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Magnetized Kantowski–Sachs bulk viscous string cosmological models with decaying vacuum energy density $\Lambda(t)$

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Abstract: In this paper, we deal with bulk viscous magnetized Kantowski–Sachs string cosmological models in the presence of time variable cosmological term Λ. To obtain a deterministic model, we assume the conditions $\sigma \propto \theta$ and $\zeta\theta =$ constant, where σ is the shear, θ the expansion in the model, and ζ the coefficient of bulk viscosity. The value of cosmological constant for the models is found to be small and positive, which is supported by the results from recent observations (SN 1a). The behavior of the model in the presence and absence of magnetic fields together with physical and geometrical aspects is also discussed.

Key words: Kantowski–Sachs universe, bulk viscous, magnetized cosmic string, variable cosmological constant

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1. Introduction

Recent observations of the type 1a supernova (SN 1a) established that our universe is currently accelerating [1–4] and the observations (SN 1a) of high confidence level [5–7] have further confirmed this. In addition, measurements of the cosmic microwave background (CMB) anisotropies [8–10] and large-scale structure (LSS) [11–13] strongly indicate that our universe is dominated by a component with negative pressure, dubbed as dark energy. Numerous dynamical dark energy models have been proposed in the literature, such as quintessence [14], phantom [15], k-essence [16], and tachyon [17]. However, the simplest and most theoretically appealing candidate for dark energy is vacuum energy (or the cosmological constant Λ) with a constant equation of state parameter equal to -1. Recent observations indicate that $\Lambda \sim 10^{-55}$ cm^{-2} while the particle physics prediction for Λ is greater than this value by a factor of order 10^{120}. This discrepancy is known as the cosmological constant problem. The simplest way out of this problem is to consider a varying cosmological term, which decays from a huge value at initial times to the small value observed in these days in an expanding universe [18–22].

In recent years, there has been considerable interest in string cosmology. It is generally assumed that after the big bang the universe may have undergone a series of phase transitions as its temperature lowered below some critical temperature as predicted by grand unified theories [23–25]. It can give rise to topologically stable defects such as strings, domain walls, and monopoles. Among these cosmological structures, cosmic strings is the most interesting consequences [26], because it is believed that cosmic strings give rise to density perturbations, which lead to formation galaxies [27]. The general relativistic treatment of strings has been initially given by Letelier [28] and Stachel [29]. Letelier [30] obtained massive string cosmological models in Bianchi type-I and

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Kantowski–Sachs space-times. Afterwards, Banerjee et al. [31] investigated an axially symmetric Bianchi type-I string dust cosmological model in the presence and absence of a magnetic field. The exact solutions of string cosmology for Bianchi type-II, VI, VIII, and IX space-times have been studied by Krori et al. [32] and Wang [33,34].

On the other hand, the matter distribution is satisfactorily described by perfect fluid due to large-scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. It is well known that when neutrino decoupling occurs during radiation era and decoupling of radiation with matter takes place during recombination era, the matter behaves like a viscous fluid in an early stage of the universe. Bulk viscosity is associated with GUT phase transition and string creation. The effect of viscosity on the evolution of cosmological models and the role of viscosity in avoiding the initial big bang singularity have been studied by several authors [35–39].

The study of magnetic fields provides an effective way to understand the initial phases of cosmic evolution. Primordial magnetic fields of cosmological origin have been discussed by Asseo and Sol [40] and Madsen [41]. Melvin [42] suggested, in the cosmological solution for dust and electromagnetic fields, that during the evolution of the universe, the matter was in highly ionized state and smoothly coupled with electromagnetic and consequently formed a neutral matter as a result of expansion of the universe. Hence, in a string dust universe the presence of magnetic fields is not unrealistic. Recently, the occurrence of magnetic fields on galactic scales and their importance for a variety of astrophysical phenomena has been pointed out by several authors such as Chakraborty [43], Tikekar and Patel [44], and Singh and Singh [45].

The advances in particle physics applied to the early universe have resulted in an interest in solutions to the Einstein equations with somewhat usual properties. Therefore, a set of articles developed to the new Kantowski–Sachs models has appeared. Weber [46, 47] has performed a qualitative study of the Kantowski–Sachs [48] cosmological models. Lorenz [49], Gron [50], Matravers [51], and Krori et al. [52] have also studied cosmological models for the Kantowski–Sachs space-time. Recently, Wang [53] discussed the Kantowski–Sachs string cosmological model with bulk viscosity in general relativity. Pradhan and Yadav [54] have investigated Kantowski–Sachs models with variable Newtonian gravitational constant (G) and cosmological term (Λ), whereas Katore [55] has studied the magnetized Kantowski–Sachs inflationary universe in general relativity.

Motivated by the above discussions, in this paper, we have focused upon the problem of establishing a formalism for studying a new integrability of magnetized Kantowski–Sachs bulk viscous string cosmological models with decaying vacuum energy density $\Lambda(t)$ in general relativity. The behavior of the models in the presence and absence of magnetic fields are also discussed.

2. Metric and field equations

We consider the Kantowski–Sachs space-time metric in the form

$$ds^2 = -dt^2 + A^2 dr^2 + B^2(d\theta^2 + \sin^2\theta \, d\varphi^2), \tag{1}$$

where A and B are the function of cosmic time t only.

The energy momentum tensor for a cloud string with a magnetic field in a co-moving coordinate system is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \zeta\theta(u_i u_j - g_{ij}) + E_{ij}, \tag{2}$$

where the vector u_i describes the cloud 4-velocity and x_i represents a direction of anisotropy, i.e. the string satisfies the relations

$$u^i u_i = -x^i x_i = -1, \quad u^i x_i = 0. \tag{3}$$

Here ρ is the rest energy of the cloud strings with massive particles attached to them. It is given by $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the density of tension that characterizes the strings. The energy momentum for the magnetic field is

$$E_{i\,j} = \frac{1}{4\pi}\left(F_{i\,k}F_{j\,l}g^{k\,l} - \frac{1}{4}g_{i\,j}F^{k\,l}F_{k\,l}\right) \quad , \tag{4}$$

where $F_{i\,j}$ is the electromagnetic field tensor satisfying Maxwell's equations

$$F_{[i\,j;k]} = 0, \quad \left(F^{i\,j}\sqrt{-g}\right)_{;\,j} = 0. \tag{5}$$

In commoving coordinates, the incident magnetic field is taken along x-axis, with the help of Maxwell's equations (5); the only nonvanishing component of $F_{i\,j}$ is

$$F_{23} = I = Cons\tan t. \tag{6}$$

In order to determine the system completely, we consider Takabayasi's equation of state [56],

$$\rho = k\lambda, \tag{7}$$

where k is constant.

The Einstein's field equations (in gravitational unit $c = 1$, $8\pi\,G = 1$)

$$R_{i\,j} - \frac{1}{2}g_{i\,j}R = -8\pi\,GT_{i\,j} - g_{i\,j}\Lambda, \tag{8}$$

For metric (1), the field equation (8) with the help of Eqs. (2)–(7) takes the form

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = \lambda + \zeta\theta + \frac{I^2}{8\pi\,B^4\sin^2\theta} + \Lambda, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \zeta\theta - \frac{I^2}{8\pi\,B^4\sin^2\theta} + \Lambda, \tag{10}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = \rho + \frac{I^2}{8\pi\,B^4\sin^2\theta} + \Lambda. \tag{11}$$

Here, and also in what follows, a dot indicates ordinary differentiation with respect to cosmic time t.

3. Solution in presence of bulk viscosity and magnetic field

The research on exact solutions is based on some physically reasonable restrictions used to simplify the Einstein equations. Eqs. (9)–(11) are 3 independent equations involving 5 unknowns, viz. A, B, λ, ζ, and Λ. To solve the system completely, we need 2 extra conditions. Firstly, we assume that the coefficient of bulk viscosity ζ is inversely proportional to the expansion θ. This condition leads to

$$\zeta\theta = L\,(cons\tan t). \tag{12}$$

The motive behind assuming this condition is explained in the literature [57–59]. Secondly, we consider the expansion scalar θ is proportional to the shear σ, i.e. $\theta \propto \sigma$ [60]. This condition leads to

$$A = B^n, \tag{13}$$

where $n > 0$ a constant.

From Eqs. (9)–(11), with the help of (12) and (13), eliminating λ and Λ, we have

$$\ddot{B} + \alpha B^{-1}\dot{B}^2 = \frac{\beta}{B} + \frac{\gamma M}{B^3} + \delta LB, \tag{14}$$

where

$$\alpha = \frac{(k-1)(1-n^2) - 2n}{2-(1-k)(1-n)}, \quad \beta = \frac{(1-k)}{2-(1-k)(1-n)},$$

$\gamma = \frac{2(k-1)}{2-(1-k)(1-n)}$, $\delta = \frac{1}{2-(1-k)(1-n)}$, and $M = \frac{I^2}{8\pi \sin^2\theta}$.

To solve Eq. (14), we denote $\dot{B} = \eta$, then $\ddot{B} = \eta\frac{d\eta}{dB}$, and Eq. (14) can be reduced to the first order differential equation in the following form:

$$2\eta\frac{d\eta}{dB} + 2\alpha B^{-1}\eta^2 = \frac{2\beta}{B} + \frac{2\gamma M}{B^3} + 2\delta LB. \tag{15}$$

Eq. (15) can be further written as

$$\frac{d}{dB}\left(\eta^2 B^{2\alpha}\right) = 2\beta B^{2\alpha-1} + 2\gamma MB^{2\alpha-3} + 2\delta LB^{2\alpha+1}. \tag{16}$$

Thus we obtain

$$dt = \left[\frac{\beta}{\alpha} + \frac{\gamma MB^{-2}}{(\alpha-1)} + \frac{\delta LB^2}{(\alpha+1)} + CB^{-2\alpha}\right]^{-\frac{1}{2}} dB, \tag{17}$$

where C is the constant of integration. Hence the line element (1) becomes

$$ds^2 = -\left[\frac{\beta}{\alpha} + \frac{\gamma MB^{-2}}{(\alpha-1)} + \frac{\delta LB^2}{(\alpha+1)} + CB^{-2\alpha}\right]^{-1} dB^2 + B^{2n}dr^2 + B^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{18}$$

Under suitable transformation of coordinates, Eq. (18) reduces to

$$ds^2 = -\left[\frac{\beta}{\alpha} + \frac{\gamma MT^{-2}}{(\alpha-1)} + \frac{\delta LT^2}{(\alpha+1)} + CT^{-2\alpha}\right]^{-1} dT^2 + T^{2n}dr^2 + T^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{19}$$

where $\alpha \neq \pm 1$ and the cosmic scale $B = T$ can be determined by Eq. (19).

3.1. Some physical and geometric features

For the model (19), the other physical parameters can be easily obtained. The expressions for the energy density ρ, the string tension density λ, the particle energy density ρ_p, the coefficient of bulk viscosity ζ, and the cosmological term Λ are given by

$$\rho = \left\{2n\frac{\beta}{\alpha} + (2n+1)\frac{\gamma M}{(\alpha-1)}T^{-2} + (2n-1)\frac{\delta L}{(\alpha+1)}T^2 + C(2n+\alpha)T^{-2\alpha} - 2MT^{-2}\right\}T^{-2} + L, \tag{20}$$

$$\lambda = \frac{1}{k}\rho, \tag{21}$$

$$\rho_p = \rho - \lambda = \left(1 - \frac{1}{k}\right)\rho, \tag{22}$$

$$\zeta = \frac{L}{(n+2)}\left[\frac{\beta}{\alpha T^2} + \frac{\gamma\, MT^{-4}}{(\alpha-1)} + \frac{\delta\, L}{(\alpha+1)} + CT^{-2\alpha-2}\right]^{-\frac{1}{2}}\quad, \tag{23}$$

$$\Lambda = \left[1 + \frac{\beta}{\alpha} + \frac{2\gamma\, MT^{-2}}{(\alpha-1)} + (\alpha+1)CT^{-2\alpha}\right]T^{-2} + MT^{-4} - L, \tag{24}$$

The spatial volume V of universe is given by

$$V = T^{n+2}, \tag{25}$$

The spatial volume $V \to 0$ when $T \to 0$, and $V \to \infty$ when $T \to \infty$.

The physical quantities expansion scalar θ and shear scalar σ are given by

$$\theta = u^i_{;i} = (n+2)\left[\frac{\beta}{\alpha T^2} + \frac{\gamma\, MT^{-4}}{(\alpha-1)} + \frac{\delta\, L}{(\alpha+1)} + CT^{-2\alpha-2}\right]^{\frac{1}{2}}, \tag{26}$$

$$\sigma^2 = \frac{1}{2}\sigma_{i\,j}\sigma^{i\,j} = \frac{2(n-1)}{3}\left[\frac{\beta}{\alpha T^2} + \frac{\gamma\, MT^{-4}}{(\alpha-1)} + \frac{\delta\, L}{(\alpha+1)} + CT^{-2\alpha-2}\right], \tag{27}$$

Hence

$$\lim it_{T\to\infty}\left(\frac{\sigma^2}{\theta^2}\right) = \frac{4(n-1)^2}{3(n+2)^2} = cons\tan t. \tag{28}$$

Thus the model does not approach isotropy for large values of T.

The Hubble parameter H and the deceleration parameter q are given by

$$H = \frac{(n+2)}{3}\left[\frac{\beta}{\alpha T^2} + \frac{\gamma\, MT^{-4}}{(\alpha-1)} + \frac{\delta\, L}{(\alpha+1)} + CT^{-2\alpha-2}\right]^{\frac{1}{2}}, \tag{29}$$

$$q = -1 + \frac{3}{(n+2)}\frac{\left\{\frac{\beta}{\alpha T^3} + \frac{2\gamma\, MT^{-5}}{(\alpha-1)} + C(\alpha+1)T^{-2\alpha-3}\right\}}{\left\{\frac{\beta}{\alpha T^2} + \frac{\gamma\, MT^{-4}}{(\alpha-1)} + \frac{\delta\, L}{(\alpha+1)} + CT^{-2\alpha-2}\right\}^{\frac{3}{2}}}. \tag{30}$$

3.2. Discussion

The model (19) represents the string magnetized Kantowski–Sachs universe with bulk viscosity. From Eq. (20), we observe that the energy condition $\rho \geq 0$ given by Hawking and Ellis [61] leads to

$$\left\{\frac{2n\beta}{\alpha} + \left(\frac{2n\gamma+\gamma}{\alpha-1} - 2\right)\frac{M}{T^2} + C(2n+\alpha)\frac{1}{T^{2\alpha}}\right\}\frac{1}{T^2} \geq \left\{\frac{(1-2n)\delta}{(\alpha+1)} - 1\right\}L. \tag{31}$$

The expansion scalar (θ) is infinite at $T = 0$ and $\theta \to \frac{\delta L}{(\alpha+1)}$ when $T \to \infty$, provided $\alpha + 1 > 0$. The energy density $\rho \to \infty$ at $T = 0$ and $\rho = L\left\{1 + \frac{\delta(2n-1)}{\alpha+1}\right\}$ when $T \to \infty$ provided $\alpha + 1 > 0$. The spatial volume

$V \to 0$ when $T \to 0$ and $V \to \infty$ when $T \to \infty$. Since $\lim it_{T\to\infty}\left(\frac{\sigma}{\theta}\right) = cons\tan t$, the model does not approach isotropy for the whole range of time. Further, when $1 < k < 2$, we have $\frac{\rho_p}{|\lambda|} < 1$, and in this case the strings dominate over the particles. However, when $k > 2$ or $k < 0$, we have $\frac{\rho_p}{|\lambda|} > 1$; therefore, the massive string dominates the universe in the process of evolution. Figure 1 shows that the cosmological term Λ is a decreasing function of time and this approaches a small value as time increases (i.e. in the present epoch). Recent cosmological observations [1,3,4,6,62–64] suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda\left(G\hbar/c^3 = 10^{-123}\right)$. These observations on magnitude red-shift of the type 1a supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Moreover, the recent observations [1–6,65,66] reveal that the value of deceleration parameter (q) is confined in the range $-1 \leq q < 0$ and the present day universe is undergoing an accelerated expansion. Figure 2 shows that the value of q lies in the range $-1 \leq q < 0$, which is consistent with recent observations.

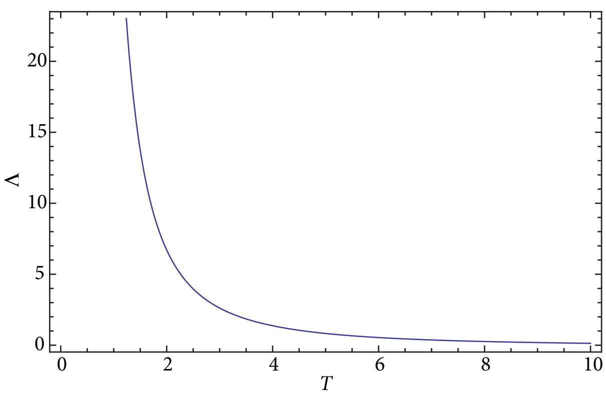

Figure 1. The variation of Λ vs. T for the model (19) with parameters $\alpha = 21$, $\beta = 50$, $L = 0.09$, $M = 17$ and $\gamma = 12$.

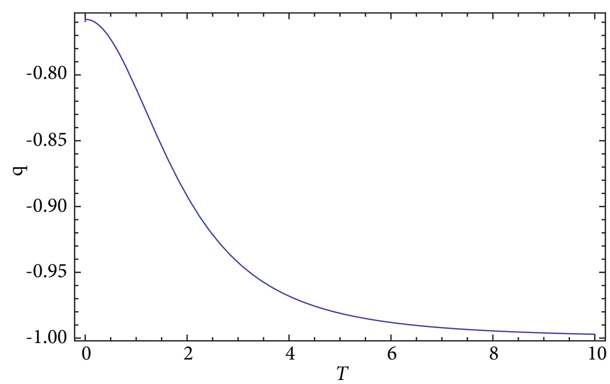

Figure 2. The variation of q vs. T for the model (19) with parameters $\alpha = 0.01$, $\beta = 0.02$, $\gamma = 2$, $\delta = 4$, $n = 0.7$, $L = 1$ and $M = 0.06$.

3.3. Bulk viscous model in absence of magnetic field

In absence of a magnetic field ($M = 0$), we obtain a string cosmological model with bulk viscosity and in this case the model (19) reduces to the form

$$ds^2 = -\left[\frac{\beta}{\alpha} + \frac{\delta\, LT^2}{(\alpha+1)} + CT^{-2\alpha}\right]^{-1} dT^2 + T^{2n}dr^2 + T^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{32}$$

The expressions for the energy density ρ, the string tension density λ, the particle energy density ρ_p, the coefficient of bulk viscosity ζ, and the cosmological constant Λ are given by

$$\rho = \left\{2n\frac{\beta}{\alpha} + (2n-1)\frac{\delta L}{(\alpha+1)}T^2 + C(2n+\alpha)T^{-2\alpha}\right\}T^{-2} + L, \tag{33}$$

$$\lambda = \frac{1}{k}\rho, \tag{34}$$

$$\rho_p = \rho - \lambda = \left(1 - \frac{1}{k}\right)\rho, \tag{35}$$

$$\zeta = \frac{L}{(n+2)}\left[\frac{\beta}{\alpha T^2} + \frac{\delta L}{(\alpha+1)} + CT^{-2\alpha-2}\right]^{-\frac{1}{2}}, \tag{36}$$

$$\Lambda = \left[1 + \frac{\beta}{\alpha} + (\alpha+1)CT^{-2\alpha}\right]T^{-2} - L, \tag{37}$$

The physical quantities expansion scalar θ and shear scalar σ are given by

$$\theta = u^i_{;i} = (n+2)\left[\frac{\beta}{\alpha T^2} + \frac{\delta L}{(\alpha+1)} + CT^{-2\alpha-2}\right]^{\frac{1}{2}}, \tag{38}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{2(n-1)}{3}\left[\frac{\beta}{\alpha T^2} + \frac{\delta L}{(\alpha+1)} + CT^{-2\alpha-2}\right], \tag{39}$$

Hence,

$$\lim it_{T\to\infty}\left(\frac{\sigma^2}{\theta^2}\right) = \frac{4(n-1)^2}{3(n+2)^2} = cons\tan t. \tag{40}$$

Thus the model does not approach isotropy for large values of T.

The Hubble parameter H and the deceleration parameter q are given by

$$H = \frac{(n+2)}{3}\left[\frac{\beta}{\alpha T^2} + \frac{\delta L}{(\alpha+1)} + CT^{-2\alpha-2}\right]^{\frac{1}{2}}, \tag{41}$$

$$q = -1 + \frac{3}{(n+2)}\frac{\left\{\frac{\beta}{\alpha T^3} + C(\alpha+1)T^{-2\alpha-3}\right\}}{\left\{\frac{\beta}{\alpha T^2} + \frac{\delta L}{(\alpha+1)} + CT^{-2\alpha-2}\right\}^{\frac{3}{2}}}. \tag{42}$$

From (37), we observe that the cosmological constant is a decreasing function of time and it approaches a small and positive value for large T (i.e. the present epoch) as shown in Figure 3, which is supported by the results from recent type 1a supernova observations [1,3,4,6,62–64]. Figure 4 shows the value of q confined anywhere in the range $-1 \leq q < 0$, which is consistent with recent observations [1–6,65,66] and the negative value indicates that our model (32) is accelerating.

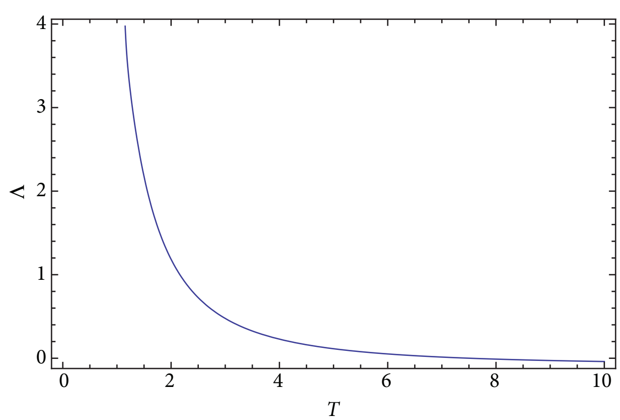

Figure 3. The variation of Λ vs. T for the model (32) with parameters $\alpha = 21$, $\beta = 50$, $L = 0.09$, and $\gamma = 12$.

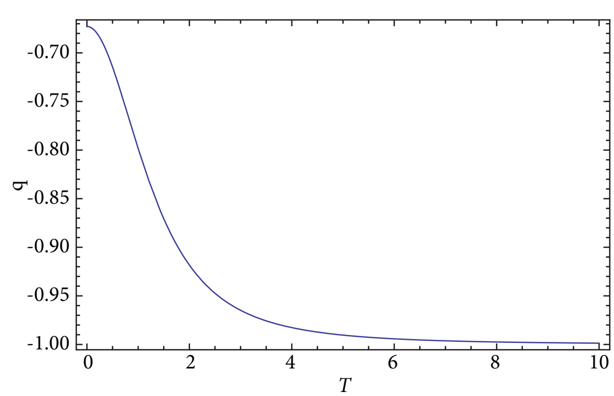

Figure 4. The variation of q vs. T for the model (32) with parameters $\alpha = 0.01$, $\beta = 0.02$, $\delta = 4$, $n = 0.7$, and $L = 2$.

3.4. Discussion

For the model (32), the energy condition $\rho \geq 0$ given by Hawking and Ellis [61] leads to

$$\left\{\frac{2n\beta}{\alpha} + C(2n+\alpha)\frac{1}{T^{2\alpha}}\right\}\frac{1}{T^2} \geq \left\{\frac{(1-2n)\delta}{(\alpha+1)} - 1\right\}L. \tag{43}$$

Eqs. (34) and (35) show that when $k \geq 1$ the particle density $\rho_p \geq 0$ and string tension density $\lambda \geq 0$; however, $\rho_p > 0$ and $\lambda < 0$, when $k < 0$. Since $\lim it_{T\to\infty}\left(\frac{\sigma}{\theta}\right)$ = constant the model does not approach isotropy for large values of T. The energy density $\rho \to \infty$ at $T = 0$ and $\rho = L\left\{1 + \frac{\delta(2n-1)}{\alpha+1}\right\}$ when $T \to \infty$, provided $\alpha + 1 > 0$. Further, when $1 < k < 2$, we observe $\frac{\rho_p}{|\lambda|} < 1$, and in this case the strings dominate over the particles. However, when $k > 2$ or $k < 0$, we have $\frac{\rho_p}{|\lambda|} > 1$; therefore, the massive string dominates the universe in the process of evolution.

From Eq. (36), we have seen that the cosmological term Λ, being very large at initial times, relaxes to a genuine cosmological constant at the late times, which is in accordance with the observations [1,3,4,6,62–64]. The recent observations of SN 1a show that the present universe is accelerating and the value of the deceleration parameter lies somewhere in the range $-1 \leq q < 0$.

4. Conclusion

In this paper we have studied Kantowski–Sachs bulk viscous string cosmological model in the presence and absence of an electromagnetic field. To be able to obtain a more general model we assume that the coefficient of bulk viscosity is inversely proportional to the expansion, i.e. $\zeta\theta =$constant, and expansion is proportional to shear, i.e. $\theta \infty \sigma$. The presence of a magnetic field affects energy density (ρ), string particle density (ρ_p), tension density (λ), coefficient of bulk viscosity (ζ), cosmological term (Λ), and expansion as well as acceleration of the universe. The bulk viscosity plays a greater role in the evolution of the properties of the models. The presence of viscosity helps us to get a universe with accelerated expansion. Hence the presence of bulk viscosity should be taken into account in the description of the time evolution of the properties of the universe. In all the deterministic models, $\frac{\sigma}{\theta} = cons\tan t$ implies that the models do not approach isotropy. In the presence and absence of a magnetic field, the cosmological term Λ in the models decreases as a function of time and approaches a small value at late time. The values of cosmological "constant" for the models are found to be small and positive, which is supported by the results from supernova observations recently obtained by the High-Z Supernova 1a Team and Supernova Cosmological Project [1–4,6,62–64].

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References

[1] Perlmutter, S.; Gabi, S.; Goldhaber, G.; Goobar, A.; Groom, D. E.; Hook, I. M.; Kim, A. G.; Kim, M. Y.; Lee, J. C.; Pain, R.; et al. *Astrophys. J.* **1997**, *483*, 565–581.

[2] Perlmutter, S.; Aldering, G.; Della Valle, M.; Deustua, S.; Ellis, R. S.; Fabbro, S.; Fruchter, A.; Goldhaber, G.; Groom, D. E.; Hook, I. M.; et al. *Nature* **1998**, *391*, 51–54.

[3] Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R. A.; Nugent, P.; Castro, P. G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D. E.; et al. *Astrophys. J.* **1999**, *517*, 565–585.

[4] Riess, A. G.; Filippenko, A. V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P. M.; Gilliland, R. L.; Hogan, C. J.; Jha, S.; Kirshner, R. P.; et al. *Astron. J.* **1998**, *116*, 1009–1038.

[5] Tonry, J. L.; Schmidt, B. P.; Barris, B.; Candia, P.; Challis, P.; Clocchiatti, A.; Coil, A. L.; Filippenko, A. V.; Garnavich, P.; Hogan, C. J.; et al. *Astrophys. J.* **2003**, *594*, 1–24.

[6] Riess, A. G.; Strolger, L. G.; Tonry, J.; Casertano, S.; Ferguson, H. C.; Mobasher, B.; Challis, P.; Filippenko, A. V.; Jha, S.; Li, W.; et al. *Astron. J.* **2004**, *607*, 665–687.

[7] Clocchiatti, A.; Schmidt, B. P.; Filippenko, A. V.; Challis, P.; Coil, A. L.; Covarrubias, R.; Diercks, A.; Garnavich, P.; Germany, L.; Gilliland, R.; et al. *Astrophys. J.* **2006**, *642*, 1–21.

[8] Bennett, C. L.; Halpern, M.; Hinshaw, G.; Jarosik, N.; Kogut, A.; Limon, M.; Meyer, S. S.; Page, L.; Spergel, D. N.; Tucker, G. S.; et al. *Astrophys. J. Suppl.* **2003**, *148*, 1–27.

[9] de Bernardis, P.; Ade, P. A. R.; Bock, J. J.; Bond, J. R.; Borrill, J.; Boscaleri, A.; Coble, K.; Crill, B. P.; De Gasperis, G.; Farese, P. C.; et al. *Nature* **2000**, *404*, 955–959.

[10] Hanany, S.; Ade, P.; Balbi, A.; Bock, J.; Borrill, J.; Boscaleri, A.; de Bernadis, P.; Ferreira, P. G.; Hristov, V. V.; Jaffe, A. H.; et al. *Astrophys. J.* **2000**, *545*, L5–L9.

[11] Tegmark, M.; Blanton, M. R.; Strauss, M. A.; Hoyle, F.; Schlegel, D.; Scoccimarro, R.; Vogeley, M. S.; Weinberg, D. H.; Zehavi, I.; Berlind, A.; et al. *Astrophys. J.* **2004**, *606*, 702–740.

[12] Tegmark, M.; Strauss, M. A.; Blanton, M. R.; Abazajian, K.; Dodelson, S.; Sandvik, H.; Wang, X.; Weinberg, D. H.; Zehavi, I.; Bahcall, N. A.; et al. *Phys. Rev. D* **2004**, *69*, 103501–103526.

[13] Spergel, D. N.; Verde, L.; Peiris, H. V.; Komatsu, E.; Nolta, M. R.; Bennett, C. L.; Halpern, M.; Hinshaw, G.; Jarosik, N.; Kogut, A; et al. *Astrophys. J. Suppl.* **2003**, *148*, 175–194.

[14] Ratra, B.; Peebles, P. J. E. *Phys. Rev. D* **1988**, *37*, 3406–3427.

[15] Caldwell, R. R. *Phys. Lett. B* **2002**, *545*, 23–29.

[16] Armendariz-Picon, C.; Mukhanov, V.; Steinhardt, P. J. *Phys. Rev. Lett.* **2000**, *85*, 4438–4441.

[17] Padmanabhan, T. *Phys. Rev. D* **2002**, *66*, 021301–021304.

[18] Everett, A. E. *Phys. Rev. D* **1981**, *24*, 858–868.

[19] Ozer, M.; Taha, O. *Phys. Lett. B* **1986**, *171*, 363-365.

[20] Chen, W.; Wu, Y. S. *Phys. Rev. D* **1990**, *41*, 695–698.

[21] Carvalho, J. C.; Lima, J. A. S.; Waga, I. *Phys. Rev. D* **1992**, *46*, 2404–2407.

[22] Arbab, A. I.; Abdel-Rahman, A. M.-M. *Phys. Rev. D* **1994**, *50*, 7725–7728.

[23] Kibble, T. W. B. *J. Phys. A. Math. Gen.* **1976**, *9*, 1387–1398.

[24] Kibble, T. W. B. *Phys. Rep.* **1980**, *67*, 183–199.

[25] Vilenkin, A. *Phys. Rep.* **1985**, *121*, 263–315.

[26] Zel'dovich, Ya. B. *Mon. Not. R. Astron. Soc.* **1980**, *192*, 663–667.

[27] Vilenkin, A. *Phys. Rev. D* **1981**, *24*, 2082–2089.

[28] Letelier, P. S. *Phys. Rev. D* **1979**, *20*, 1294–1302.

[29] Satchel, J. *Phys. Rev. D* **1980**, *21*, 2171–2181.

[30] Letelier, P. S. *Phys. Rev. D* **1983**, *28*, 2414–2419.

[31] Banerjee, A.; Sanyal, A. K.; Chakraborty, S. *Pramana J. Phys.* **1990**, *34*, 1–11.

[32] Krori, K. D.; Chaudhury, T.; Mahanta, C. R.; Mazumdar, A. *Gen. Relativ. Gravit.* **1990**, *22*, 123–130.

[33] Wang, X. X. *Chin. Phys. Lett.* **2003**, *20*, 615–617.

[34] Wang, X. X. *Chin. Phys. Lett.* **2003**, *20*, 1202–1205.

[35] Misner, C. W. *Nature* **1967**, *214*, 40–41.

[36] Weinberg, S. *Astrophys. J.* **1971**, *168*, 175–194.

[37] Murphy, G. L. *Phys. Rev. D* **1973**, *8*, 4231–4233.

[38] Beesham, A. *Phys. Rev., D* **1993**, *48*, 3539–3543.

[39] Krori, K. D.; Mukherjee, A. *Gen. Relativ. Gravit.* **2000**, *32*, 1429–1438.

[40] Asseo, E.; Sol, H. *Phys. Rep.* **1987**, *148*, 307–436.

[41] Madsen, M. S. *Mon. Not. R. Astron.* **1989**, *237*, 109–117.

[42] Melvin, M. A. *Ann. New York Acad. Sci.* **1975**, *262*, 253–274.

[43] Chakraborty, S. *Ind. J. Pure Appl. Phys.* **1991**, *29*, 31–33.

[44] Tikekar, R.; Patel, L. K. *Gen. Relativ. Gravit.* **1992**, *24*, 397–404.

[45] Singh, G. P.; Singh, T. *Gen. Relativ. Gravit.* **1999**, *31*, 371–378.

[46] Weber, E. *J. Math. Phys.* **1984**, *25*, 3279–3285.

[47] Weber, E. *J. Math. Phys.* **1985**, *26*, 1308–1310.

[48] Kantowski, R.; Sachs, R. K. *J. Math. Phys.* **1966**, *7*, 443–446.

[49] Lorenz, D. *J. Phys. A. Math. Gen.* **1983**, *16*, 575–584.

[50] Grøn, Ø. *J. Math. Phys.* **1986**, *27*, 1490–1493.

[51] Matravers, D. R. *Gen. Relativ. Gravit.* **1988**, *20*, 279–288.

[52] Krori, K. D.; Goswami, A. K.; Purkayastha, A. D. *J. Math. Phys.* **1995**, *36*, 1347–1352.

[53] Wang, X. X. *Astrophys. Space Sci.* **2005**, *298*, 433–440.

[54] Pradhan, A.; Yadav, V. K. *Int. J. Mod. Phys. D* **2002**, *11*, 893–912.

[55] Katore, S. D. *Bulg. J. Phys.* **2010**, *37*, 144–151.

[56] Takabayasi, T. *Quantum Mechanics, Determination, Causality and Particles*, Reidal: Dordrecht, the Netherlands, 1976, p. 179.

[57] Bali, R.; Pradhan, A. *Chin. Phys. Lett.* **2007**, *24*, 585–588.

[58] Saha, B. *Mod. Phys. Lett. A* **2005**, *20*, 2127–2143.

[59] Yadav, V. K.; Yadav, L. *Rom. J. Phys.* **2013**, *58*, 64–74.

[60] Tiwari, R. K.; Dwivedi, U. *FIZIKA (Zegreb)*, **2010**, *19*, 1–8.

[61] Hawking, S. W.; Ellis, G. F. R. *The Large Scale Structure of Spacetime*, Cambridge University Press, Cambridge, UK, 1973, p. 91.

[62] Garnavich, P. M.; Kirshner, R. P.; Challis, P.; Tonry, J.; Gilliland, R. L.; Smith, R. C.; Clocchiatti, A.; Diercks, A.; Filippenko, A. V.; Hamuy, M.; et al. *Astrophys. J.* **1998**, *493*, L53–L57.

[63] Garnavich, P. M.; Jha, S.; Challis, P.; Clocchiatti, A.; Diercks, A.; Filippenko, A. V.; Gilliland, R. L.; Hogan, C. J.; Kirshner, R. P.; Leibundgut, B.; et al. *Astrophys. J.* **1998**, *509*, 74–79.

[64] Schmidt, B. P.; Suntzeff, N. B.; Phillips, M. M.; Schommer, R. A.; Clocchiatti, A.; Kirshner, R. P.; Garnavich, P.; Challis, P.; Leibundgut, B.; Spyromilio, J.; et al. *Astrophys. J.* **1998**, *507*, 46–63.

[65] John, M. V. *Astrophys. J.* **2004**, *614*, 1–5.

[66] Knop, R. A.; Aldering, G.; Amanullah, R.; Astier, P.; Blanc, G.; Burns, M. S.; Conley, A.; Deustua, S. E.; Doi, M.; Ellis, R.; et al. *Astrophys. J.* **2003**, *598*, 102–137.