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Amplitude modulation of ion-acoustic waves in magnetized electron-positron-ion plasma with q-nonextensive electrons and positrons

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Abstract: Using a standard reductive perturbation technique, a nonlinear Schrödinger equation has been derived that describes the nonlinear evolution of ion-acoustic waves in magnetized electron-positron-ion plasma with electrons and positrons following q-nonextensive distribution. It is shown that excitation of both bright and dark envelope solitary structures is possible in the model plasma under consideration. The conditions for the excitation and structure of these envelope solitary waves are shown to depend sensitively on the strength of the external magnetic field, obliqueness of wave propagation relative to the external magnetic field, and other plasma parameters such as the q-nonextensivity of plasma species, positron concentration, and electron-positron temperature ratio.

Key words: Ion-acoustic waves, envelope solitary waves, q-nonextensive distribution

1. Introduction

The widespread existence of electron-positron-ion (e-p-i) plasma in astrophysical, space, and laboratory environments [1–12] inspired many researchers to study the phenomenon of nonlinear wave propagation in various e-p-i plasma systems. Data obtained from space satellites such as POLAR, S3-3, and FAST [13–16] suggest the presence of multicomponent e-p-i plasmas in the high altitude cusp, polar cap, and plasma sheet boundary. On the surface of fast rotating neutron stars and magnetars and in the pulsar magnetosphere [1,2], a strong magnetic field exists, and this magnetic field has a significant impact on nonlinear wave propagation, particularly in the low-frequency ion-acoustic region. For this, in recent years an interest has been developed to study nonlinear propagation of ion-acoustic waves in magnetized e-p-i plasmas [17–25]. Most of these studies considered Maxwellian distribution for electrons and positrons, but it is well known that the extremely turbulent nature of space plasma leads to a non-Maxwellian distribution for plasma species. There have been many experimental, space, and satellite observations [26–42] that endorse the presence of non-Maxwellian plasma species in space and astrophysical environments. This non-Maxwellian distribution of plasma species fits excellently with the power law distribution explained by Cairn [43] and Vasyliunas [28]. Considering this reality, Alinejad et al. [20,44] investigated the characteristics of ion-acoustic solitary waves in magnetized e-p-i plasma with superthermal and nonthermal particle distribution. Large-amplitude ion-acoustic solitary wave profiles have been investigated by El-Tantawy et al. [45] in magnetized e-p-i plasma with κ-distributed electrons and positrons. Their findings endorse that magnetized e-p-i plasma with non-Maxwellian species gives a trustworthy

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justification to the physics of nonlinear electrostatic waves in space and laboratory plasmas where wave damping produces a power law tail of plasma distribution.

A complex regime of cosmological or astrophysical plasmas can be characterized as a physical system driven in far-from-equilibrium states. Since Tsallis q-nonextensive particle distribution [46,47] is supposed to be the most general distribution that can describe this complexity more appropriately, in recent years some authors [48–50] have studied nonlinear wave propagation in e-p-i plasma with q-nonextensive particle distribution. During the last few years q-nonextensive distribution has been successfully applied to describe many non-Maxwellian systems such as self-gravitating polytrophic systems, solar neutrino deficit problem, galaxy clusters, wave propagation in plasma, and microwave radiation in cosmic regions [50–57]. In the context of wave propagation in plasma, recently Ferdousi et al. [21] considered ion-acoustic solitary waves in magnetized e-p-i plasma with both electrons and positrons having q-nonextensive distribution and studied the properties of oblique propagation of solitary waves. Similar research was also done by Alinejad [50]. Sahu [48] reported that both the profiles of compressive and rarefactive solitary waves and double layers are affected by the nonextensivity of plasma species. Regarding modulational instability of ion-acoustic waves in e-p-i plasmas, very few results have been reported in the literature. Jehan et al. [24] studied modulational instability of ion-acoustic waves in magnetized e-p-i plasma considering Boltzmann's distribution for electrons and positrons. Eslami et al. [56] investigated the modulational instability of ion-acoustic waves in unmagnetized e-p-i plasma having q-nonextensive distribution for electrons and positrons.

As far as we know, no one has reported an investigation of amplitude modulation for ion-acoustic solitary waves in magnetized e-p-i plasma with q-nonextensive electrons and positrons. The purpose of this paper is to address this problem. In this paper we have considered magnetized e-p-i plasma having Tsallis q-nonextensive distribution for both the electrons and positrons and studied the formation of ion-acoustic envelope solitons. For this purpose we have derived a nonlinear Schrödinger (NLS) equation by using a standard reductive perturbation technique and studied how the external magnetic field and nonextensivity of electrons and positrons affect the stability/instability of the wave and the structure of envelope solitons. In order to study numerically the effect of nonextensivity on the characteristics of the nonlinear wave structure we have used, following suggestion of Verheest [57], a value of q in the range of $\frac{1}{3} < q < 1$. The limit $q = 1$ corresponds to Boltzmann statistics and $q \neq 1$ corresponds to the case of Tsallis statistics. For $q > 1$ there is a thermal cut-off on the maximum value allowed for the velocity of an electron and it makes this range unsuitable to model superthermality in the plasma. However, for $q < 1$, this is not the case, and it can be used to model superthermality. Normalization of the distribution function requires that $q > -1$ and the requirement of finite energy further restricts the range to $\frac{1}{3} < q < 1$ [57].

2. Basic equations

We consider collisionless e-p-i plasma with q-nonextensive electrons and positrons in the presence of an external magnetic field. The normalized equations governing ion dynamics in such plasma are the following [21]:

$$\frac{\partial n_i}{\partial t} + \overrightarrow{\nabla}.(n_i \overrightarrow{u_i}) = 0, \tag{1}$$

$$\frac{\partial \vec{u}_i}{\partial t} + \left(\overrightarrow{u_i}.\overrightarrow{\nabla}\right)\overrightarrow{u_i} = -\overrightarrow{\nabla}\phi + \omega_{ci}\left(\vec{u}_i \times \hat{z}\right), \tag{2}$$

where n_i and $\overrightarrow{u_i}$ are respectively the normalized concentration and velocity of the positive ions; $\omega_{ci} = \frac{eB_0}{m_i}$ is the ion gyrofrequency. The density n_i is normalized by the equilibrium electron density n_{e0}, velocity $\vec{u}_i$ is normalized by the ion-acoustic speed $C_s = \sqrt{\frac{k_B T_e}{m_i}}$, all length by the electron Debye length $\lambda_{De} = \sqrt{\frac{k_B T_e}{4e^2 n_{e0}}}$, time by $\frac{\lambda_{De}}{C_s}$, and electrostatic potential ϕ by $\frac{k_B T_e}{e}$.

In the above we have assumed that phase speed is much larger than ion thermal speed but smaller than the thermal speed of electrons and positrons [22] and the external constant magnetic field is directed along the z-axis, i.e. $\overrightarrow{B_0} = B_0 \hat{z}$. The propagation is considered to be in the (x, z) plane.

The equilibrium charge neutrality condition is $n_{e0} = n_{i0} + n_{p0}$, where n_{s0} is the unperturbed number density of the particle species s(s equals e for the electrons, i for the ions, and p for the positrons). The equilibrium charge neutrality condition in normalized form will be $\frac{n_{i0}}{n_{e0}} = 1-\chi$, where $\chi = \frac{n_{p0}}{n_{e0}}$. The normalized number densities of q-nonextensive electrons and positrons are given by [21,46,48,50]:

$$n_e = \left[1 + (q-1)\,\phi\right]^{\frac{1+q}{2(q-1)}} \tag{3}$$

and

$$n_p = \chi\left[1 - (q-1)\,\sigma_P \phi\right]^{\frac{1+q}{2(q-1)}}, \tag{4}$$

where $\sigma_p = T_e/T_p$ is the ratio between electron and positron temperatures; parameter q stands for the strength of nonextensivity. The nonextensive distribution is unnormalizable under the condition $q < -1$ and for $q \to 1$ the distribution approaches the well-known Maxwell–Boltzmann distribution.

Simplifying Eqs. (3) and (4), Poisson's equation can be written as:

$$\nabla^2\phi = \left(\frac{q+1}{2}\right)\left[1 + \chi\sigma_P\right]\phi + \frac{(q+1)\,(q-3)}{4}\left(1 + \chi\sigma_p^2\right)\phi^2 + \frac{(q+1)\,(q-3)\,(3q-5)}{16}\left(1 + \chi\sigma_p^3\right)\phi^3 - n_i. \tag{5}$$

3. Derivation of the NLS equation

To study the nonlinear evolution of the wave we make the following Fourier expansions for the field quantities [58]:

$$F = \varepsilon^2 F_0^{'} + \sum_{s=1}^{\infty} \varepsilon^s \{F_s \exp(is\psi) + F_s^* \exp(-is\psi)\}, \tag{6}$$

where F stands for the field quantities n_i, $\overrightarrow{u_i}$, and ϕ; $F_0^{'}$ and F_s are assumed be functions ξ and τ where $\xi = \varepsilon(l_x x + l_y y + l_z z - C_g t)$ and $\tau = \varepsilon^2 \tau$; ε is a small parameter measuring weakness of dispersion and nonlinearity; l_x, l_y, l_z are the directional cosines of the wave vector $\vec{k}$ along the x, y, and z directions, respectively; $C_g = \frac{d\omega}{dk}$ is the group velocity; and $\psi = k_x \hat{x} + k_z \hat{z} - \omega t$. In the x-z plane wave vector $\vec{k}$ will be given by $\vec{k} = \hat{x} k \sin\theta + \hat{z} k \cos\theta$.

Now substituting the expansions of Eq. (6) into Eqs. (1), (2), and (5) and then equating from both sides the coefficients of $\exp(\mathrm{i}\psi)$, $\exp(2\mathrm{i}\psi)$ and terms independent of ψ, we obtain three sets of equations, which we call respectively I, II, and III (see the Appendix).

To solve these equations we make the following perturbation expansion for the z-component of the field quantities:

$$X = X^{(1)} + \varepsilon X^{(2)} + \varepsilon^2 X^{(3)} + \cdots \tag{7}$$

For x-component of the field quantities it is:

$$X = \sqrt{\varepsilon} X^{(1)} + \sqrt[3]{\varepsilon} X^{(2)} + \sqrt[5]{\varepsilon} X^{(3)} + \cdots \tag{8}$$

Solving the lowest-order equations obtained from the set of equations I, we get the following solutions for the first harmonic quantities in the lowest order:

$$u_{ix_1}^{(1)} = \frac{k_x \omega}{\omega^2 - \omega_{ci}^2} \alpha.$$

For ion-acoustic waves$\omega << \omega_{c_i}$ and we can write:

$$u_{ix_1}^{(1)} \approx -\frac{k_x \omega}{\omega_{ci}^2} \alpha, \tag{9}$$

$$u_{iy_1}^{(1)} \approx i \frac{k_x}{\omega_{ci}} \alpha, \tag{10}$$

$$u_{iz_1}^{(1)} = \frac{k_z}{\omega} \alpha, \tag{11}$$

and

$$n_{i1}^{(1)} = q_1 + k^2, \tag{12}$$

in which

$$\alpha = \phi_1^{(1)} \tag{13}$$

and

$$a_1 = \frac{q+1}{2} \left(1 + \chi \sigma_P\right). \tag{14}$$

The linear dispersion relation obtained for the low frequency ion-acoustic wave is

$$\omega^2 = \frac{k_z^2 \omega_{ci}^2 \left(1 - \chi\right)}{\omega_{ci}^2 \left(a_1 + k^2\right) + k_x^2 \left(1 - \chi\right)}, \tag{15}$$

and its group velocity is

$$C_g = \frac{d\omega}{dk} = \frac{1}{\omega} \left[\frac{\omega_{ci}^4 k \left(1 - \chi\right) \cos^2\theta \left(a_1 + 2k^2\right)}{\left\{\omega_{ci}^2 \left(a_1 + k^2\right) + k_x^2 \left(1 - \chi\right)\right\}^2} \right]. \tag{16}$$

In Figures 1 and 2 we show graphically the linear dispersion relation with wave frequency ω plotted as a function of the wave vector k for various values of ω_{ci} and q. Figure 1 shows that the wave frequency increases with increase in the strength of magnetic field while Figure 2 shows that the wave frequency decreases with increase in the value of q in the range of $\frac{1}{3} < q < 1$. It is also numerically found that the wave frequency depends sensitively on the obliqueness of wave propagation. In the case of unmagnetized plasma, ω_{ci} must be zero and then the wave propagation is one-dimensional in nature. Considering this situation we get the same dispersion relation as obtained by Eslami et al. [56]. By putting q = 1 (Boltzmann distribution) in the linear dispersion relation of Eq. (15) we find that our result reduces to that obtained by Jehan et al. [24].

First harmonic quantities in the second order can be obtained by substituting the perturbation expansions of Eqs. (7) and (8) in the set of equations I and solving order ε^2 equations. Thus, we obtain the following:

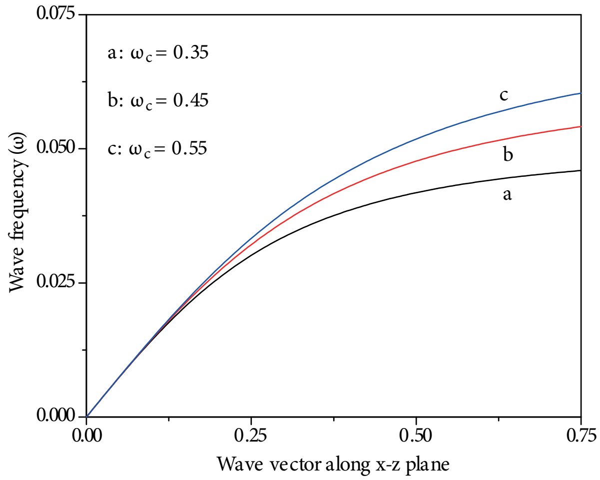

Figure 1. Linear dispersion relation for different magnetic field strength (ω_{c_i} = 0.35, 0.45, and 0.55); other parameters are q= 0.35, θ = 30^0, σ_P = 0.3, and χ= 0.3.

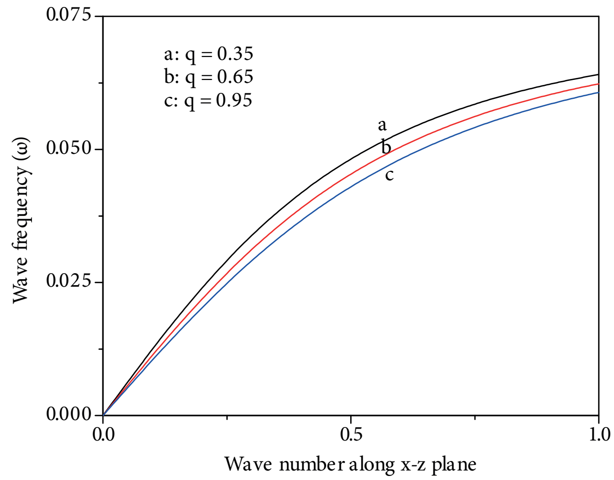

Figure 2. Linear dispersion relation for different values of nonextensivity (q = 0.35, 0.65, and 0.95); other parameters are ω_{c_i} = 0.5, θ = 30^0, σ_P = 0.5, and χ= 0.4.

$$\phi_1^{(2)} = 0, \tag{17}$$

$$u_{iz1}^{(2)} = \frac{i}{k_z\left(1-\chi\right)}\left[\left(1-\chi\right)\frac{k_z}{\omega} - 2\omega k - C_g\left(a_1 + k^2\right)\right]\frac{d\alpha}{d\xi},$$

$$n_{iz1}^{(2)} = -2ik\frac{d\alpha}{d\xi}.$$

The second harmonic quantities in the lowest order obtained from the set of equations II after substituting the expansions of Eq. (6) are the following:

$$\phi_2^{(1)} = \frac{k_z^2\left[\left(1-\chi\right)A + 2\omega\left(a_1+k^2\right)\right] - 2\omega^3\omega_{ci}^2 a_2}{2\omega\left[\omega^2\left(a_1+4k^2\right) - k_z^2\left(1-\chi\right)\right]}\alpha^2,$$

$$u_{ix2}^{(1)} = \frac{k_x A}{\omega_{ci}^2}\alpha^2 - \frac{4k_x\omega}{\omega_{ci}^2}\phi_2,$$

$$u_{iy2}^{(1)} = i\left[\frac{k_x A}{2\omega_{ci}\omega}\alpha^2 - \frac{\omega_{ci}u_{ix2}}{2\omega}\right],$$

$$u_{iz2}^{(1)} = \frac{k_z}{\omega}\phi_2 + \frac{k_z}{2\omega^2}A.\alpha^2,$$

$$n_{i2}^{(1)} = \left(a_1 + 4k^2\right)\phi_2 + a_2\alpha^2, \tag{18}$$

where

$$A = \frac{k_z^2}{\omega} - \frac{k_x^2\omega}{\omega_{ci}^2},$$

$$a_2 = \frac{\left(1+\chi\sigma_p^2\right)\left(q+1\right)\left(q-3\right)}{4}. \tag{19}$$

The zeroth harmonic components generated through nonlinear self-interaction of the finite amplitude wave are obtained from the set of equations III after substituting the expansions of Eq. (6):

$$\phi_0^{(1)} = \frac{2k_z\left[(1-\chi)\,k_z + 2\left(a_1+k^2\right)\omega\right] - C_g a_2 \omega^2 \omega_{ci}^2}{\omega^2\left[C_g^2 a_1 - 2\,(1-\chi)\right]}\alpha\alpha^*,$$

$$u_{ix0}^{(1)} = \frac{\phi_0}{C_g} - \frac{2k_z k_x}{C_g \omega_{ci}^2}\alpha\alpha^*,$$

$$u_{iy0}^{(1)} = i\frac{2k_z k_x}{C_g \omega_{ci}\omega}\alpha\alpha^*,$$

$$u_{iz0}^{(1)} = \frac{\phi_0}{C_g} + \frac{2k_z^2}{C_g \omega^2}\alpha\alpha^*, \tag{20}$$

$$n_{i0}^{(1)} = a_1\phi_0 + a_2\alpha\alpha^*.$$

Now to derive the desired NLS equation we need first harmonic quantities in the third order. Collecting coefficients of ε^3 from both sides of the set of equations I after substituting the perturbation expansions of Eqs. (7) and (8), we get a set of equations for first harmonic quantities in the third order, from which after proper elimination we obtain the following NLS equation:

$$i\frac{\partial\alpha}{\partial\tau} + P.\frac{\partial^2\alpha}{\partial\xi^2} = Q.\alpha^2\alpha^*, \tag{21}$$

where

$$P = \frac{\omega^2}{\left(a_1+k^2\right)\omega^2 - (1-\chi)\,k_z^2}\left[\left[(1-\chi)\left\{\frac{k_z}{\omega} - \frac{k_x\omega}{\omega_{ci}^2}\right\} - 2\omega k\right]\left(\frac{1}{k_z} - \frac{C_g}{\omega}\right) - 3\omega - 2kC_g\right] \tag{22}$$

and

$$Q = \frac{\omega^2}{\left(a_1+k^2\right)\omega^2 - (1-\chi)\,k_z^2}\left[F_1 + \frac{(1-\chi)\,k_z}{\omega}F_3 - \omega F_4\right], \tag{23}$$

in which

$$\begin{aligned} F_1 \;&=\; \left(a_1+4k^2\right)A\phi_2 + a_1 A\phi_0 + \frac{\left(a_1+k^2\right)}{C_g}\left\{k_x\left(\phi_0 - \frac{2k_z k_x}{\omega_{ci}^2}\right) + k_z\left(\phi_0 + \frac{2k_z^2}{\omega^2}\right)\right\} \\ &\quad + \left(a_1+k^2\right)\left[\frac{k_x^2\left(A-4\omega\phi_2\right)}{\omega_{ci}^2} + \frac{k_z^2}{\omega}\left(\phi_2 + \frac{A}{2}\right)\right], \end{aligned} \tag{24}$$

$$\begin{aligned} F_3 \;&=\; \frac{k_z}{\omega}A.\left(\phi_2 + \frac{A}{2\omega^2}\right) + \frac{k_z}{\omega}\left[\frac{k_x^2\left(A-4\omega\phi_2\right)}{\omega_{ci}^2} + \frac{k_z^2}{\omega}\left(\phi_2 + \frac{A}{2\omega^2}\right)\right] \\ &\quad + \left(\phi_0 + \frac{2k_z^2}{\omega^2}\right)\frac{1}{C_g}\left\{\frac{2k_z^2}{\omega} - \frac{k_x^2\omega}{\omega_{ci}^2}\right\} + \frac{k_x k_z}{\omega C_g}\left\{\phi_0 - \frac{2k_x k_z}{\omega_{ci}^2}\right\}, \end{aligned} \tag{25}$$

$$F_4 = a_2\left(\phi_2 + \phi_0\right) + a_3, \tag{26}$$

and

$$a_3 = \frac{(q+1)(q-3)(3q-5)\left(1+\chi\sigma_p^3\right)}{16}. \tag{27}$$

The NLS equation of Eq. (21) describes the nonlinear evolution of the amplitude of ion-acoustic waves in magnetized e-p-i plasma with q-nonextensive electrons and positrons. The NLS equation (Eq. (21)) has been studied extensively in connection with the nonlinear propagation of different wave modes in plasma.

Thus, a uniform wave train may be modulationally stable or unstable depending on the sign of the product of the group dispersive and the nonlinearity coefficient, i.e. PQ. It has been shown that the wave becomes modulationally unstable for PQ $<$ 0 and stable for PQ $>$ 0.

In the unstable region the growth rate of instability has a maximum value g_m given by:

$$g_m = |Q|\,\alpha_0^2. \tag{28}$$

For the unstable wave packet (PQ $<$ 0), the ion-acoustic wave generates bright solitons, and for the stable wave packet ($PQ > 0$) it generates dark solitons. Thus, the sign of the product PQ determines the stability/instability profile of ion-acoustic waves as well as the type of soliton structure. The width of the solitons is found to be proportional to $|P|$ and soliton amplitude is inversely proportional to $|Q|$. Thus, we find that the nonlinear evolution of the wave depends on the product PQ. As the coefficients P and Q depend on different plasma parameters such as nonextensive parameter q, positron concentration χ, ion gyrofrequency ω_{ci}, obliqueness of propagation, and electron-positron temperature ratio σ_p, product PQ can have both positive and negative values over different parametric regions. Thus, these physical parameters are expected to significantly influence the stability character of the modulated ion-acoustic wave.

4. Results and discussion

It has been found numerically that there is a critical value of the wave number below which the wave is stable ($PQ > 0$) and above which the wave is unstable (PQ < 0). The value of the critical wave number is found to depend on different physical parameters. In the unstable region ($PQ < 0$) it can be show that the ion-acoustic wave can propagate as an envelope bright soliton. On the other hand, in the stable region ($PQ > 0$), the wave can propagate in the form of an envelope hole called a dark soliton.

In order to investigate the stability profile we have numerically investigated the ratio P/Q in terms of different parameters involved. In Figure 3 we show the variation of P/Q with wave number for different values of the ion gyrofrequency (ω_{ci}), keeping other plasma parameters constant. It shows that increase in ω_{ci}, i.e. increase in magnetic field strength, lowers the value of the critical wave number separating stable and unstable regions. Thus, increase in the strength of the magnetic field tends to destabilize the wave. It is also noticed that as ω_{ci} increases the width of both the dark and bright solitons increases. These observations agree with those obtained by Jehan et al. [24] and are found to be true only for $0.36 < \omega_{ci} < 0.70$. For values of ω_{ci} beyond this range (i.e. $\omega_{ci} < 0.36$ or $\omega_{ci} > 0.70$), the magnetic field is shown to have an opposite effect.

To study the effect of nonextensivity on the nonlinear properties of ion-acoustic waves we plot in Figure 4 P/Q as a function of k for different values of the nonextensive parameter q, keeping other plasma parameters constant. It is seen that as q increases in the range of $\frac{1}{3} < q < 1$ the critical wave number separating the stable and unstable regions decreases. Also, with increase in q, the width of the dark soliton increases and that of the bright soliton decreases. Similar results were obtained by Eslami et al. [56] for the nonextensive parameter in the range of q $<$ 0 for unmagnetized plasma.

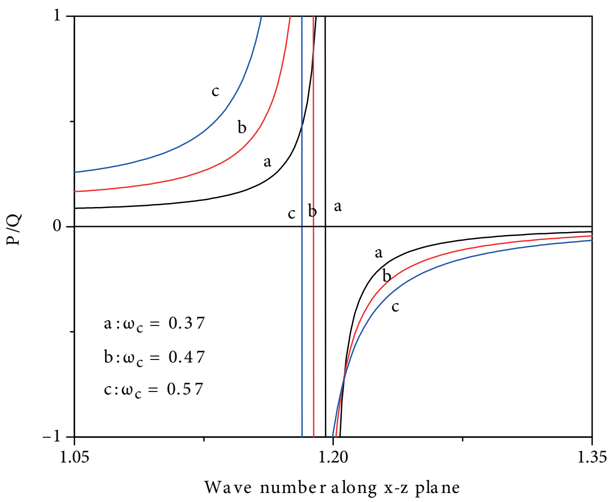

Figure 3. P/Q vs. wave number (k) plot for different magnetic field strength (ω_{c_i} = 0.37, 0.47, and 0.57); other parameters are q = 0.35, θ = 10^0, σ_P = 0.25, and χ=0.25.

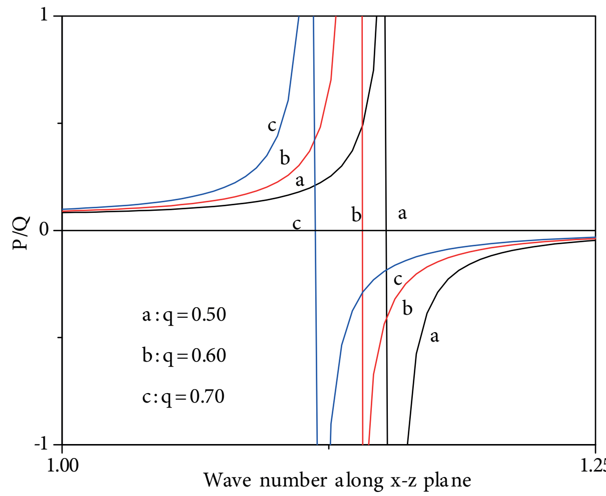

Figure 4. P/Q vs. wave number (k) plot for different values of nonextensivity (q = 0.50, 0.60, and 0.70); other parameters are ω_{c_i} = 0.57, θ = 10^0, σ_P = 0.5, and χ=0.45.

We have also studied the dependence of modulational instability growth rate $|Q|$ on various physical parameters. In Figure 5 we show the dependence of the instability growth rate on the strength of the magnetic field and nonextensive parameter q. It shows that for a given value of q the instability growth rate increases with increase in magnetic field strength. Also, for a given magnetic field, the instability growth rate decreases with increase in nonextensive parameter q in the range of $\frac{1}{3} < q < 1$. One possible physical explanation for this is that as q approaches 1 in the range of $\frac{1}{3} < q < 1$ the electrons and positrons evolve from nonextensive distribution towards equilibrium Maxwellian distribution. Consequently, there is a relatively smaller number of energetic particles, which results in lower growth rate of instability.

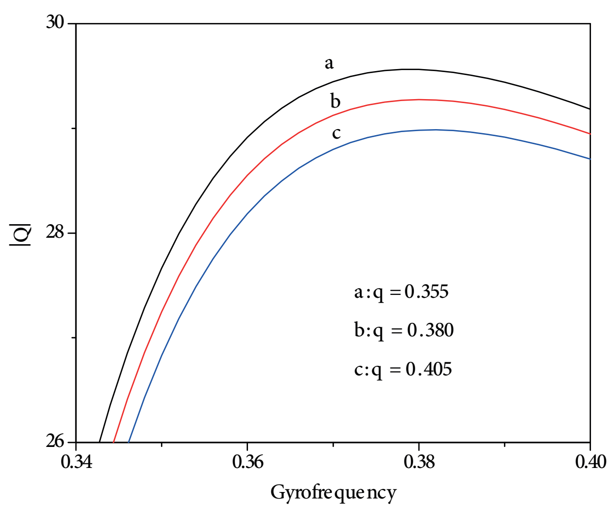

Figure 5. Instability growth rate $|Q|$ vs. ω_{c_i} plot for different values of nonextensivity (q= 0.355, 0.380, and 0.405); other parameters are θ = 10^0, σ_P = 0.5, and χ = 0.4.

To conclude, we have studied both analytically and numerically the modulational instability of low-frequency ion-acoustic waves in magnetized e-p-i plasma having q-nonextensive electrons and positrons. It has been shown that the presence of an external magnetic field and the nonextensivity of plasma species play important roles in determining the critical wave number above which the wave becomes modulationally unstable and also in determining the nature and structure of envelope solitons excited in the model plasma under study. It has been observed that increase in the strength of the magnetic field or increase in the value of the q-parameter in the range of $\frac{1}{3} < q < 1$ widens the instability region of the wave in the k-space. It was also noticed that increase in the strength of the magnetic field increases the width of both the dark and bright solitons. On the other hand, increase in the value of the q parameter in the range of $\frac{1}{3} < q < 1$ increases the width of dark solitons and decreases that of the bright soliton.

Furthermore, our results in the limit of no magnetic field qualitatively agree with that obtained by Eslami et al. [56]. It may be noted that Eslami et al. [56] considered unmagnetized plasma, whereas in the present paper we have considered magnetized plasma. For numerical analysis they used the range $q < 0$ and $q > 1$, but we have used the most relevant range, $\frac{1}{3} < q < 1$. Our results also agree with those obtained by Jehan et al. [24] in the limit of Boltzmann's distributions ($q \to 1$) for electrons and positrons.

Finally, we would like to point out that the present results may be helpful to understand the amplitude modulation of ion-acoustic waves and formation of bright and dark solitons in magnetized e-p-i plasma having q-nonextensive electrons and positrons, which are common in many space and astrophysical plasma environments.

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Appendix

Set of equations I:

$$\varepsilon\left(-i\omega-\varepsilon C_g\frac{\partial}{\partial\xi}+\varepsilon^2\frac{\partial}{\partial\tau}\right)n_i^{(1)}+\varepsilon\left(1-\chi\right)\left(ik_x+\varepsilon\frac{\partial}{\partial\xi}\right)u_{ix}^{(1)}+\varepsilon\left(1-\chi\right)\left(ik_z+\varepsilon\frac{\partial}{\partial\xi}\right)u_{iz}^{(1)}=$$

$$-\varepsilon^3 ik_x\left[n_{i0}^{(1)}u_{ix1}^{(1)}+u_{ix0}^{(1)}n_{i1}^{(1)}+n_{i1}^{*(1)}u_{ix2}^{(1)}+u_{ix1}^{*(1)}n_{i2}^{(1)}\right]$$

$$-\varepsilon^3 ik_z\left[n_{i0}^{(1)}u_{iz1}^{(1)}+u_{iz0}^{(1)}n_{i1}^{(1)}+n_{i1}^{*(1)}u_{iz2}^{(1)}+u_{iz1}^{*(1)}n_{i2}^{(1)}\right] \tag{A1}$$

$$\varepsilon\left(-i\omega-\varepsilon C_g\frac{\partial}{\partial\xi}+\varepsilon^2\frac{\partial}{\partial\tau}\right)u_{ix}^{(1)}-\omega_{ci}u_{iy1}^{(1)}+\varepsilon\left(ik_x+\varepsilon\frac{\partial}{\partial\xi}\right)\phi_1^{(1)}=$$

$$-\left[2\varepsilon^3 ik_x\left(u_{ix0}^{(1)}u_{ix1}^{(1)}+u_{ix2}^{(1)}u_{ix1}^{*(1)}\right)+\varepsilon^3 ik_z\left(u_{iz0}^{(1)}u_{ix1}^{(1)}+u_{ix0}^{(1)}u_{iz1}^{(1)}+u_{iz2}^{(1)}u_{ix1}^{*(1)}+u_{ix2}^{(1)}u_{iz1}^{*(1)}\right)\right] \tag{A2}$$

$$\varepsilon\left(-i\omega-\varepsilon C_g\frac{\partial}{\partial\xi}+\varepsilon^2\frac{\partial}{\partial\tau}\right)u_{iy}^{(1)}-\omega_{ci}u_{ix1}^{(1)}+\varepsilon\left(ik_y+\varepsilon\frac{\partial}{\partial\xi}\right)\phi_1^{(1)}=$$

$$-\varepsilon^3 ik_x\left(u_{iy0}^{(1)}u_{ix1}^{(1)}+u_{ix0}^{(1)}u_{iy1}^{(1)}+u_{iy2}^{(1)}u_{ix1}^{*(1)}+u_{ix2}^{(1)}u_{iy1}^{*(1)}\right)$$

$$-\varepsilon^3 ik_z\left(u_{iz0}^{(1)}u_{iy1}^{(1)}+u_{iy0}^{(1)}u_{iz1}^{(1)}+u_{iz2}^{(1)}u_{iy1}^{*(1)}+u_{iy2}^{(1)}u_{iz1}^{*(1)}\right) \tag{A3}$$

$$\left(-i\omega-\varepsilon C_g\frac{\partial}{\partial\xi}+\varepsilon^2\frac{\partial}{\partial\tau}\right)u_{iz}^{(1)}+\varepsilon\left(ik_x+\varepsilon\frac{\partial}{\partial\xi}\right)\phi_1^{(1)}=$$

$$-\left[2\varepsilon^3 ik_z\left(u_{iz0}^{(1)}u_{iz1}^{(1)}+u_{iz2}^{(1)}u_{iz1}^{*(1)}\right)+\varepsilon^3 ik_x\left(u_{iz0}^{(1)}u_{ix1}^{(1)}+u_{ix0}^{(1)}u_{iz1}^{(1)}+u_{iz2}^{(1)}u_{ix1}^{*(1)}+u_{ix2}^{(1)}u_{iz1}^{*(1)}\right)\right] \tag{A4}$$

$$-\varepsilon\left(k_x^2+k_y^2+k_z^2\right)\phi_1-\varepsilon\left(\frac{q+1}{2}\right)\phi\left(1+\chi\sigma_p\right)+\varepsilon n_{i1}=-3\varepsilon^3\frac{\partial^2\phi_1}{\partial\xi^2}+$$

$$2\varepsilon^3\left[\phi_0\phi_1+\phi_2\phi_1^*\right]\left[\frac{(q+1)(q-3)\left(1+\chi\sigma_p^2\right)}{8}\right]+\left[\frac{(q+1)(q-3)(3q-5)\left(1+\chi\sigma_p^3\right)}{48}\right]\varepsilon^3\phi_1^2\phi_1^* \tag{A5}$$

Set of equations II:

$$-i\omega n_{i2}^{(1)}+ik_x\left(1-\chi\right)u_{i2x}^{(1)}=-\left[ik_z\left(1-\chi\right)u_{i2z}^{(1)}+ik_x n_{i1}^{(1)}u_{ix1}^{(1)}+ik_z n_{i1}^{(1)}u_{iz1}^{(1)}\right] \tag{A6}$$

$$-2i\omega u_{ix2}^{(1)}+2ik_x\phi_2^{(1)}-\omega_{ci}u_{iy2}^{(1)}=-\left[ik_x\left(u_{i1x}\right)^2+ik_z u_{ix1}^{(1)}u_{iz1}^{(1)}\right] \tag{A7}$$

$$-2i\omega u_{iy2}^{(1)}+2ik_y\phi_2^{(1)}-\omega_{ci}u_{ix2}^{(1)}=-\left[ik_x u_{ix1}^{(1)}u_{iy1}^{(1)}+ik_z u_{iz1}^{(1)}u_{iy1}^{(1)}\right] \tag{A8}$$

$$-2i\omega u_{iz2}^{(1)}+2ik_z\phi_2^{(1)}=-\left[ik_z\left(u_{i1z}\right)^2+ik_x u_{ix1}^{(1)}u_{iz1}^{(1)}\right] \tag{A9}$$

$$-4\left(k_x^2+k_y^2+k_z^2\right)\phi_2^{(1)}-\left(\frac{q+1}{2}\right)\phi_2^{(1)}\left(1+\chi\sigma_p\right)+n_{i2}^{(1)}=$$

$$2\left[\frac{(q+1)(q-3)\left(1+\chi\sigma_p^2\right)}{8}\right]\phi_1^2 \tag{A10}$$

Set of equations III:

$$-C_g n_{i0} + (1-\chi)\left(u_{ix0} + u_{iz0}\right) = -\left[n_{i1}^{(1)} u_{ix1}^{*} + u_{ix1}^{(1)} n_{i1}^{*} + n_{i1}^{(1)} u_{iz1}^{*} + u_{iz1}^{(1)} n_{i1}^{*}\right] \tag{A11}$$

$$-C_g u_{ix0} + \phi_0 = -\left[2u_{ix1}^{(1)} u_{ix1}^{*} + \left(u_{iy1}^{(1)} u_{ix1}^{*} + u_{ix1}^{(1)} u_{iy1}^{*}\right) + \left(u_{iz1}^{(1)} u_{ix1}^{*} + u_{ix1}^{(1)} u_{iz1}^{*}\right)\right] \tag{A12}$$

$$-C_g u_{iy0} = -\left[2u_{iy1}^{(1)} u_{iy1}^{*} + \left(u_{iy1}^{(1)} u_{ix1}^{*} + u_{ix1}^{(1)} u_{iy1}^{*}\right) + \left(u_{iz1}^{(1)} u_{iy1}^{*} + u_{iy1}^{(1)} u_{iz1}^{*}\right)\right] \tag{A13}$$

$$-C_g u_{iz0} + \phi_0 = -\left[2u_{iz1}^{(1)} u_{iz1}^{*} + \left(u_{iz1}^{(1)} u_{ix1}^{*} + u_{ix1}^{(1)} u_{iz1}^{*}\right) + \left(u_{iz1}^{(1)} u_{ix1}^{*} + u_{ix1}^{(1)} u_{iz1}^{*}\right)\right] \tag{A15}$$

$$-\left(\frac{q+1}{2}\right)\phi_0^{(1)}\left(1+\chi\sigma_p\right) + n_{i0}^{(1)} = 2\left[\frac{(q+1)(q-3)\left(1+\chi\sigma_p^2\right)}{8}\right]\phi_1\phi_1^{*} \tag{A15}$$