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It is sufficient to set the cosmological constant to zero or to a small number at an initial time

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Abstract: I point out a simple but usually overlooked fact about the cosmological constant problem: to solve the cosmological constant problem it is sufficient to find a symmetry or mechanism that sets the cosmological constant to zero or to a tiny value at some time in the past, provided that general relativity is the relevant theory of gravity, and the energy–momentum tensor (excluding the part of the form of a cosmological constant) is conserved. The relevant symmetry or mechanism need not be applicable today. Any additional cosmological constant term induced by a phase transition in the energy–momentum tensor in this case is compensated by a shift in the cosmological constant term of gravitational origin.

Key words: Cosmological constant, general relativity, cosmology

1. Introduction

Einstein's field equations are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{1}$$

where $R_{\mu\nu}$, $g_{\mu\nu}$, Λ, G, c, and $T_{\mu\nu}$ are the Ricci tensor, the metric tensor, the cosmological constant, Newton's gravitational constant, the speed of light, and the energy–momentum tensor, respectively, and $R = g^{\mu\nu}R_{\mu\nu}$. The most standard explanation for the accelerated expansion of the universe is a positive cosmological constant $\Lambda > 0$. Λ may be considered either as an integration constant following from Bianchi identities or as a special term, $T^{(\Lambda)}_{\mu\nu}$ in $T_{\mu\nu}$ with $T^{(\Lambda)}_{\mu\nu} \propto g_{\mu\nu}$ or as a combination of both. There are many potential theoretical contributions to $T^{(\Lambda)}_{\mu\nu}$, ranging from the minima of the potentials of spontaneously broken symmetries (e.g., Higgs mechanism) to zero-point energies of quantum fields [1, 2, 3]. However, the values of these contributions are much larger than the values implied by the observations. For example, the contribution of the Higgs potential is $\sim 10^{55}$ times larger than the value deduced from observations. This problem is called the (old) cosmological constant problem (CCP). There are numerous studies in the literature to solve this problem [2, 3]. In the evaluation of the proposed models an important fact about cosmological problem may be overlooked, that is, it is sufficient for a symmetry or a mechanism to be applicable in the past (while not being relevant today) to solve the CPP. In other words, it may have set the cosmological constant to zero or to a tiny value in the past and this is sufficient for the solution of the CCP although that symmetry or mechanism may not be surviving today. This option is a quite plausible option in the light of the presence of models in the literature where

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the universe was Minkowskian in the past [4, 5, 6, 7] so that supersymmetry, scale symmetry [2, 3], or metric reversal symmetry [8, 9, 10, 11, 12, 13, 14] or a dynamical mechanism based on the running of the vacuum energy [15, 16, 17] was applicable. I will show below that once the cosmological constant is set to zero (or to a small number) in the past then additional contributions to vacuum energy (of the form of cosmological constant) do not weigh provided that Einstein field equations are the relevant equations for gravitational field, and the energy–momentum tensor (excluding the part of the form of a cosmological constant) is conserved.

As is well known, after taking the trace of both sides of (1) one gets

$$-R - 4\Lambda = \frac{8\pi G}{c^4} T \tag{2}$$

where $T = g^{\mu\nu} T_{\mu\nu}$. After using (2) in (1), Eq. (1) may be written as

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T \right) \tag{3}$$

which is obtained from (1) by adding $\frac{1}{4} g_{\mu\nu} R$ to both sides and then using Eq. (2) on the right-hand side. In other words, Eq. (3) is another way of writing the Einstein equations provided that it is supplemented by (2). This form is more suitable for our discussion given below.

Eq. (3) is nothing but the form of the gravitational field equations in the case of unimodular gravity [18, 19, 20]. Eq. (3) is invariant under the shift of $R_{\mu\nu}$ or $T_{\mu\nu}$ by a cosmological constant term. This may suggest that writing the Einstein equations in the form of Eq. (3) solves the cosmological constant but this is not true since (3) is supplemented by (2). In other words, the cosmological constant is not removed, it is hidden in (2), i.e. the unimodular gravity does not remove the cosmological constant, it only hides it [21, 22]. Although Eq. (3) does not solve the cosmological constant problem, it has some virtue that is mentioned above; it is not affected by the shift of either $R_{\mu\nu}$ or $T_{\mu\nu}$ by a cosmological constant term, i.e. under either of the transformations

$$T_{\mu\nu} \to T_{\mu\nu} + g'_{\mu\nu} \frac{c^4}{8\pi G} \Delta\Lambda^{(e)} \tag{4}$$

$$R_{\mu\nu} \to R_{\mu\nu} + g'_{\mu\nu} \Delta\Lambda^{(g)} \tag{5}$$

where $\Delta\Lambda^{(e)}$ ($\Delta\Lambda^{(g)}$) is the contribution to the cosmological constant due to the source term (the curvature term), and the prime denotes a possible transformation of the metric tensor. The invariance of Eq. (3) under (4) or (5) only corresponds to the fact that the cosmological constant is a constant; therefore, it does not change with time. In other words, if the cosmological constant problem is set to some value (e.g., to zero or to a tiny value) by some mechanism or some symmetry at some initial time then it will remain so provided that general relativity is the relevant theory of gravity.

The invariance of the (total) cosmological constant under (4) or (5) may be seen in the original form of the Einstein equations as well. Although this fact is less apparent in the original form of the Einstein equations it gives additional insight into this phenomenon. Note that a CC may be either of geometrical origin (i.e. of gravitational origin) or may be due to the energy–momentum tensor. One may consider that a CC of geometrical origin contributes to the left-hand side of the Einstein equations while a CC of energy–momentum tensor origin contributes to the right-hand side of the Einstein equations. Although what matters gravitationally is the sum of these two contributions, conceptually there is a distinction between these two contributions. For example, the contribution of Higgs potential to CC is through the energy–momentum tensor, while the piece of an extra dimensional curvature scalar that depends only on extra dimensions gives a contribution to the 4-dimensional CC that is geometrical in origin. The transformation in (4) (that shifts the CC due to the energy–momentum tensor) may either induce a cosmological constant of geometrical origin or may transform the Einstein tensor in (1). Let us determine what happens if the transformation (4) is performed. Provided that the value of CC is set to some initial value Λ_0 in the past, any additional contribution at later times (through some phase transformation, e.g., by a Higgs potential of QCD condensate formation) cannot take place instantly since an instant transformation is not physical. Therefore, the evolution of CC from Λ_0 to $\Lambda_0 + \Delta\Lambda$ implies time dependence of CC for the times between Λ_0 and $\Lambda_0 + \Delta\Lambda$. An energy–momentum tensor with a time varying cosmological constant component is not a solution of the Einstein equations (unless $\frac{8\pi G}{c^4}$ is time dependent or the energy–momentum tensor for the usual matter is not conserved) [16]. This implies that the transformation (4) can only induce a CC of geometrical origin on the left-hand side of the Einstein equations if we adopt the standard theory. This may be seen explicitly as follows: under the transformation (4) Eq. (1), in general, would become

$$R'_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R' - g_{\mu\nu}\Lambda - g_{\mu\nu}\Delta\Lambda^{(g)} = \frac{8\pi G}{c^4}T_{\mu\nu} + g_{\mu\nu}\Delta\Lambda^{(e)} \tag{6}$$

where $'$ denotes a possible transformation in Ricci tensor, and $\Delta\Lambda^{(g)}$ denotes a possible CC of gravitational origin induced by the change in the energy–momentum tensor due to (4). A variable cosmological constant is not a solution of the Einstein equations provided that general relativity is the correct theory of gravitation and we assume that the matter part of the energy–momentum tensor is conserved. Hence we have

$$R'_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R' = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \tag{7}$$

This, in turn, (after using (1)) implies that we should have

$$\Delta\Lambda = \Delta\Lambda^{(e)} + \Delta\Lambda^{(g)} = 0 \tag{8}$$

i.e. $\Delta\Lambda^{(g)}$ is induced on the left-hand side of the Einstein equations to compensate the shift of CC by $\Delta\Lambda^{(e)}$ on the right-hand side of the Einstein equations. In other words, once the cosmological constant of the universe is set to some initial value then an additional contribution to the energy–momentum tensor of the form of CC at a later time is canceled out by a CC of gravitational origin provided that nature obeys general relativity exactly and the energy–momentum tensor (excluding the part of the form of a CC) is conserved.

In fact this is the result of a simple fact: the cosmological constant is a constant and so it remains zero once it is set to zero at some initial time. Although this seems self-evident it points out a simple and important consequence: it is sufficient to find a symmetry (e.g., supersymmetry, scale symmetry, metric reversal symmetry) or a mechanism that sets $\Lambda = 0$ or Λ to a small value at some initial time; then it will remain so for later times. For example, it is shown in [10] that metric reversal symmetry is a good symmetry of vacuum while it is not a symmetry of nonvacuum states in general in 4 dimensions. One may argue that at the beginning of the universe the universe was an empty Minkowskian space, and matter is created later, e.g., as implied in [4, 5]. Hence a zero or a tiny CC may be forced due to a symmetry at the beginning of the universe, and then that value of

CC is unaffected by additional contributions induced by phase transitions, e.g., by the one leading to a nonzero vacuum expectation value for the Higgs potential or by the one leading to QCD condensate (since these terms need a time varying CC term in some intermediate time and this, in turn, induces zero total contribution to CC as explained above) provided that general relativity is the correct theory of gravity, and the energy–momentum tensor (excluding its part of the form of a CC term) is conserved. In the case of modified theories of gravity [23, 24] (e.g., the scalar–tensor theories, the chameleon mechanism, bimetric theories, f(R) theories) one does not expect a considerable change in this conclusion. The predictions of successful theories of modified gravity should be very close to those of general relativity because general relativity (at present) is consistent with all observations [24]. Therefore, it is plausible to assume that a modified gravity theory may induce only very small contributions to CC due to these phase transitions. Note that the argument in this paper may be the reason why Higgs potential and QCD condensate give no (or a tiny) contribution to CC, and hence may provide a significant step towards the solution of the CCP.

In summary, in this study it is pointed out that the total CC remains the same even after an additional contribution to CC of energy–momentum tensor origin (e.g., through phase transitions) since this contribution is canceled out by a CC that is induced in the gravitational sector provided that general relativity is the theory of gravitation and the energy–momentum tensor (excluding the part of the form of CC) is conserved (which seems to be the case in all known physically relevant cases). Therefore, once the cosmological is set to some value (e.g., to zero or to a small value) through some symmetry or some mechanism at an initial time it will remain so even when there are phase transitions inducing cosmological constant type contributions in the energy–momentum tensor provided that the standard physical theories are adopted. This observation may be useful in the direction of solution of the CCP.

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