\eta - \eta'$-mesons and the remaining $\beta^P_{q\bar{q}}$-parameters of the unflavored pseudoscalar sector wave functions. Taking into account the fact that $|q\bar{q}\rangle$ is orthogonal for different quarks flavors, for various $V(P) \to P(V)\gamma$ decays one can get a representation of the decay constant in PiQM (see Table 2).

$$\beta^P_{u\bar{u}} = (280.60 \pm 25.07)\ \text{MeV},\ \beta^P_{d\bar{d}} = (277.95 \pm 25.07)\ \text{MeV},\ \beta^P_{s\bar{s}} = (494.54 \pm 19.66)\ \text{MeV}, \tag{5.1}$$

$$\theta_P = (-9.1 \pm 2.4)^\circ,\ \alpha_G = (9.5 \pm 3.4)^\circ,\ \phi_G = (-28.8 \pm 3.4)^\circ.$$

In Table 1 we use the abbreviation

$$I(q\bar{q}) = \int \text{dk}\ \text{k}^2\ \Phi(\text{k}, \beta^V_{q\bar{q}})\Phi^*(\text{k}, \beta^P_{q\bar{q}}) \left(\frac{(\omega_{m_q}(\text{k}) + 2\ m_q)}{3\ \omega^2_{m_q}(\text{k})} + \kappa_q \left(-\frac{1}{3}\ \frac{m_q^2 + \omega_{m_q}(\text{k})(m_q + \omega_{m_q}(\text{k}))}{m_q\ \omega^2_{m_q}(\text{k})} \right) \right), \tag{5.2}$$

which was obtained from (2.19) taking into account $m_q = m_{\bar{q}}$ and $|e_q| = |e_{\bar{q}}|$. Using basic parameters of the model (see Section 3) and relations (4.9) and (5.2), one can obtain the following values for mixing angles and $\beta^P_{q\bar{q}}$-parameters from the condition of compliance of theoretical calculations with experimental data (see Table 2).

Note that for the calculation we used mixing angle value $\theta_V = (31.92 \pm 0.2)^\circ$ [1], which for ω-meson decay into the $\ell^+\ell^-$-pair leads to

$$\beta^V_{s\bar{s}} = (336.56 \pm 1.38)\ \text{MeV}. \tag{5.3}$$

We start the analysis of obtained values for mixing $\theta_P(\phi_P)$-angle: it is well known that naive mixing scheme (4.6) leads to a value of the angle $\theta_P \approx -14^\circ$, which is confirmed in the works devoted to calculations on lattice QCD [39], chiral models [40], and light-front PiQM calculation [41]. Nevertheless, the recent tendency to take into account the gluonium content leads to the results $\theta_P \in (-20^\circ; -10^\circ)$: in [31] for SU(3)-breaking effects the calculation used the value $\theta_P \approx -17^\circ$, while s hidden local symmetry model obtained $\theta_P \approx -11^\circ$ [42], and analysis of the experimental data gives the value of angle $\theta_P \approx -12^\circ$ [8]. Thus, the problem of fixing the θ_P-angle of the pseudoscalar sector does not have an exact solution. The value obtained in our work of $\theta_P = (-9.1 \pm 2.4)^\circ$ or (see (4.4)) $\phi_P = \theta_P + \arctan\sqrt{2} = (54.3 \pm 2.1)^\circ$ lies within reasonable limits compared to other approaches and models; in addition, using the obtained values α_G and ϕ_G (see (5.1)), from (4.9) one can obtain the value $|Z_{\eta'}| = 0.48 \pm 0.12$, which, according to the authors, is a good result, as in [8], using a global fit for $V(P) \to P(V)\gamma$ decays, the obtained value was $|Z_{\eta'}| = 0.34 \pm 0.04$.

For the analysis of model assumption (3.3) we use experimental data on $\tau^{\pm} \to \rho^{\pm}\nu_\tau$ from [28], $f_{\rho^+}^{(\mathrm{exp})} = (209.3 \pm 1.5)$ MeV. Since $f_{\rho^0} = f_{\rho^+}/\sqrt{2}$ [43], one can get that $f_{\rho^0} = (148.39 \pm 0.04)$ MeV, which is in good agreement with our approach. Note that the decay constant $f_{\rho^0}^{(exp.)}$ obtained from the analysis of the $e^+e^- \to \pi^+\pi^-$ reaction [28] has the value (156.42 ± 14.44) MeV, which differs from (148.39 ± 0.04) MeV.

Table 3. Comparing experimental $\Gamma^{(exp.)}$ and theoretical $\Gamma^{(th.)}$ for $V(P) \to P(V)\gamma$-decay taking into account anomalous quark magnetic moments.

Radiative decay	$\Gamma^{(exp.)}$, keV	$\Gamma^{(th.)}$, keV
$\omega \to \pi^0\gamma$	687 ± 19 [8] 713.9 ± 19.9 [28]	704 ± 13
$\omega \to \eta\gamma$	5.8 ± 1.1 [28]	6.8 ± 2.3
$\phi \to \pi^0\gamma$	5.5 ± 0.2 [28]	5.6 ± 2.1
$\phi \to \eta\gamma$	55.4 ± 1.2 [28]	55.7 ± 2.5
$\phi \to \eta\prime\gamma$	0.27 ± 0.01 [28]	0.27 ± 0.14
$\rho^0 \to \pi^0\gamma$	77 ± 28 [44]	83 ± 4
$\rho^0 \to \eta\gamma$	44.7 ± 3.2 [28]	44.8 ± 3.5
$\eta\prime \to \rho^0\gamma$	56.7 ± 2.7 [28]	56.9 ± 2.3
$\eta\prime \to \omega\gamma$	5.1 ± 0.4 [28]	5.1 ± 0.8

The remaining parameters of the model are analyzed in Tables 3 and 4 (symbol $^{(\dagger)}$ marks the reactions from which model parameters were obtained).

Table 4. Comparing experimental $f_{V,P}^{(exp.)}$ [28] and theoretical values of leptonic decay constants.

Decay channel	$f_{V,P}^{(exp.)}$, MeV	$f_{V,P}^{(th.)}$, MeV
$^{(\dagger)}\pi^+ \to \ell\tilde{\nu}_\ell$	131.61 ± 0.17	131.61 ± 0.11
$^{(\dagger)}K^+ \to \ell\tilde{\nu}_\ell$	156.87 ± 0.78	156.87 ± 0.43
$^{(\dagger)}\tau \to \rho^+\tilde{\nu}_\tau$	209.3 ± 1.5	209.3 ± 0.5
$^{(\dagger)}\tau \to K^{*+}\tilde{\nu}_\tau$	205.3 ± 6.2	205.3 ± 1.3
$^{(\dagger)}\omega \to e^+e^-$	46.82 ± 8.12	46.82 ± 2.74
$\rho^0 \to e^+e^-$	156.42 ± 14.44	148.36 ± 3.12
$\phi \to e^+e^-$	76.21 ± 1.23	76.24 ± 3.72

6. Conclusion and outlooks

This work is devoted to the calculation of the integral representation of radiative decay constants of pseudoscalar and vector mesons. In the course of the work the authors, based on previously obtained results of quark masses and $\beta_{q\bar{Q}}$-parameters of wave functions, estimated the values of anomalous quark magnetic moments. These values were compared and they do not contradict with the other magnitudes in different models and approaches.

As a result, using gluonium content, we obtained the values of the radiative decay constant for unflavored mesons followed by the analysis of pseudoscalar θ_P-mixing angle value. The analysis showed that the proposed model describes the experimental data on the radiative decays of light unflavored mesons well.

It should be noted that the results of the model, based on the point-form of PiQM, are self-consistent, which makes it possible to use the model for the research of decay constant $g_{VP\gamma^*}$ for square transfer momentum $t = q^2$.

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